Numerical Studies on Braess-Like Paradoxes for Non-Cooperative Load Balancing in Distributed Computer Systems

Said Fathy El-Zoghdy,* Hisao Kameda,† and Jie Li‡

Abstract

Distributed computer systems consists of nodes (hosts, computers) and a communication means that connects nodes. Jobs arrive at each node and can be forwarded through the communication means to the other nodes for remote processing. Numerical examples of a Braess-like paradox in which adding capacity to a distributed computer system may degrade the performance of all users in the system have been reported. Unlike the original Braess paradox, in the models examined, this behavior occurs only in the case of finitely many users and not in the case of infinite number of users, and the degree of performance degradation can increase without bound. This study examines numerically, some examples around the Braess-like paradox. From the numerical examples, it is observed that, in the class optimum, the worst ratio of performance degradation in the paradox is largest (*i.e.,* the worst performance is obtained) in the complete symmetry case with the arrival rate is closest to the processing rate. And, as the system parameter setting gradually departs the above-mentioned symmetric case without keeping any kind of symmetries, the worst ratio of performance degradation decreases rapidly. It decreases slowly (more slowly) if the system parameter setting departs the complete symmetry while keeping the individual (overall) symmetry property. Indeed, it is also observed that in complete symmetry, if the communication means of type (C) is used, the worst ratio of performance degradation may increase without bound as the arrival rate gets very close to the processing rate.

keywords Braess paradox, distributed decision, distributed computer systems, load balancing, Nash equilibrium, non-cooperative game, performance optimization, Wardrop equilibrium.

1 Introduction

In many systems including communication networks in distributed computer systems, transportation flow networks, *etc.* we have several distinct objectives for performance optimization. Among them, we have three typical optima corresponding to three typical decision schemes:

(1) [Completely centralized decision scheme]: All jobs are regarded to belong to one group that has only one decision maker. The decision maker seeks to optimize a single performance measure such as the total cost over all jobs (*e.g.,* the mean response time of the entire system). In the literature, the corresponding solution concept is referred to as a system optimum, overall optimum, cooperative optimum or social optimum. In this paper, we shall refer to it as the *overall optimum*.

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(2) [Completely distributed decision scheme]: Each of infinitely many jobs (or the user of each) optimizes its own cost (*e.g.*, its own expected response time), independently of the others. In this optimized situation, each job cannot expect any further benefit by changing its own decision. It is also assumed that the decision of a single job has a negligible impact on the performance of the entire system. In the literature, the corresponding solution concept is referred to as an individual optimum, Wardrop equilibrium, or user optimum. In this paper, we shall refer to it as the *individual optimum*.

(3) [Intermediately distributed decision scheme]: Infinitely many jobs are classified into a finite number ($N(>1)$) of classes or groups, each of which has its own decision maker and is regarded as one player or user. Each decision maker optimizes non-cooperatively its own cost (*e.g.*, the expected response time) over only the jobs of its own class. The decision of a single decision maker of a class has a non-negligible impact on the performance of other classes. In this optimized situation, each of a finite number of classes or players cannot receive any further benefit by changing its decision. In the literature, the corresponding solution concept is referred to as a class optimum, Nash equilibrium, or user optimum. In this paper, we shall refer to it as the *class optimum*.

Note that (3) is reduced to (1) when the number of players reduces to 1 ($N = 1$) and approaches (2) when the number of players becomes infinitely many ($N \to \infty$) [1].

Intuitively, we can think that the total processing capacity of a system will increase when the capacity of a part of the system increases, and so we expect improvements in performance objectives accordingly in that case. The famous Braess paradox tells us that this is not always the case; *i.e.*, adding capacity to the system may sometimes lead to the degradation in the benefits of all users in an individual optimum [3, 11, 10, 4, 9].

The Braess paradox has attracted the attention of researchers in the field of software multi-agent systems (see [2] for example) and in the theory of computing (see [19] for example).

