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Foreword

The present volume contains full papers and extended abstracts accepted for the International Workshop "Networking Games and Management" held in the Institution of the Russian Academy of Sciences, Institute of Applied Mathematical Research KarRC RAS, Petrozavodsk, Russia, June 30 – July 2, 2012.

The emphasis of the seminar is on the following topics:

- networking games and management,
- optimal routing,
- price of anarchy,
- auctions,
- negotiations.

23 papers from Russia, Finland, Spain, Turkey, the Netherlands were submitted and included into the volume.

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On behalf of the Organization Committee
Professor Vladimir V. Mazalov

Contents

Богданова О.К., Парилина Е.М.
Эмпирическое исследование социальной сети «vk.com» 8

V. V. Breer
Equilibrium Condition Evaluation of Large Network of Anonimous Conformity Agents 11

J. V. Chirkova
Price of Anarchy in Machine Load Balancing Game with 3 Machines 14

D.N. Fedyanin
On a reflective network structure of agents' beliefs 15

R. Flores, E. Molina, J. Tejada
Pyramidal value for directed graph restricted games 19

M. V. Goubko
Hierarchy optimization: theory and applications 20

V.V. Mazalov, A.V. Gurtov
Queueing System with On-Demand Number of Servers 23

A. A. Ivashko
Optimal strategy in a Buying-Selling Problem 24

E. E. Ivashko
Game of Lobbying in Social Networks 26

A. B. Khmelnitskaya, D. Talman
Tree, web and average web value for cycle-free directed graph games 30

С.А. Корниенко, Г.А. Угольницкий
Модель оценки качества в производственных системах с учетом коррупции 31

S. Kumacheva
Corruption And Inspectors' Mistakes in Tax Control 36

A. Lukyanenko
Cloud computing ecosystem 39

В.Д. Матвеенко, О.А. Нев
Игровая модель приращения знаний 41

N. N. Nikitina
Evaluation of a probabilistic backfilling algorithm for job scheduling on a computing cluster 44

T. E. Nosalskaya
On a Discrete Model of the Cake Division Problem 45

Д. А. Новиков
Теория игр и мультиагентные системы 47

E.M. Parilina
Dynamic Scenarios of Data Transmission in Wireless Networks 51

А. В. Щипцова
Пространственная конкуренция на рынке товаров двух видов 54

A. A. Sedakov, L. A. Petrosyan
Cooperation in Stochastic Network Gamesm 56

A. Sergeeva
The problem of big delay service under choice of different pricing schemes 59

И. А. Серяков
Теоретико игровая транспортная задача на сети с заданными пропускными способностями 62

A.N. Rettieva
Coalitional Stability in Management Problems 64

A. A. Vasin, Y. Sosina, G. W. Weber
Evaluation of Market Power in Local and Two-node Markets 66

Эмпирическое исследование социальной сети «vk.com»

Богданова О.К., Парилина Е.М.

Санкт-Петербургский государственный университет,
Санкт-Петербург, Россия

E-mail: bogdanova.o.k@gmail.com, elena.parilina@gmail.com

Рассмотрим социальную сеть, состоящую из конечного числа агентов, и смоделируем процесс распространения нового продукта на рынке, который можно представить данной сетью.

Пусть $N = \{1, \ldots, n\}$ — множество агентов рассматриваемой социальной сети. Каждый агент может существовать только в одном из двух различных состояний: он может быть либо активным, либо восприимчивым. Активные агенты уже обладают продуктом, а восприимчивые — нет, но под воздействием внешних факторов (влияние соседей, реклама и т. д.) могут этот продукт приобрести и таким образом перейти в активное состояние. Тогда в рассматриваемой сети возможны следующие состояния: $(0, n), (1, n-1), \ldots, (n, 0)$. Здесь каждая пара $(i, n-i)$, $i = \overline{0, n}$ — это состояние сети, где i — число активных агентов, $n-i$ — число восприимчивых агентов. Опишем способ перехода сети из одного состояния в другое.

Допустим, на агентов сети влияет два фактора: реклама продукта и мнение соседей. Степень воздействия рекламы обозначим через $\lambda = \lambda(c)$, где $c \in [0, c_0]$ — количество средств, вложенных в рекламу, $c \geqslant 0$ — неубывающая функция. Вероятность перехода из восприимчивого состояния в активное (т. е. вероятность приобретения продукта) обозначим через $p = p(\lambda, \theta)$, где $\theta = \dfrac{a}{k}$ — отношение числа активных соседей a к общему числу соседей $k = const$, $p \in [0, 1]$ — неубывающая по θ функция. Предположим, что обратный переход (из активного состояния в восприимчивое) невозможен.

