On a reserved territory approach for a resource management problem [1]

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Abstract. We consider the approach for a resource management problem with a reserved territory. The optimal values of parameters are obtained. The numerical modelling for various kinds of payoffs determining a share of prohibited area of reservoir's territory is carried out.

Keywords: resource management problem, numerical modelling.

1. Introduction

A dynamic game model of bioresource managment problem (fish catching) is considered. The centre (state), which determines a share of prohibited for catching (reserved) area of a reservoir, and players (fishing firms), which make fish catching are the participants of this game. Each player is independent decision maker, being guided by maximization of the profit of fish sale. In traditional statement (see Clark, 1985, Ehtamo and Hamalainen, 1993, Hamalainen, Kaitala and Haurie, 1984, Haurie and Tolwinski, 1984, Tolwinski, Haurie and Leitmann, 1986) center's task is catch regulation by introduction quotas on fishing. In the given work center's task - choice of optimal share of reserved territory for maintenance of stable population development in a reservoir in long-term prospect and possible fishing level's definition, sufficient for demand satisfaction.

2. Game model for one player

Let us consider the center which determines a share of reserved area of a reservoir, denoted by s, $0 \le s \le 1$. Share, on which fishing is allowed, is equal to $1-s$ accordingly. We consider the player's strategy which exploits the fish stock during T time periods. The dynamics of the fishery is described by the equation:

$$x'(t) = F(x(t)) - qE(t)(1-s(t))x(t),\ 0 \le t \le T,\ x(0) = x_0\,, \tag{1}$$

where $x(t) \ge 0$– size of the population at time t; F–natural growth function of the population; $E(t) \ge 0$– firm's fishing efforts measured by the number of vessels involved in fishing at time t, $s(t)$–share of prohibited for fishing (reserved) area of a reservoir and $q > 0$–catch-ability coefficient related to the unit fishing effort of the firm.

Assume, that population evolves in accordance with Ferhulst model of the form:

$$F(x) = rx(1 - x/K)\,,$$

where r–the intrinsic growth rate, and K–maximal natural object capacity.

Then player's profit is of the form:

$$\begin{aligned} J &= g(x(T)) + \\ &+ \int_0^T e^{-\rho t}[\Pi(q, s(t), x(t), E(t)) \cdot qE(t)(1-s(t))x(t) - c^0 E(t)]dt\,, \end{aligned}$$

where ρ–discount rate, c^0–catching costs for one vessel and Π–price function, specified as

$$\Pi(q, s(t), x(t), E(t)) = p - kqE(t)(1-s(t))x(t),\ p, k > 0\,.$$

Function $g(x)$ describes the salvage value of the stock at time T and

$$g'(x) \ge 0,\ g''(x) \le 0\,.$$

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We are interested in the optimal solution of the following problem:

$$\begin{cases} \max(J(E(t)))\,, \\ \text{where } x(t) \text{ is define from } (1)\,. \end{cases} \tag{2}$$

Theorem 1. *Let $E^*(t)$, $x^*(t)$, $\lambda(t)$ satisfy the equation (1) and next equalities are fulfilled*

$$E^*(t) = \frac{(b - q\lambda(t))(1 - s(t))x^*(t) - c}{a(1 - s(t))^2 x^*(t)^2}\,,\ 0 \le t \le T\,,$$

$$\begin{aligned} \lambda'(t) &= -E^*(t)(1 - s(t))(b - aE^*(t)(1 - s(t))x^*(t)) - \\ &- \lambda(t)(F'(x^*(t)) - qE^*(t)(1 - s(t)))\,,\ 1 \le t \le T\,, \end{aligned}$$

$$\lambda(T) = g'_x(x^*(T))\,.$$

Then $E^*(t)$ – solution of the problem (2).

Numerical modelling of the problem for various kinds of the function $s(t)$ is carried out and the optimal values of $E^*(t), x^*(t), \lambda(t)$ are obtained.

We examine the following functionals determining center's gain:

1.

$$I_1 = -\int_0^T (x(t) - \bar{x}(t))^2 dt\,,$$

where $\bar{x}(t)$–size of the population which is optimal for reproduction.

In this case I_1 is center's costs for population's reconstruction.

The rest of functionals types can be:

2.

$$I_2 = -\int_0^T (U(t) - \hat{x}(t))^2 dt\,,$$

3.

$$I_3 = -\int_0^T |U(t) - \hat{x}(t)| \cdot \alpha \cdot \Pi(q, s(t), x(t), E(t)) dt\,,$$

where $U(t) = qE(t)(1 - s(t))x(t)$–player's catch at time t, $\hat{x}(t)$–consumer level determinated by demand, $\Pi(q, s(t), x(t), E(t)) = p - kqE(t)(1 - s(t))x(t),\ p, k > 0\,.$

In these cases center's profit is considered as costs for demand satisfaction, expressed in case c) in monetary units, taking into account transport costs.

For various kinds of the function $s(t)$ the values of J, I_1, I_2 and I_3 are found. Among solutions determined by these payoffs, which compose the Pareto-optimal class, Nash equilibrium solution is obtained. Also Kalai-Smorodinsky equilibrium is defined (see Kalai and Smorodinsky, 1975).