We can expect that, in a class optimum (*i.e.*, Nash equilibrium) a similar type of paradox may occur (with large N), whenever it occurs for the individual optimum ($N \to \infty$). Indeed in [14], Korilis found examples wherein the Braess-like paradox appears in a class optimum where all user classes are identical in the same topology for which the original Braess paradox (for the individual optimum) was in fact obtained. Furthermore in [15], he also obtained a sufficient condition under which the Braess paradox should not occur in a more general model that has one source-destination pair and identical user classes.

As it is known that the class optimum converges to the individual optimum as the number of classes becomes large [1], it is natural to expect the same type of paradox in the class optimum context (for a large number of classes), whenever it occurs for the individual optimum, although it never occurs in the overall optimum where the total cost is minimized.

In [12], Kameda have obtained, however, numerical examples where a paradox similar to Braess's appears in the class optimum but does not occur in the individual optimum in the same environment. These cases look quite strange if we note that such a paradox should never occur in the overall optimum and if we regard the class optimum as an intermediate between the overall optimum and the individual optimum.

The methods and algorithms for obtaining the optima and the equilibria are described in [7, 5, 13, 8, 6]. Some related results on class optima are given in [16].

In [22], it has been shown that in the Braess network and in extended Braess networks [9, 10], the ratio of the performance degradation is bounded and less than 2. Also, in [21], it has been shown that worst ratio of performance degradation may increase without bound in class optimization where the values of parameters of all classes are identical and that behavior does not occur for the overall and individual optima, in the same setting of the system parameters (To the best of our knowledge, [21] is the first paper that reported such a case where the worst ratio of performance degradation can increase without bound.) After we read that paper, some questions arises like under what conditions in class optimization we have this strange behavior?, If we slightly change the system parameters setting to represent asymmetric system model, what will happen to that strange behavior? It will increase (decrease)? If it is increases (decreases), this increasing (decreasing) will be rapidly or slowly? and finally, What will be the overall tendency of the worst ratio of performance degradation? In this paper, through a number of numerical examples for the Braess-like paradox wherein adding a resource to a system leads to the performance degradation to all users in the class optimum for load balancing policy, we found that in the class optimum, the worst ratio of performance degradation in the paradox is largest (*i.e.*, the worst performance is obtained) in the complete symmetry case with the arrival rate is closest to the processing rate. And, as the system parameter setting gradually departs the above-mentioned symmetric case without keeping any kind of symmetries, the worst ration of performance degradation decreases rapidly. It decreases slowly (more slowly) if the system parameter setting departs the complete symmetry while keeping the individual (overall) symmetry property. Indeed, it is also observed that in complete symmetry, if the communication means of type (C) is used, the worst ratio of performance degradation may increase without bound as the arrival rate gets very close to the processing rate.

This paper is organized as follows. Section 2 presents the description and the assumptions of the model studied in this paper. Section 3 describes the results of numerical examination. Section 4 summarizes this paper.

2 Model Description and Assumptions

We consider a distributed computer system model that consists of two nodes (host computers or processors) connected with a communication means as shown in figure 1. Nodes are numbered 1 and 2. Each node consists of a single exponential server with service rate $\mu_i (i = 1, 2)$. We classify jobs arriving at node i into class i, $i = 1, 2$. Node i has external Poisson arrival with rate ϕ_i, out of which the rate x_{ii} of jobs are proceeded at node i. The rate x_{ij} of jobs is forwarded upon arrival through the communication means to another node j $(\neq i)$ to be processed there, and the results of processing those jobs are returned back through the communication means to node i. Then it follows that $x_{ii} + x_{ij} = \phi_i (i \neq j)$, $x_{ij} \geq 0$, $i, j = 1, 2$. We denote the vector $(x_{11}, x_{12}, x_{21}, x_{22})$ by $\mathbf{x}$. We denote the set of $\mathbf{x}$'s that satisfy the constrains by $\mathbf{C}$ and let $\Phi = \phi_1 + \phi_2$. Each node has one decision maker, also numbered i, $i = 1, 2$. Within these constrains, a set of values of x_{ij},$(i, j = 1, 2)$ are chosen to achieve the optimization. The load on node i is $x_{ii} + x_{ji} (i \neq j)$ and is denoted by β_i. The expected processing $(including\ queueing)$ time of a job that is proceeded at node i, is $1/(\mu_i - \beta_i)$ for $\beta_i < \mu_i$(otherwise it is infinite) and denoted by $D_i(\beta_i)$.