Процесс распространения продукта в социальной сети опишем с помощью цепи Маркова с матрицей вероятностей перехода

$$\Pi = \begin{pmatrix} C_n^0 p^0 (1-p)^n & C_n^1 p^1 (1-p)^{n-1} & \dots & C_n^n p^n (1-p)^0 \\ 0 & C_{n-1}^0 p^0 (1-p)^{n-1} & \dots & C_{n-1}^{n-1} p^{n-1} (1-p)^0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 1 \end{pmatrix}$$

и конечным пространством состояний $\{(0, n), (1, n-1), \dots, (n, 0)\}$.

Предположим, что существует некоторая фирма, распространяющая новый продукт в рассматриваемой социальной сети. Целью этой фирмы является максимизация прибыли в долгосрочном периоде. Допустим, вид функций $\lambda(c)$ и $p(\lambda, \theta)$ известен. Тогда прибыль фирмы опишем функцией $b(\lambda, \theta, c, r) = \sum_{i=1}^{k} b_i \delta^{i-1} D(r_i)$, где $b_i = E[a_i - a_{i-1}] r_i - c_i$ — ожидаемая прибыль на i–ом шаге, a_i — число активных соседей на i–ом шаге, $\delta \in (0, 1)$ — фактор дисконтирования (обесценивания), c_i — издержки фирмы на рекламу на i–ом шаге, $D(r_i)$ — функция спроса, $r_i \in [r_0, R]$ — цена единицы продукта на i–ом шаге, r_0 — себестоимость единицы продукта. Стратегией рассматриваемой фирмы будет вектор $\{(r_i, c_i)\}$, $i = \overline{1, k}$, где (r_i, c_i) —стратегия фирмы на i–ом шаге, k — общее число шагов.

Таким образом, задача заключается в нахождении оптимальной стратегии фирмы, т. е. стратегии, максимизирующей ее общую прибыль.

Целью настоящего исследования является изучение вида функции $p(\lambda, \theta)$. В качестве функции $p(\lambda, \theta)$ можно использовать какую-любо функцию распределения. Для подбора вида распределения была исследована социальная сеть «vk.com». В этой социальной сети были получены три выборки по 300 агентов.

Для имеющихся выборок с помощью критерия согласия хи-квадрат Пирсона были проверены гипотезы о виде распределения генеральной совокупности. В качестве тестируемых распределений были выбраны распределение Вейбулла, гамма-распределение и логнормальное распределение. После оценки параметров для всех трех выборок подтвердилась гипотеза о логнормальном распределении исследуемой случайной величины.

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Equilibrium Condition Evaluation of Large Network of Anonimous Conformity Agents

Vladimir V. Breer

Institute of Control Sciences
Moscow, Russia

E-mail: `breer@live.ru`

There are several mathematical models describing the agent's behavior of conformity, i.e. an agent follows the behavior of the others. Among them there is a bunch of models that have been investigated since [3] was published by M. Granovetter. They are called threshold models of collective behavior. These models are developed for situations where an agent has two alternatives one of which she chooses according to her utility function (so called binary decisions). This utility function depends upon the proportion of the others that chose particular alternative. Let's name this proportion *social pressure*. If this social pressure is greater than the threshold characterized the agent, she chooses the same alternative. Otherwise she chooses the opposite one. This is done in order to maximize agent's utility.

The classical example is the decision of joining a riot or not. Here the agent's threshold is the proportion of the group he would have to see join before he would do so. The cost of joining a riot is declines as the riot size increases, since the probability of being ceased is smaller as the riot size increases. Each agent has her own level of cost or threshold she would pay for taking part in riot. According to this behavior it is important to investigate the percentage of agents taking part in the riot in equilibrium.

If we consider a model with large (infinite) number of agents, then we have to use stochastic methods of evaluation the condition of this large network.

Consider index set *of agents* $N = \{1, 2, ..., n\}$, where each agent has two choises - *to be active or inactive*. These choises of the agent i denote by the binary variable $x_i \in \{0; 1\}$, where choice 1 means, that the player is active, whereas choice 0 – the player is inactive. *Condition of finite network* defined by *vector of actions* of all agents $x \in \{0, 1\}^n$. Vector of external agents' actions relative to the agent i denote by $x_{-i} = (x_1, x_2, ..., x_{i-1}, x_{i+1}, ..., x_n) \in \{0, 1\}^{n-1}$.

