# Dynamic games in resource management problems

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## Introduction

The primary aim of rational resource exploitation consists in sustainable development of a renewable resource. A new method of ecological regulation is proposed, where the center determines the optimal share of the exploited territory to maintain the stable resource evolution in the long-term prospect. Cooperation plays an important role in resource management problems as it leads to a sparing mode of exploitation. We present two methodological schemes to maintain cooperation: incentive equilibrium and time-consistent imputation distribution procedure. Another meaningful applied problem is to determine cooperative behavior in asymmetric models, where players have different discount factors and planning horizons. We apply Nash bargaining schemas to construct cooperative strategies and payoffs in such models. The results of numerical modelling for different resources are presented.

## Reserved territory approach

Divide the exploitation area into two parts, $s(t)$ and $1-s(t)$. The center determines the size of the reserved territory $0 \le s(t) \le 1$, $n$ players exploit the resource on open area (Mazalov, Rettieva, 2006).

The dynamics of the renewable resource has the form

$$x'(t) = f(x(t), u(t, s(t)), 0 \le t \le T,\ x(0) = x_0\,, \qquad (1)$$

where $x(t) \ge 0$ – the size of the resource at time $t$, $f(x(t), u(t, s(t)))$ – natural growth function, $u_i(t, s(t)) \ge 0$ – the strategy (exploitation rate) of player $i$ at time $t$, $i = 1, \dots, n$, $u(t, s(t)) = (u_1(t, s(t)), \dots, u_n(t, s(t)))$.

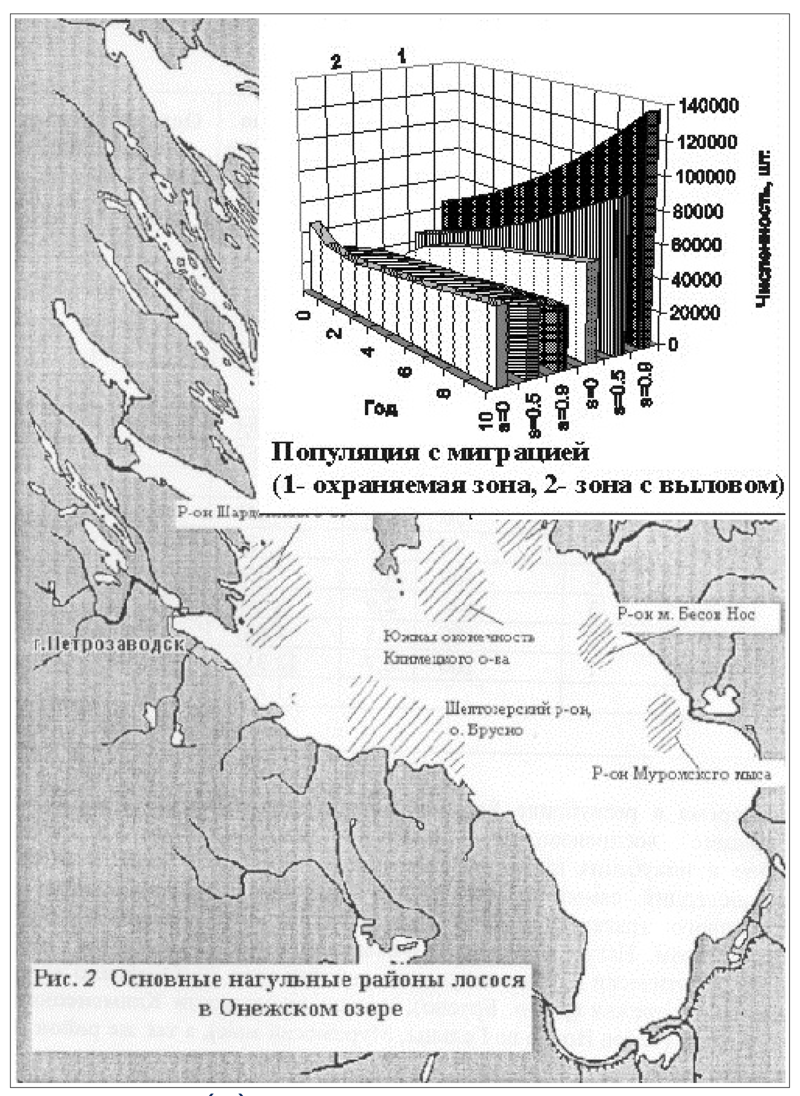
The exploitation rate is proportional to the size of the resource on the open for exploitation territory, i.e. $x(t)(1 - s(t))$.