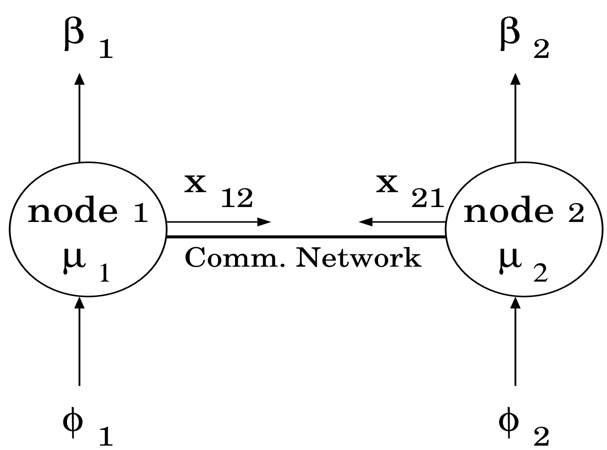

Figure 1: A distributed Computer System

The expected communication $(including\ queueing)$ time of forwarding a job arriving at node i to node j $(i \neq j)$, is denoted by $G_{ij}(\mathbf{x})$. Thus the expected response time of a job that arrives at node i is

$$T_i(\mathbf{x}) = \frac{1}{\phi_i} \sum_k x_{ik} T_{ik}(\mathbf{x}), \tag{1}$$

where

$$T_{ii}(\mathbf{x}) = D_i(\beta_i), \tag{2}$$
$$T_{ij}(\mathbf{x}) = D_j(\beta_j) + G_{ij}(\mathbf{x}), \text{ for } j \neq i. \tag{3}$$

Then the overall expected response time is

$$T(\mathbf{x}) = \frac{1}{\Phi} \sum_i \phi_i T_i(\mathbf{x}) \tag{4}$$

Also, We have the following assumptions:

Assumption 1 *We assume that the expected processing* $(including\ queueing)$ *time of a job that is processed at node* i *(*$D_i(\beta_i)$*), is a strictly increasing, convex and continuously differentiable function of* β_i*.,*

Assumption 2 We assume that the mean communication delay $(including$ queueing $delay)$ for forwarding jobs arriving at node i to node j $(G_{ij}(\mathbf{x}))$, $(i \neq j)$, is a positive, nondecreasing, convex and continuously differentiable function of $\mathbf{x}$, and that $G_{ii}(\mathbf{x}) = 0$. We assume further that each job is forwarded at most once.

Remark 2.1 *Note that as a consequence of Assumptions* 1 *and* 2*, the functions* $T(\cdot)$, $T_i(\cdot)$ *and* $T_{ij}(\cdot)$ *are convex and differentiable with respect to* $\mathbf{x}$*.*

As to the communication means, we consider the following three types (A), (B), and (C).

(A) It consists of 2 two-way communication lines (one dedicated line for each node in the system). The two-way communication line i is used for forwarding of jobs that arrive at node i (and for sending back the processed results of these jobs).

We assume that the expected time length of forwarding and sending back a job is to be

$$G_{ij}(\mathbf{x}) = \frac{t}{1 - x_{ij}t}, \tag{5}$$

if $x_{ij}t < 1$, and otherwise infinite.

That is, we assume that each communication channel is modelled by a processor sharing server with service rate $1/t$; *i.e.*, the mean communication (without queueing) time is t, and thus, the capacity of each communication line is $1/t$.

(B) It consists of a single-channel communication line that is used commonly in forwarding and sending back of jobs that arrive at all the nodes of the system.