Agent $i \in N$ is forced by *social pressure*, which we define by $\frac{1}{n-1}\sum\limits_{j \neq i} x_j$. It is bounded $0 \leq \frac{1}{n-1}\sum\limits_{j \neq i} x_j \leq 1$. On the other hand agent $i \in N$ is charachterized by *threshold of independance* $0 < \theta_i < 1$.

Let's define the behavior of conformity agent with the help of the following *utility function*:

$$u_i\left(x, x_{-i}\right) = \left(\frac{1}{n-1}\sum_{j \neq i} x_j - \theta_i\right) x_i, i \in N. \tag{1}$$

Denote by G the following game in normal form $\left(\{0, 1\}^n, \{u_i\}_{i \in N}, N\right)$. It is shown in [1] that for the game G the structure of Nash equilibrium is very simple. Let's arrange thresholds θ_i by their value in increasing order. The structure of Nash equilibrium x^* is as follows. All the payers whose thresholds are not bigger than the value of average action $\theta_i \leq \frac{1}{n}\sum\limits_{i}^{n} x_i^*$ are active $x_i^* = 1$, the rest are inactive $x_i^* = 0$.

Consider common utility function as a sum of individual utility functions, which charachterizes the value of the network:

$$U(x) = \sum_i u_i\left(x, x_{-i}\right) = \frac{1}{n-1}\sum_{j,i:j \neq i}^{n} x_j x_i - \sum_i^n \theta_i x_i. \tag{2}$$

Let's $\{\theta_i\}_{i \in N}$ be identically distributed random valies with density of distribution $f_\theta : (0; 1) \to [0; 1]$. For large n for Nash equilibrium we can evaluate (2) as

$$U(s_n) = ns_n^2 - s_n - n \int\limits_0^{s_n} t f_\theta(t) dt, \tag{3}$$

where $s_n = \frac{1}{n} \sum\limits_i^n x_i^*$ - fraction of active agents.

As shown in [2] according to the large deviation principle we can get for the mathematical expectation of s_n where $n \to \infty$ the following :

$$Es = \lim_{n \to \infty} Z^{-1} \int\limits_0^1 y e^{-y} e^{n\left\{y^2 - \int\limits_0^y t f_\theta(t) dt - I(y)\right\}} dy, \tag{4}$$

where $I(y)$ is entropy function of the uniformly distributed random value.

As n is large, the mathematical expectation of fraction s_n (4) mainly depends upon extremal point of the function $\left\{y^2 - \int\limits_0^y t f_\theta(t) dt - I(y)\right\}$, which in turn depends upon one parameter - the distribution of thresholds f_θ.

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Price of Anarchy in Machine Load Balancing Game with 3 Machines

Julia V. Chirkova

Institute of Applied Mathematical Research,
Karelian Research Centre RAS, Petrozavodsk, Russia

E-mail: `julia@krc.karelia.ru`

The Machine Load Balancing Game with 3 machines is considered. A set of N jobs is to be assigned to a set of 3 machines with speeds $speed_i (i = 1, 2, 3)$ which are 1, r and s correspondingly. So, j-th job's pure strategy l_j is a machine's number i. The size of a job j is denoted by w_j. The running time of a job j on a machine i with a speed $speed_i$ is $w_j/speed_i$. The delay on this machine is $\sum_{j:l_j=i} w_j/speed_i$. Jobs choose machines to minimize their own delays. The social cost of a schedule is the maximum delay among all machines, i.e. *makespan*.

Pure strategies only are considered. Pure Nash Equilibrium is a schedule where no one job has a reason to choose another machine. Optimal schedule provides a minimum of the social cost. The Price of Anarchy (PoA) is a ratio between the worst Nash equilibrium social cost and the optimal social cost.

The upper bound estimation of the Price of Anarchy (PoA) for this model is obtained analytically. The exact value of PoA is obtained numerically. Estimations are visualized to compare them.

On a reflective network structure of agents' beliefs

Fedyanin Denis Nikolaevich

Institute of Control Sciences RAS, Moscow, Russia

E-mail: dfedyanin@inbox.ru

Problems of decision making in situations when decision makers (*agents*) gain depends not only on his actions, but on actions of other agents, is the subject of traditional models of game theory and the theory of collective behavior. One of the key factors in this case is a mutual awareness of agents, their possible actions and principles of decision-making.