The players' payoff functions, $i = 1, \dots, n$:

$$J_i = \int_0^T e^{-\rho t} g_i(x(t), u(t, s(t)))\, dt + G_i(x(T))\,. \qquad (2)$$

The center's gain:

$$I = \int_0^T e^{-\rho t} \gamma(x(t), u(t, s(t)))\, dt\,.$$




(a) Salmon in Onego lake.

## Cooperation maintenance

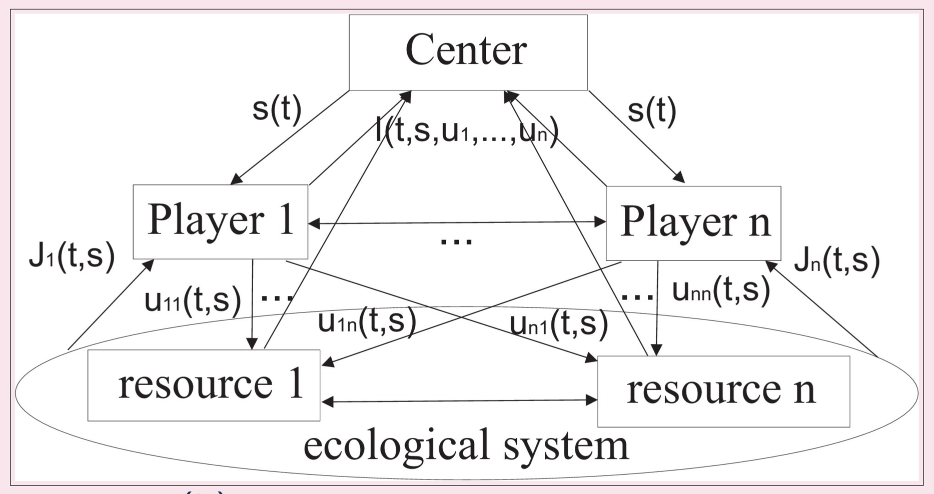
### Incentive equlibrium

Divide the water area into two parts, $s(t)$ and $1 - s(t)$, where two players exploit the resource. The dynamics of the resource and the players' payoffs have the forms (1)–(2).

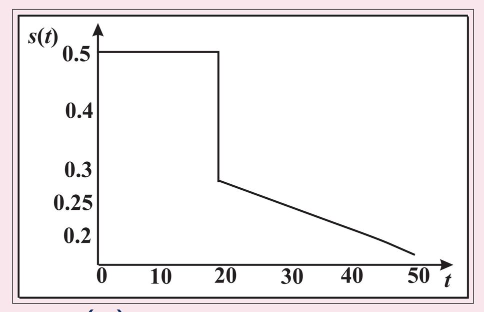
$s^c$ – the territory sharing under cooperation. Players deviating from the cooperative agreement are punished by the center via changing the exploitation territory proportionally to the value of deviation.

**Definition 1 (Mazalov, Rettieva, 2008, 2010).** *A strategy pair $(\gamma_1, \gamma_2)$ is called cooperative incentive equilibrium if*

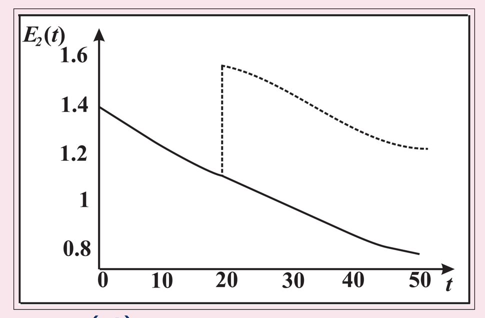
$$u_1^c(s^c) = \gamma_1(u_2^c(s^c))\,, \quad u_2^c(s^c) = \gamma_2(u_1^c(s^c))\,,$$
$$J_1(u_1^c(s^c), u_2^c(s^c)) \ge J_1(u_1(s), \gamma_2(u_1(s))) \quad \forall u_1 \in U_1\,,\ 0 \le s \le 1\,,$$
$$J_2(u_1^c(s^c), u_2^c(s^c)) \ge J_2(\gamma_1(u_2(s)), u_2(s)) \quad \forall u_2 \in U_2\,,\ 0 \le s \le 1\,.$$




(b) Cooperative incentive equilibrium.