The assumption on the line is the same as (A) except that there is only one line which is used for jobs arriving at all the nodes of the system. Thus the expected communication (with queueing) time of a job arriving at node i and being processed at node j $(\neq i)$ is expressed as

$$G_{ij}(\mathbf{x}) = \frac{t}{1 - \lambda t}, \tag{6}$$

if $\lambda t < 1$, and otherwise infinite, where $\lambda = x_{12} + x_{21}$ is the communication traffic through the line.

(C) It consists of a single or multiple communication line that has no queueing delay. Thus the expected communication time of a job arriving at node i and being processed at node j $(\neq i)$ is expressed as

$$G_{ij}(\mathbf{x}) = t. \tag{7}$$

We refer to the length of time between the instant when a job arrives at a node and the instant when it leaves one of the nodes, after all processing and communication, if any, are over as *the response time* for the job.

We have three optima, the overall, the individual, and the class.

(1)The overall optimum is given by $\bar{\mathbf{x}}$ that satisfies the following,

$$T(\bar{\mathbf{x}}) = \min T(\mathbf{x}) \quad \text{such that} \quad \mathbf{x} \in \mathbf{C}. \tag{8}$$

(2)The individual optimum is given by $\hat{\mathbf{x}}$ that satisfies the following for all i,

$$T_i(\hat{\mathbf{x}}) = \min\{T_{ii}(\hat{\mathbf{x}}), T_{ij}(\hat{\mathbf{x}})\}(i \neq j) \quad \text{such that} \;\; \hat{\mathbf{x}} \in \mathbf{C}. \tag{9}$$

(3)The class optimum (or a Nash equilibrium) is given by $\tilde{\mathbf{x}}$ that satisfies the following for all i,

$$\tilde{T}_{ik} = T_{ik}(\tilde{\mathbf{x}}) = \min_{x_{ii}, x_{ij}} T_{ik}(\tilde{\mathbf{x}}_{-(ii,ij)}; x_{ii}, x_{ij}), \text{ such that } (\tilde{\mathbf{x}}_{-(ii,ij)}; x_{ii}, x_{ij}) \in \mathbf{C}. \tag{10}$$

where $(\tilde{\mathbf{x}}_{-(ii,ij)}; x_{ii}, x_{ij})$ denotes a 4-dimensional vector in which the elements corresponding to $\tilde{x}_{ii}$ and $\tilde{x}_{ij}$ have been replaced respectively, by x_{ii} and x_{ij}.

In [17, 20] it is shown that the three problems (8),(9), and (10) are equivalent to some variational inequalities. For the existence and uniqueness of those optima the reader is referred to [17, 18]. In [13], it have been shown that no mutual forwarding of jobs occurs in overall and individual optima. Consequently, no paradox occurs for overall and individual optima.

For each set of data ϕ_i and μ_i, $i = 1, 2$, we can find some value t^∞ (depending upon the set of data) of the mean communication time such that the communication line is not used any more at equilibria if the mean communication time is larger than t^∞. For the set of data ϕ_i and μ_i, $i = 1, 2$, we increase the communication time from 0 to t^∞. For each t we compute the class optimum (Nash equilibrium).

We focus our attention on the degradation that may occur when increasing the communication capacity. To this aim we say that a Braess-like paradox occurs if the following holds:

$$\begin{array}{ll} D_i(t_1, t_2) > 0 & \text{for all i} \\ \text{for some } \mathrm{t}_1, \mathrm{t}_2 & \text{such that } 0 < \mathrm{t}_1 < \mathrm{t}_2 \end{array} \tag{11}$$

Where $D_i(t_1, t_2) = \dfrac{\tilde{T}_i(t_1) - \tilde{T}_i(t_2)}{\tilde{T}_i(t_2)}$, and $\tilde{T}_i(t)$ denotes the mean response time for class i jobs, computed at the unique (Nash) equilibrium, when the mean communication time is t.