3. Game model for two players

The case with two players (fishing firms) is considered. Here the dynamics of the fishery is described by the equation:

$$\begin{gathered} x'(t) = F(x(t)) - q_1E_1(t)(1 - s(t))x(t) - q_2E_2(t)(1 - s(t))x(t), \\ 0 \le t \le T,\ x(0) = x_0\,, \end{gathered} \tag{3}$$

where $x(t) \ge 0$– size of the population at time t; F–natural growth function of the population;$E_i(t) \ge 0$, $i = 1, 2$ – firm's fishing efforts measured by the number of vessels involved in fishing at time t, $s(t)$–share of prohibited for fishing (reserved) area of a reservoir and $q_i > 0$, $i = 1, 2$–catch-ability coefficient related to the unit fishing effort of the firm.

In this case the player's profit is of the form:

$$J_i = g_i(x(T)) + \\ + \int_0^T e^{-\rho_i t}[\Pi_i(q_i, s(t), x(t), E_i(t)) \cdot q_i E_i(t)(1-s(t))x(t) - c_i^0 E_i(t)]dt\,,$$

where ρ_i–discount rate, c_i^0–catching costs for one vessel and Π_i–price function, specified as

$$\Pi_i(q_i, s(t), x(t), E_i(t)) = p_i - k_i q_i E_i(t)(1-s(t))x(t),\ p_i, k_i > 0\,.$$

We are interested in the optimal solution of the following problem:

$$\begin{cases} \max(\mu_1 J_1(E_1, E_2) + \mu_2 J_2(E_1, E_2)) \\ \quad \text{where } x(t) \text{ is defined from (3)}\,, \end{cases} \tag{4}$$

for some $\mu_1, \mu_2 > 0,\ \mu_1 + \mu_2 = 1$

As in the case with one player optimal solution can be determine using next statement.

Theorem 2. *Let $E_1^*(t)$, $E_2^*(t)$, $x^*(t)$, $\lambda(t)$ satisfy the equation (3) and next equalities are fulfilled*

$$E_i^*(t) = \frac{(b_i - \mu_i^{-1} q_i \lambda(t))(1-s(t))x^*(t) - c_i}{a_i(1-s(t))^2 x^*(t)^2}\,,\ 0 \le t \le T\,,$$

$$\begin{aligned} \lambda'(t) &= -\sum_{i=1}^{2} \mu_i E_i^*(t)(1-s(t))(b_i - a_i E_i^*(t)(1-s(t))x^*(t)) - \\ &- \lambda(t)(F'(x^*(t)) - \sum_{i=1}^{2} q_i E_i^*(t)(1-s(t)))\,,\ 1 \le t \le T\,, \end{aligned}$$

$$\lambda(T) = \sum_{i=1}^{2} \mu_i g'_{ix}(x^*(T))\,.$$

Then $E_1^*(t), E_2^*(t)$– solution of the problem (4).

Numerical modelling of the problem for the various kinds of function $s(t)$ is carried out and the optimal values of $E_1^*(t), E_2^*(t), x^*(t), \lambda(t)$ are obtained.

For various kinds of the function $s(t)$ the optimal values of the similar to the case with one player functionals J_1,J_2, $\bar{I}_1$, $\bar{I}_2$ and $\bar{I}_3$ are received. Among solutions determined by these payoffs, which compose Pareto-optimal class, Nash equilibrium solution is obtained. Also Kalai-Smorodinsky equilibrium is defined.

4. Modelling of the general case

In the general case $s(t)$ can be an arbitrary function. The modelling of the variant in which the center can change the share of prohibited for catching area of a reservoir one or more times at an arbitrary time moment is conducted.

Let us denote the moment, when the center can change it's policy – t^*, a share of reserved area of a reservoir on the interval $[0, t^*)$ – $s_1(t)$, and on $[t^*, T]$ – $s_2(t)$. As an example you can see the modelling of this case with an arbitrary uniformly distributed functions t^*, $s_1(t)$, $s_2(t)$ on Fig.1.

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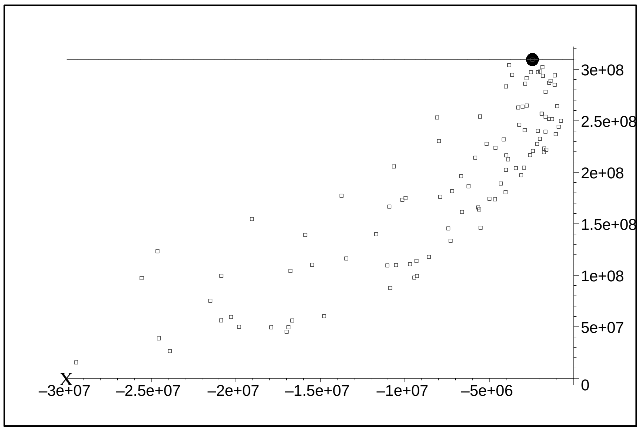

Fig. 1. Values of I_3 and J

$x(0) = 150000$ $\hat{x} = 2500$

Nash equilibrium achieve at $t^* = 95$
$s_1(t) = 0$ $s_2(t) = 0.7$
$I_3 = -2444747.669$ $J = 309403975$

On the plot the point of Nash equilibrium is denoted by full circle and the point of maximin payoffs – by X.