(c) Territory sharing.



(d) Deviator control.

### Dynamic stability and rationality conditions

The characteristic function $V(S, t)$ is the profit of coalition $S$, $S \subset N$, for every subgame started from the state $x_t^c$ at time $t$. The imputation set:

$$\xi(t) = (\xi_1(t), \dots, \xi_n(t)) : \sum_{i=1}^n \xi_i(t) = V(N, t),\ \xi_i(t) \ge V(i, t),\ i = 1, \dots, n.$$

**Definition 2 (Petrosyan, 1977).** *The vector $\beta(t) = (\beta_1(t), \dots, \beta_n(t))$ is a time-consistent imputation distribution procedure if*

$$\xi_i(0) = \int_0^\infty e^{-\rho t} \beta_i(t)\, dt\,,$$

*and for all $t \ge 0$*

$$\xi_i(0) = \int_0^t e^{-\rho\tau} \beta_i(\tau)\, d\tau + e^{-\rho t} \xi_i(t)\,, i = 1, \dots, n\,.$$

The players following the cooperative trajectory are guided by the same optimality principle at each current time and hence do not have any reasonable motivation to deviate from the cooperation agreement.

**Definition 3 (Mazalov, Rettieva, 2010).** *The imputation $\xi(t) = (\xi_1(t), \dots, \xi_n(t))$ satisfies each step rational behavior condition if*

$$\beta_i(t) + \delta V(i, t+1) \ge V(i, t)\,, i = 1, \dots, n$$

*for all $t \ge 0$, where $\beta(t) = (\beta_1(t), \dots, \beta_n(t))$ is the time-consistent imputation distribution procedure.*

The proposed condition offers an incentive to player $i$ to maintain cooperation because at every step she gains more from cooperation than from noncooperative behavior.

## Main Publications


- Mazalov V.V., Rettieva A.N. (2006) Nash equilibrium in ecological problems. *Mathematical Modelling*, 18(5).
- Mazalov V.V., Rettieva A.N. (2008) Incentive equilibrium in discrete-time bioresource sharing model. *Doklady Mathematics*, 78 (3).
- Mazalov V.V., Rettieva A.N. (2010) Fish wars and cooperation maintenance. *Ecological Modelling*, 221.
- Mazalov V.V., Rettieva A.N. (2015) Asymmetry in a cooperative bioresource management problem. *Game-Theoretic Models in Mathematical Ecology*. NY: Nova Science Publishers.
- Rettieva A.N. (2015) A bioresorce management problem with different planning horizons. *Automation and Remote Control*, 76 (5).
- Rettieva A. (2017) Cooperation in resource management problems. *Contributions to Game Theory and Management*, 10.


## Asymmetry

### Different discount factors

The payoff functions of the players are

$$J_i = \sum_{t=0}^n \delta_i^t g_i(u_{1t}, u_{2t})\,,$$

where $\delta_i \in (0,1)$ – the discount factor of player $i$, $i = 1, 2$.
Define cooperative behavior (Mazalov, Rettieva, 2015) by a recursive bargaining procedure:

$$(H_1^{ic} - H_1^{iN})(H_2^{ic} - H_2^{iN}) \to \max_{u_1^i, \dots, u_1^i, u_2^i, \dots, u_2^i}\,,$$

where $H_j^{ic}$ – cooperative payoffs of the players, $H_j^{iN}$ – noncooperative payoffs of the players, $j = 1, 2$, for $i$-step game.

### Different planning horizons

Players exploit the resource during $n_1$ and $n_2$ steps, respectively: 1) fixed $n_1 < n_2$, 2) random.

$$1)\ J_1 = \sum_{t=0}^{n_1} \delta_1^t \ln(u_{1t}^c), \quad J_2 = \sum_{t=0}^{n_1} \delta_2^t \ln(u_{2t}^c) + \sum_{t=n_1+1}^{n_2} \delta_2^t \ln(u_{2t}^a)\,,$$

where $u_i^c \ge 0$ $(i = 1, 2)$ – the cooperative strategies, $u_2^a \ge 0$ – the strategy of player 2 during individual exploitation.