For simplicity, we only consider the case where $t_2 = t^\infty$, *i.e.*, the system has no communication means, and we denote $D_i(t, t^\infty)$ by $\Delta_i(t)$. Denote $\boldsymbol{\phi} = (\phi_1, \phi_2)$ and $\boldsymbol{\mu} = (\mu_1, \mu_2)$. Thus, we define the worst ratio of performance degradation in the paradox $\Gamma(\boldsymbol{\mu}, \boldsymbol{\phi})$ as follows:

$$\Gamma(\boldsymbol{\mu}, \boldsymbol{\phi}) = \max_t\{\min_i\{\Delta_i(t)\}\}. \tag{12}$$

Denote t_0 the mean communication time, such that the previous maximum is reached.

Let us define three symmetries with respect to the system parameter setting.

Overall symmetry

If the following condition holds

$$\frac{\mu_i}{(\mu_i - \phi_i)^2} = constant, for\ all\ i, \tag{13}$$

then according to [13, 8], there will be no forwarding of jobs among nodes for any value of the communication line capacity $1/t$, for the cases (A), (B), and (C) of the communication means, when the system is at the overall optimum. If condition 13 holds, in this case, we say that we have an overall symmetry property among nodes.

Individual symmetry

If the following condition holds

$$\frac{1}{(\mu_i - \phi_i)} = constant, for\ all\ i, \tag{14}$$

it can be proved from definition (9) that at the individual optimum there will be forwarding of jobs among nodes for any value of the communication line capacity $1/t$, for the cases (A), (B), and (C) of the communication means. If condition 14 holds, in this case, we say that we have an individual symmetry property among nodes.

Complete symmetry

If both conditions (13), and (14) hold or equivalently if μ_i=constant and ϕ_i=constant, for all i, then we say that we have a complete symmetry among nodes. In complete symmetry, according to [13], there will be no forwarding of jobs occurs both in the overall and individual optima.

3 Results and discussion

The system examined here consists of two nodes (servers) and a communication means of types (B), and (C). The same model but with communication means of type (A) showed almost similar results as that obtained with the communication means of type (B). We show typical numerical examples, with changing the system parameter values for each of the four directions: complete, overall, individual, and no symmetry maintained.

Remark 3.1 Generally, the worst ratio of performance degradation obtained when the communication means of type (C) is used is larger than that obtained when the communication means of type (B) is used.

3.1 Complete Symmetry Maintained

In the two figures 2, and 3, we show the effect of changing $\mu_1 = \mu_2 = \mu$ starting from 1.001, keeping $\phi_1 = \phi_2 = 1$ (*i.e.*, the complete symmetry property is maintained) on the worst ration of performance degradation where the communication means of types (B) and (C) are used respectively.

Generally, from the two figure 2, and 3, we can say that the worst ratio of performance degradation increases as the processing rate μ gets closest to the arrival rate ϕ and it decreases as the processing rate μ gets larger than the arrival rate ϕ. (note in complete symmetry $\phi_1 = \phi_2 = \phi$, and $\mu_1 = \mu_2 = \mu$).

Also, as shown in figure 3, by using the communication means of type (C), the worst ratio of performance degradation may increase without bound (in complete symmetry) as the processing rate μ gets very close to the arrival rate ϕ.

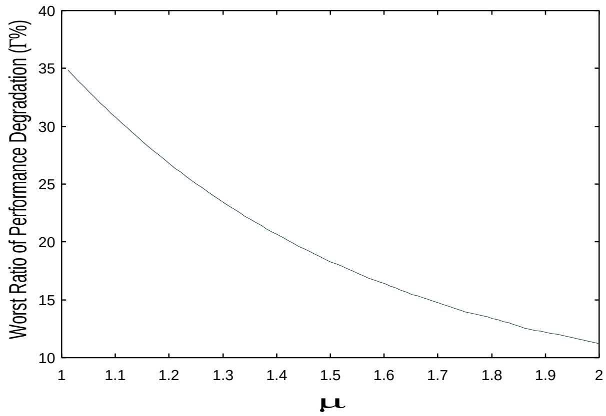

Figure 2: The worst ratio of performance degradation ($\Gamma(\%)$) given the values of $\mu_1 = \mu_2 = \mu$, and $\phi_1 = \phi_2 = 1$ (*i.e.*, the complete symmetry property is maintained) using the communication means of type (B).