The process of forming ideas about principles of decision-making of opponents is called strategic reflection. Reflection could be divided into strategic reflection and informational one [6]. Informational reflection is a process of suggestions about what values of uncertain parameters are and what estimations of them are used by opponents. In other words, an informational reflection of the information relates to the agent's knowledge about the nature of reality (what the game), and reflective reality (see what other game). Strategic reflection is a process of suggestions about principles of making decisions by opponents in the framework of the awareness. Thus, informational reflection takes place only under conditions of incomplete knowledge, and its result is used in decision-making (and sometimes it is included in strategic reflection). Strategic reflection takes place even in the case of full information, and is focused on principles of decisions of players. In other words, information and strategic reflection can be studied independently, but under conditions of incomplete knowledge they both have a place.

Reals players have limited strategic reflection [1], and not only cannot be aware of the objective functions opponents, but even if he knows them to be mistaken about their principles of decision: whether the agents are able to calculate the Nash equilibrium in the game, which of the existing Nash equilibria in a game they choose if there are several them, whether they will seek an equilibrium in pure or mixed strategies, how to eliminate the uncertainty in the absence of information of parameters of nature, etc. The real agent can also have both limited information and limited strategic reflection .

Concept of rank of reflection is often used to simulate different types of rationality of players, according to this concept there is a partition of players according to their types of rationality - ranks (reflective splitting) [5]. But in this paper we consider a special kind of structure of reflection of players - *reflective network* - which includes elements of both information and strategic reflection and widen the concept of rank of reflection. To build the attributes of each player, such as a set of actions and the objective function, we added decision rule which he uses under uncertainty and a set of players, whose set of attributes he knows precisely. Thus this paper proposes a new modeling tool a difficult strategic situation of reflection of agents - reflective network. Reflective network takes into account not only the rank of an agent of reflection, but also his awareness of certain opponents. This allows, in particular, to model decision-making structures of the active network.

Reflexive network is constructed as follows [4]: the nodes (vertices) of the network are players, and the arc of the player i to player j exists if and only if a player j belongs to the set of players, whose attributes are known to the player i. This network is completely defines the structure of knowledge and allows us to calculate the balance. Thus, the problem of searching game with incomplete awareness is completely determined by the strategy sets of players, their objective functions, their rules of decision making and reflective network.

The purpose of this paper is to discover dependence of the influence of reflexive networks and decision rules players on the properties of equilibria in games. The basic idea of the research is to construct a reflexive network with the defined strategy sets of players, their objective functions and decision rules. The purpose of such construction is to force payers (without changing their goal functions, sets of strategies, sequences of moves) to game equilibrium at a given point.

This paper demonstrates an example of successful analysis of reflexive network's influence on equilibria in certain network game, which all non-reflective equilibria are known and have been found in [2,3]. It has been proved several statements on the assumption that the set of admissible decision rules in this game has the following properties: the players who know each other's attributes, play a Nash equilibrium in pure strategies, and the actions of players whose attributes are unknown to them, they predict, based on the criteria for acceptance Hurwitz solutions with the parameter equal to the optimism either zero or one. It was also constructed optimal control by reflexive network for some cases of different distributions of optimism among the players to prevent any non-zero equilibrium in the given network game.

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Pyramidal value for directed graph restricted games

Ramon Flores

Universidad Carlos III de Madrid, Spain

E-mail: rflores@est-econ.uc3m.es

Elisenda Molina

Universidad Carlos III de Madrid, Spain

E-mail: elisenda.molina@uc3m.es

Juan Tejada

Universidad Complutense de Madrid, Spain

E-mail: jtejada@mat.ucm.es

We propose a value for cooperative TU games with partial action, when influence relations are directional: the Pyramidal Value. Based on this concept, we also propose an index of global Social Network Efficiency which measures the ability of the network to promote efficient coalition structures to form. We analyze their properties and show their behavior by means of some illustrative applications (hierarchies, symmetric games,...)

Hierarchy optimization: theory and applications[1]

Mikhail V. Goubko

Institute of Control Sciences RAS, Moscow, Russia

E-mail: mgoubko@mail.ru

Abstract

We survey the body of analytical and numerical methods for hierarchy optimization developed during the recent decade and sketch their recent applications.

Keywords: hierarchy optimization, growing decision tree, user menu design, hierarchy in firms, assembly line balancing.