To construct the cooperative behavior, apply the Nash bargaining solution (Rettieva, 2015):

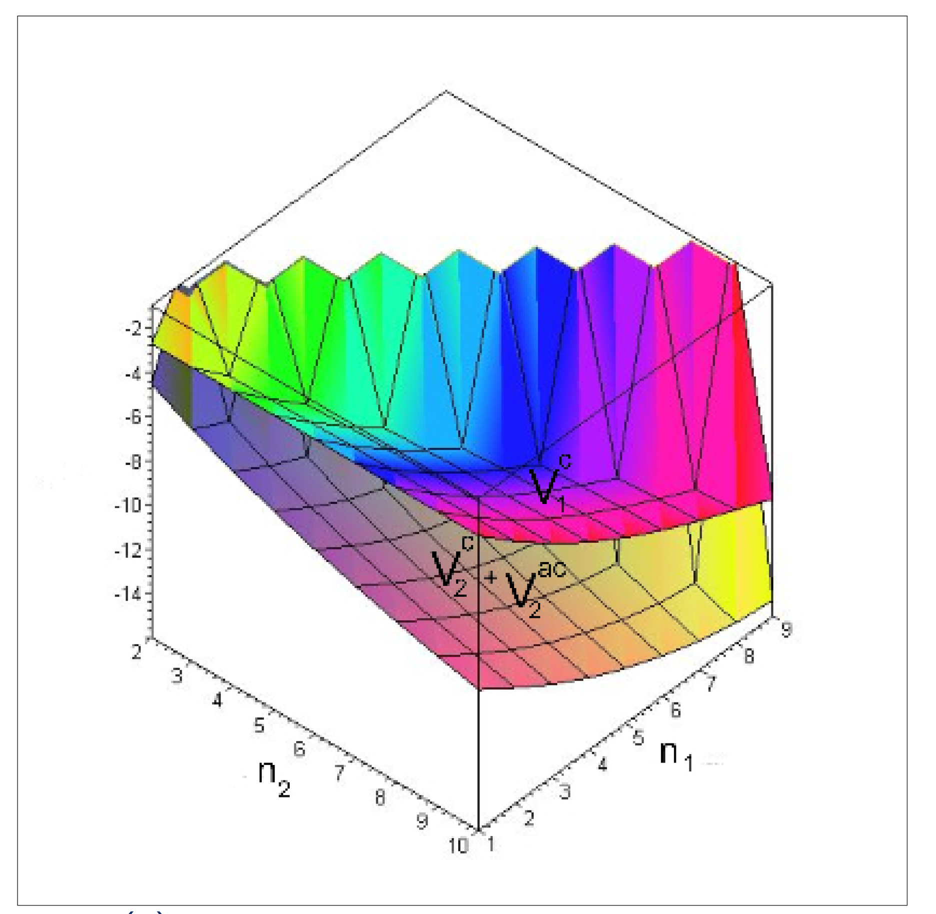
$$\left(V_1^c(x, \delta_1)[0, n_1] - V_1^N(x, \delta_1)[0, n_1]\right) \cdot$$
$$\cdot (V_2^c(x, \delta_2)[0, n_1] + V_2^{ac}(x^{cn_1}, \delta_2)[n_1, n_2] -$$
$$-V_2^N(x, \delta_2)[0, n_1] - V_2^{aN}(x^{Nn_1}, \delta_2)[n_1, n_2]) \to \max\,,$$

where $V_i^N(x, \delta_i)[0, n_1]$ – the Nash equilibrium payoffs, $V_2^{ac(aN)}(x^{cn(Nn)_1}, \delta_2)[n_1, n_2]$ – the payoff of player 2 owing to its individual harvesting after $n_1$ steps of cooperative (noncooperative) behavior.

2) The payoffs of the players are determined as

$$H_i = E\Big\{\sum_{t=1}^{n_i} \delta_i^t g_i(u_{1t}, u_{2t}) I_{\{n_i \le n_j\}} +$$
$$+ \Big(\sum_{t=1}^{n_j} \delta_i^t g_i(u_{1t}, u_{2t}) + \sum_{t=n_j+1}^{n_i} \delta_i^t g_i(u_{it}^a)\Big) I_{\{n_i > n_j\}}\Big\}\,,$$

where $i, j = 1, 2$, $i \ne j$, $u_{it}^a$ – the strategy of player $i$ when the partner leaves the game, $i = 1, 2$.
To define cooperative behavior, we employ the Nash bargaining solution; the role of the status quo points belongs to the noncooperative payoffs of players (Rettieva, 2017).




(e) Cooperative payoffs for different horizons.