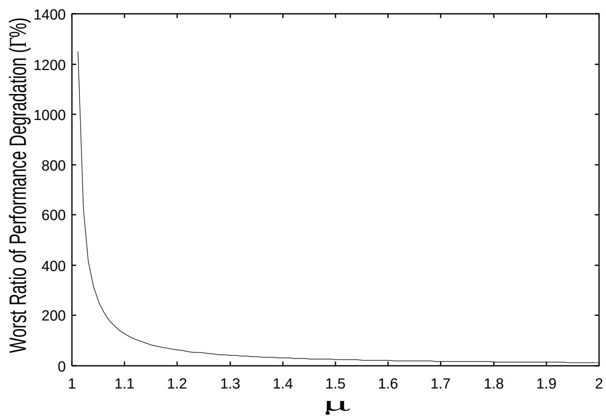

Figure 3: The worst ratio of performance degradation ($\Gamma(\%)$) given the values of $\mu_1 = \mu_2 = \mu$, and $\phi_1 = \phi_2 = 1$(*i.e.*, the complete symmetry property is maintained) using the communication means of type (C).

3.2 Overall Symmetry Maintained

The two figures 4, and 7 show how the worst ratio of performance degradation depends on the values of the system parameters μ_1, and μ_2. Given the values of μ_1, and μ_2, and ϕ_1, and ϕ_2 are given by $\frac{\mu_i}{(\mu_i - \phi_i)^2} = 5$ (*i.e.*, overall symmetry property is maintained) using the communication means of types (B) and (C) respectively.

From the two figures 4, and 7, it is observed that the worst ratio of performance degradation gets it's maximum value (*i.e.*, the worst performance is obtained) when $\mu_1 = \mu_2$, and thus, $\phi_1 = \phi_2$ (*i.e.*, in complete symmetry), and it increases as μ_1, and μ_2 increase. It decreases slowly as the system parameter setting gradually departs the complete symmetry while keeping the overall symmetry property.

3.3 Individual Symmetry Maintained

The two figures 5, and 8 show how the worst ratio of performance degradation depends on the values of the system parameters μ_1, and μ_2. Given the values of μ_1, and μ_2, and ϕ_1, and ϕ_2 are given by

$\frac{1}{\mu_i - \phi_i} = 0.25$ (*i.e.*, the individual symmetry property is maintained) by using the communication means of types (B) and (C) respectively.

From the two figures 5, and 8, it is observed that the worst ratio of performance degradation gets it's maximum value (*i.e.*, the worst performance is obtained) when $\mu_1 = \mu_2$, and thus, $\phi_1 = \phi_2$ (*i.e.*, in complete symmetry), and it increases as μ_1, and μ_2 increase. It decreases a little bit rapidly than that obtained in the overall symmetry case as the system parameter setting gradually departs the complete symmetry while keeping the individual symmetry property.

3.4 No Symmetry Maintained

The two figures 6, and 9 show how the worst ratio of performance degradation depends on the values of the system parameters μ_1,and μ_2. Given the values of μ_1, and μ_2, and ϕ_1, and ϕ_2 are given by $\frac{\phi_i}{\mu_i - \phi_i} = 4$ (*i.e.*, no symmetry property is maintained) by using the communication means of types (B) and (C) respectively.

From the two figures 6, and 9, it is observed that the worst ratio of performance degradation gets it's maximum value (*i.e.*, the worst performance is obtained) when $\mu_1 = \mu_2$, and thus, $\phi_1 = \phi_2$ (*i.e.*, in complete symmetry), and it increases as μ_1, and μ_2 increase. It decreases rapidly than that obtained in the individual and overall symmetry cases as the system parameter setting gradually departs the complete symmetry without keeping any kind of symmetries.