The problems of hierarchy optimization arise in different areas, from computer science to management. The well-known examples include the problems of an optimal prefix code, decision tree growing, communication network optimization, etc. They are studied separately, although allowing for a uniform description in terms of a general mathematical framework. This framework provides a common language to model different problems of hierarchy optimization, develop universal solution methods, and adopt and generalize local approaches.

An hierarchy optimization problem is to minimize a cost function by the choice of an admissible hierarchy (usually, the set of admissible hierarchies is too large for exhaustive search to be possible). The concepts of the abstract hierarchy and the sectional cost function suggested by Voronin and Mishin in 2001 provide a basis for viable study. The hierarchy is understood as an acyclic graph (generally, not a tree) with the given set of leafs and the sole root. The set of leafs is fixed by the problem setting, be

[1]The research is supported by the grant 10-07-00129 of Russian Foundation for Basic Research.

it an alphabet in the coding problem or a set of base stations in telecom. The hierarchy cost function is called sectional if it sums up the costs of hierarchy nodes, while the node cost depends on the tuple of groups of leaves reachable from the children of this node.

Analytical methods of hierarchy optimization include conditions to narrow the set of potential solutions, and estimates of the attributes of the optimal hierarchy. Numeric methods reduce to exact and approximate algorithms for optimal hierarchy search. Analytical estimates of hierarchy cost are intensively used in algorithms to branch and cut the search space.

Sectional cost functions cover a wide range of applications, but are concise enough to allow for comprehensive deductions about the optimal hierarchy shape – when a hierarchy is tall or flat, tree-shaped, or resembles a conveyor belt. Also numerous algorithms were developed by Mishin to seek an optimal hierarchy, a tree, or a conveyor for sectional cost functions.

Nevertheless, the optimal hierarchy problem for a sectional cost function has no efficient solution in general. Homogenous cost functions provide an interesting subclass, which allows for a complete solution of an optimal hierarchy problem. The optimal hierarchy is proved to be uniform, the closed-form solution is derived for optimal hierarchy cost and shape (span of control and skewness), and efficient algorithms were developed to construct nearly-optimal hierarchies.

The suite of optimization techniques developed allowed solving the following hierarchy optimization problems in different areas.

In data mining – for the problem of growing a decision tree (used in classification and machine learning) a new combinatory lower-bound estimate for classification costs is suggested along with new efficient algorithms for building a decision tree.

In human-computer interaction – for the problem of hierarchical menu design a mathematical model was proposed for menu structure optimization and efficient algorithms were developed and implemented in the convenient computer-based design tool.

In management – the models of a management hierarchy were explored to address the fundamental issues of the theory of the firm. The closed-form expressions are obtained for hierarchy maintenance costs, span of control, the number of layers, managers' efforts and compensation, which enable comparative static analysis and model identification from data.

In production planning – an assembly line balancing problem was reduced to a special case of a hierarchy optimization problem; new algorithms were suggested for the generalized setting.

In the report we discuss the latest results, recent applications and perspectives. Potential applications of the theory and techniques are very diverse. To name a few: for information theory – handy taxonomies and classifiers, efficient schemes for hierarchical calculus; for business administration – methods for supply networks structure optimization.

Queueing System with On-Demand Number of Servers

V.V. Mazalov

Institute of Applied Mathematical Research,
Karelian Research Centre RAS, Petrozavodsk, Russia

E-mail: vmazalov@krc.karelia.ru

A.V. Gurtov

Centre for Wireless Communications
University of Oulu

E-mail: gurtov@ee.oulu.fi

Keywords: queuing theory, modeling.

We consider a queuing system where the number of active servers changes depending on the length of the queue. As a practical example of such system, we consider the security check queue at the airports. The number of active servers increases when the queue grows by k customers and decreases accordingly. That allows to save server re- sources while maintaining acceptable performance (average queuing time and its variation) for customers. We obtain a closed-form solution for the serving time, queue length and average number of servers.

To validate the model we have selected the data of Dallas Fort international airport, the 8th largest in the world. Our simulation model shows a close match with analytic results. Cost savings in the number of open servers are achievable while providing acceptable waiting time for the customers.

Optimal strategy in a Buying-Selling Problem

Anna A. Ivashko

Institute of Applied Mathematical Research,
Karelian Research Centre RAS, Petrozavodsk, Russia

E-mail: aivashko@krc.karelia.ru

Keywords: optimal stopping, urn scheme, ballot problem

In this paper, the following optimal double stopping problem on trajectories is considered. Suppose that there is an urn containing m balls of value -1 and p balls of value $+1$. The player is allowed to draw ball randomly, without replacement, one by one. The value -1 is attached to minus ball and value $+1$ to plus ball. Determine sequence $Z_0 = 0$, $Z_n = \sum\limits_{k=1}^{n} X_k$, $1 \leq n \leq m+p$, where X_k is the value of the ball chosen at the k-th draw. The player observes the values of the balls and wants to make two stops. The aim of player is to maximize the expected gain, the gain is difference between maximum and minimum values of the trajectory formed by $\{Z_n\}_{n=0}^{m+p}$ (net gain problem).

