Some recent algorithms to compute the nucleolus of structured cooperative games

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Abstract

Nucleolus is one of the fundamental solution concepts in cooperative game theory. There has been considerable progress in locating the nucleolus in recent years. These algorithms work efficiently for certain structured classes of cooperative games. Though the data of a cooperative game grows exponentially in size with the number of players, assignment games, and balanced connected games, grow only polynomially in size, on the number of players. The algorithm for locating the nucleolus for assignment games is based on an efficient graph theoretic algorithm which counts the longest paths to each vertex and trimming of cycles to quickly arrive at the lexicographic geometric center. Connected games are solved by the technique of feasible direction, motivated by the algorithm for the assignment games. The sellers market corner of the core for assignment games has its counterpart, in balanced connected games at the lexmin vertex.

Nucleolus has also been characterized via a set of axioms based on subgame consistency. This is exploited for standard tree games to arrive at an efficient and intuitively explainable algorithm.

Assignment Games with Stable Core

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Abstract

We prove that the core of an assignment game (Shapley and Shubik, 1972) is stable (i.e., it is the unique von Neumann-Morgenstern solution) if and only if there is a matching between the two types of players such that the corresponding entries in the underlying matrix are all row and column maximums.

We identify other easily verifiable matrix properties and show their equivalence to various known sufficient conditions for core-stability.

By these matrix characterizations we found that on the class of assignment games, largeness of the core, extendability and exactness of the game are all equivalent conditions, and strictly imply the stability of the core.

In turn, convexity and subconvexity are equivalent, and strictly imply all aformentioned conditions.

On the nucleolus of permutation games

Abstract

Permutation games are totally balanced transferable utility cooperative games arising from certain sequencing and assignment optimization problems.

We prove that for permutation games the Davis-Maschler bargaining set coincides with the core.

Our second result is that the kernel is a (possibly non-convex) subset of the least core.

Special attention is paid to cyclic permutation games, an important subclass which contains the well-known class of assignment games.

We show how the nucleolus of an n-player cyclic permutation game can be computed by a known $\mathcal{O}(n^4)$ algorithm directly from the underlying data.