4 Conclusion

We have presented a number of numerical examples for the Braess-like paradox wherein adding a communication capacity to the system for the sharing of jobs between nodes leads to the performance degradation for all users in the class optimum for load balancing. These examples are shown in the model of load balancing between two distinct servers. From these examples, it is observed that in the class optimum, the worst ratio of performance degradation in the paradox is largest (*i.e.*, the worst performance is obtained) in the complete symmetry case with the arrival rate is closest to the processing rate. And, as the system parameter setting departs the above-mentioned symmetric case without keeping any kind of symmetries, the worst ration of performance degradation decreases rapidly. It decreases slowly (more slowly) if the system parameter setting departs the complete symmetry while keeping the individual (overall) symmetry property. Indeed, it is also observed that in complete symmetry, if the communication means of type (C) is used, the worst ratio of performance degradation may increase without bound as the arrival rate gets very close to the processing rate. If the results observed in this study hold generally, we think that more exhaustive research into these problems is worth pursuing in order to gain insight into the optimal design of distributed computer systems, communication networks, *etc.* .

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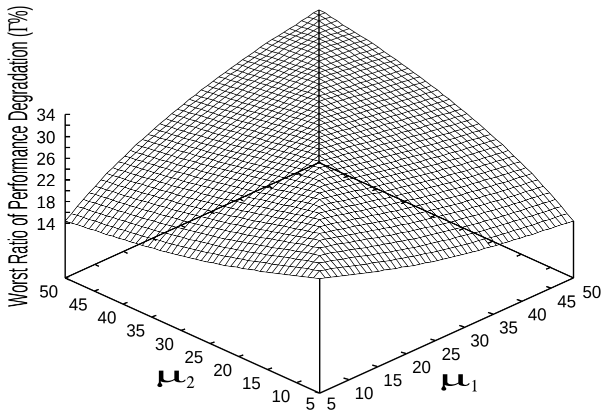

Figure 4: The worst ratio of performance degradation($\Gamma(\%)$) given the values of μ_1, and μ_2, and ϕ_1, and ϕ_2 are given by $\frac{\mu_i}{(\mu_i - \phi_i)^2} = 5$ (*i.e.*, overall symmetry property is maintained) using the communication means of type (B).

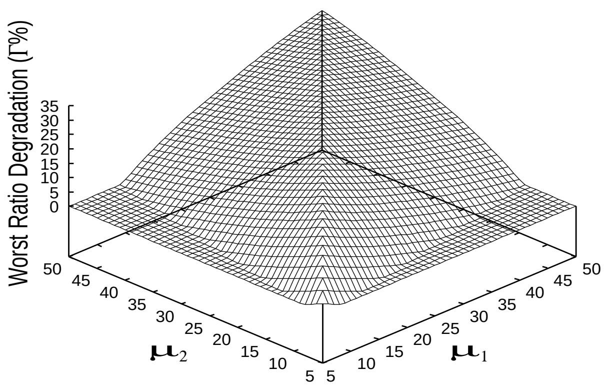

Figure 5: The worst ratio of performance degradation ($\Gamma(\%)$) given the values of μ_1, and μ_2, and ϕ_1, and ϕ_2 are given by $\frac{1}{\mu_i - \phi_i} = 0.25$ (*i.e.*, individual symmetry property is maintained) using the communication means of type (B).

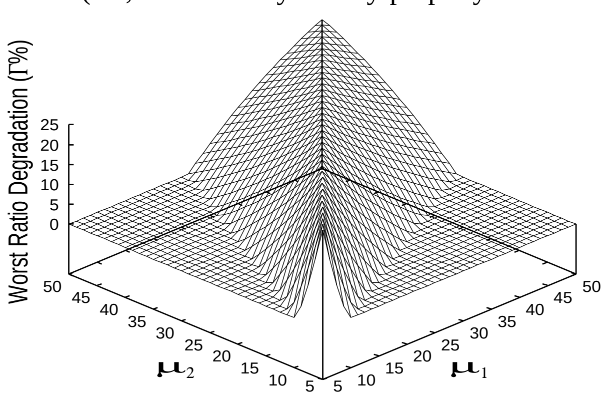

Figure 6: The worst ratio of performance degradation ($\Gamma(\%)$) given the values of μ_1, and μ_2, and ϕ_1, and ϕ_2 are given by $\frac{\phi_i}{\mu_i - \phi_i} = 4$ (*i.e.*, no symmetry property is maintained) using the communication means of type (B).