This urn scheme could be considered as the buying-selling problem. Here the value of the ball is change of the cost of an asset. The first stop means the buying of an asset and the second stop is the selling of an asset. In net gain problem the player wants to maximize the difference between costs.

The urn schemes with one stop was considered by Shepp L. (1969) (net gain problem), Tamaki M. (2001) (max-problem), Mazalov V.V., Tamaki M. (2007) (duration problem).

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Game of Lobbying in Social Networks[1]

Evgeny E. Ivashko

Institute of Applied Mathematical Research,
Karelian Research Centre RAS, Petrozavodsk, Russia

E-mail: ivashko@krc.karelia.ru

Abstract

The paper deals with model of information lobbying in social networks. The social network is defined by the matrix that shows influences of network's agents to each over. All the agents have initial beliefs. Every step each agent's opinion influenced by all other agents. The final opinion of the agents is the result of multistep interactions with the other agents. There are players who are interested in certain final opinion. They can lobby their positions at every step impacting the agents' opinions. The players pay for lobbying. The price depends on value of the player's impact to the agents. Each player aims to maximize her own payoff that depends on final opinion and price to pay.

Keywords: social network, information impact, lobby, game theory.

Consider a mathematical model of social networks introduced in [1]–[4]. Let the social network built by n agents, each agent has an opinion on certain question. The opinion of the i-th agent in the time t is defined by value $x_i^t \in [0,1]$, $i \in N = \{1, 2, ..., n\}$, $t = 0, 1, 2, ...$ An information impact of agent i to agent j is defined by value $a_{ij} \geq 0$, $i, j \in N$. The impact matrix $A =\| a_{ij} \|_{N\times N}$ is non-negative and stochastic by rows:

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$\sum_{j=1}^{n} a_{ij} = 1$. The agents have initial beliefs $x^0 = \left(x_i^0\right)_{i \in N}$. At each step the agent i changes her opinion in view of opinions of the other agents:

$$x_i^t = \sum_{j \in N} a_{ij} x_j^{t-1}, \text{ or } x^t = Ax^{t-1}, t = 1, 2, ...; i \in N.$$

Interactions of the agents repeat until they have the common opinions:

$$x = A(A(...A(Ax^0)...)) = A^\infty x^0, \text{ where } A^\infty = \lim_{t \to \infty} (A)^t.$$

It is known that the matrix A^∞ has following form:

$$A^\infty = \begin{pmatrix} \pi_1 & \pi_2 & \pi_3 \\ \pi_1 & \pi_2 & \pi_3 \\ \pi_1 & \pi_2 & \pi_3 \end{pmatrix},$$

where

$$\begin{cases} \pi_1 & = & \pi_1 a_{1,1} + \pi_2 a_{2,1} + \pi_3 a_{3,1} \\ \pi_2 & = & \pi_1 a_{1,2} + \pi_2 a_{2,2} + \pi_3 a_{3,2} \\ \pi_3 & = & \pi_1 a_{1,3} + \pi_2 a_{2,3} + \pi_3 a_{3,3} \end{cases}$$

Suppose, there is a player (outside the network) who can lobby in the social network. The lobbying impact $u_{t,i}$ of the player at the step t is the affecting on the agent's i opinion:

$$x_i^t = x_i^{t-1} - u_{t-1,i}, \text{ where } u_{t,i} \in [-1, 1]; i \in N; t = 0, 1, 2, ...$$

It is shown in [4] that the final opinion depends on sum value of the impact not on it's distribution in time, i.e.

$\forall m, r \in N, \forall \{u_0, u_1, ..., u_m\}$, where
$u_t = (u_{t,1}, u_{t,2}, ..., u_{t,n}), u_{t,i} < \infty, i \in N, t = 0, 1, ..., m,$

$$A(...(A(...A(x - u_0) - u_1) - u_m)...) = A(...A \underbrace{\left(A\left(...Ax^0...\right) - \sum_{t=0}^{m} u_t \right)}_{\text{r}} ...)$$

Also, all the impact at the specific time t^* should be concentrated on the most valuable person k^*, where

$$(t^*, k^*) \in \arg \max_{k \in N, t=0,1,...} w_k^t, w_k^t = \sum_{i \in N} \left(A^t\right)_{k,i}.$$

Consider the problem of 3 agents and K players. Let the player k pays $C(u^k) = u^k(u^k)^T$ for the lobbying u^k, $k \in K$.