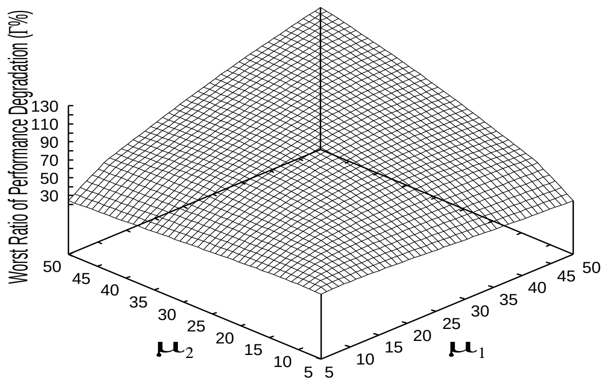

Figure 7: The worst ratio of performance degradation ($\Gamma(\%)$) given the values of μ_1, and μ_2, and ϕ_1, and ϕ_2 are given by $\dfrac{\mu_i}{(\mu_i - \phi_i)^2} = 5$ (*i.e.*, overall symmetry property is maintained) using the communication means of type (C).

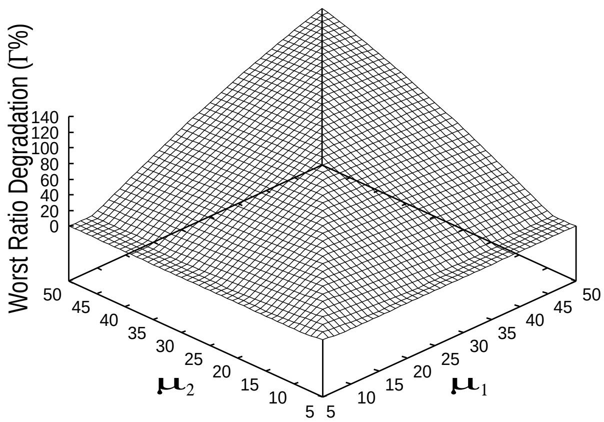

Figure 8: The worst ratio of performance degradation ($\Gamma(\%)$) given the values of μ_1, and μ_2, and ϕ_1, and ϕ_2 are given by $\dfrac{1}{\mu_i - \phi_i} = 0.25$ (*i.e.*, individual symmetry property is maintained) using the communication means of type (C).

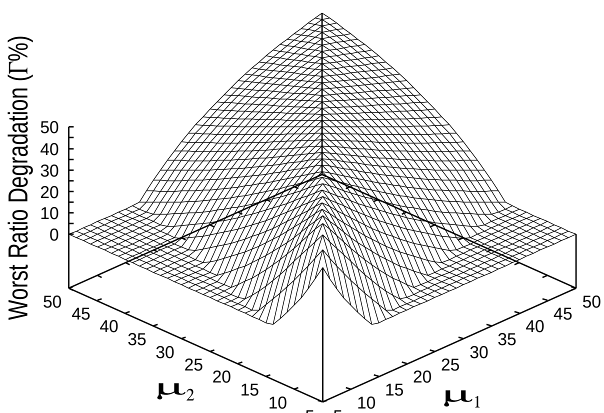

Figure 9: The worst ratio of performance degradation ($\Gamma(\%)$) given the values of μ_1, and μ_2, and ϕ_1, and ϕ_2 are given by $\dfrac{\phi_i}{\mu_i - \phi_i} = 4$ (*i.e.*, no symmetry property is maintained) using the communication means of type (C).