Using the previous formulas, the final opinion of the agents in the social network has following form:

$$B_u^\infty = A^\infty \left(x^0 - \sum_{k \in K} u^k \right).$$

Let the payoff function is $H_k(u) = -\left(B^\infty - b_{aim}^k\right)^2 - C_k(u)$, $k \in K$. The player aims to maximize her payoff.

Then the optimal control is following:

$$\begin{cases} u_3^1 &= \pi_3 \frac{\pi_1 b_1^0 + \pi_2 b_2^0 + \pi_3 b_3^0 - b_{aim}^1 - \sum_{i>1} u^i}{\pi_3^2 + 2}, \\ & \cdots \\ u_3^K &= \pi_3 \frac{\pi_1 b_1^0 + \pi_2 b_2^0 + \pi_3 b_3^0 - b_{aim}^k - \sum_{i<k} u^i}{\pi_3^2 + 2}. \end{cases}$$

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Tree, web and average web value for cycle-free directed graph games

Anna B. Khmelnitskaya

Saint-Petersburg State University, Saint-Petersburg, Russia

E-mail: a.khmelnitskaya@math.utwente.nl

Dolf Talman

Tilburg University, Tilburg, The Netherlands

On the class of cycle-free directed graph games with transferable utility solution concepts, called web values, are introduced axiomatically, each one with respect to some specific choice of a management team of the graph. We provide their explicit formula representation and simple recursive algorithms to calculate them. Additionally the efficiency and stability of web values are studied. Web values may be considered as natural extensions of the tree and sink values as has been defined correspondingly for rooted and sink forest graph games. In case the management team consists of all sources (sinks) in the graph a kind of tree (sink) value is obtained. In general, at a web value each player receives the worth of this player together with his subordinates minus the total worths of these subordinates. It implies that every coalition of players consisting of a player with all his subordinates receives precisely its worth. We also define the average web value as the average of web values over all management teams in the graph. As application the water distribution problem of a river with multiple sources, a delta and possibly islands is considered.

Модель оценки качества в производственных системах с учетом коррупции

С.А. Корниенко, Г.А. Угольницкий

Южный федеральный университет, Ростов-на-Дону, Россия

Рассмотрим производственную систему и агента, который, управляя параметрами производства, выбирает стратегию, максимизирующую его прибыль. Производственная система, представляет собой дерево D=(X,U), ориентированное в направлении корня, X-множество производственных процессов (вершины), U-множество потоков продукции между этими процессами (дуги). Агент может изменять параметры процессов таким образом, что поток продукции, определяемый как совокупность затрат и показателей качества, на выходе всей системы будет меняться. Пусть целевая функция агента имеет следующий вид:

$$p(q) - r \xrightarrow[(q,r)\in S]{} \max \qquad (1)$$

$q = (q_1, q_2, ..., q_m)$ – вектор показателей качества, $0 \le q_i \le 1, i = \overline{1..m}$

r – затраты на производство

$p(q)$ – доход от реализации продукции с показателями качества q

S – множество допустимых стратегий производства. Каждым элементом этого множества является пара (q, r), соответствующая такой стратегии производства, при которой достигается вектор показателей качества q при затратах r. При этом S является множеством, оптимальным по Парето, в том смысле, что лучшему значению показателя качества соответствуют большие затраты, то есть для любой пары стратегий:

$$(q_1, r_1) \in S, (q_2, r_2) \in S, (q_1 > q_2) \Rightarrow (r_1 > r_2)$$

$$(q_1 > q_2) \stackrel{\text{def}}{=} \forall i \in \{1, 2, ..., m\}\ (q_{1i} \geq q_{2i}),\ \exists j : (q_{1j} \geq q_{2j})$$

Проблема нахождения множества S для данной производственной сети представляет отдельный интерес и исследована в работе [3]

Условие гомеостаза означает, что некоторые из существенных показателей функционирования системы (в данном случае – показатели качества) должны принимать значения из заданных диапазонов. Условие допуска по показателю i запишем в следующем виде:

$$q_i \geq a_i,\ a_i \in [0, 1]. \tag{2}$$

a_i определяет минимально допустимое значение соответствующего показателя. Если же на показатель не наложено никаких ограничений, то соответствующее a_i можно считать равным нулю.

Перейдем к двухуровневой системе с двумя игроками: агентом и контроллером, который следит за выполнением условий гомеостаза, однако за взятку может ослаблять его.

Целевая функция агента g_2 – прибыль, которая является разностью между функцией дохода от новых показателей качества и суммой новых затрат на производство, выплаченной взятки и функцией штрафа за изменение стратегии. Она принимает вид:

$$q_2(a, b, q, r) = p(q) - r - b - d(q, q_T) \xrightarrow[b,(q,r)\in S, q_i \geq a(b)]{} \max \tag{3}$$

i – номер проверяемого контролером показателя

b – величина взятки контролеру

$a(b)$ – новая граница допуска по показателю i

q – новый вектор показателей качества

r – новые затраты на производство

S – множество допустимых стратегий

d – функция штрафа за переход от старой стратегии, обеспечивающей показатели качества q_T к новой стратегии.

Целевая функция контролера также соответствует его прибыли. Из нее исключена фиксированная зарплата и фактически она совпадает с размером взятки, выплачиваемой агентом.

$$q_1(a, b) = b \xrightarrow[a(b)]{} \max \tag{4}$$

Контролер управляет выбором a(b). На эту функцию накладываются следующие ограничения:

$$a(0) = a_0,\ q_T \le a(b) \le a_0 \tag{5}$$

Рассматриваемая игра является иерархической в том смысле, что первым ход делает контролер, сообщающий функцию отклика на взятку, затем ход делает агент, выбирая сумму взятки и значения управляемых параметров. Естественно, что контролер, делая первый ход, заинтересован в максимизации своего гарантированного выигрыша после выбора стратегии ведомым, благожелательность которого стоит под вопросом.

Введем функцию

$$u(a) = \max_{(q.r)\in S,\ q_i \ge a} (p(q) - r - d(q, q_T)) \tag{6}$$

Тогда задачу (3-5) можно записать в следующем виде:

$$\begin{array}{l} g_1(a,b) = b \underset{a(b)}{\longrightarrow} \max \\ g_2(a,b) = u(a) - b \underset{b}{\rightarrow} \max \\ b \ge 0,\ q_t \le a \le a_0,\ a(0) = a_0 \end{array} \tag{7}$$

Эта модель соответствует игре Гермейера Г2. В роли ведущего выступает контролер, в роли ведомого – агент. Используя теорему Гермейера, находим решение этой игры. Оптимальная стратегия ведущего:

$$a_\epsilon = \begin{cases} q_T & b = u(q_T) - u(a_0) - \epsilon \\ a_0 & b < u(q_T) - u(a_0) - \epsilon \end{cases} \tag{8}$$

Решением игры будет пара $(a_\epsilon(b), u(q_T) - u(a_0))$, а значения целевых функций при этом будут равны:

$$\begin{array}{l} g_1^* = u(q_T) - u(a_0) - \epsilon \\ g_2^* = u(a_0) + \epsilon \end{array} ,\epsilon > 0 \text{ сколь угодно мало} \tag{9}$$

Теперь рассмотрим трехуровневую иерархическую систему, в которую включается также центр, который назначает функции штрафа, как за дачу, так и за получение взятки. Целевые функции участников определим следующим образом (g_0 - целевая функция центра):

$$\begin{aligned} &g_0(a,b,c) = c + z(a) - l_1^*(a,b,c) - l_2^*(a,b,c) \xrightarrow[c,l_1,l_2]{} \min \\ &g_1(a,b) = b - l_1(a,b,c) \xrightarrow[a(b)]{} \max \\ &g_2(a,b) = u(a) - b) - l_2(a,b,c) \xrightarrow[b]{} \max \end{aligned} \qquad (10)$$

c – затраты центра на антикоррупционные меры

$z(a)$ – убытки центра от нарушения условия гомеостаза, $z(a_0) = 0$;

$l_1(a,b,c)$ – штраф, накладываемый на контролера

$l_2(a,b,c)$ – штраф, накладываемый на агента

$l_i(a_0, 0, c) = l_i(a, b, 0) = 0, \ i = 1, 2.$

l_1^*, l_2^*– функции, показывающие, какая часть штрафа имеет экономическую выгоду для центра.

Рассмотрим следующий вид функций штрафа:

$$l_i(a,b,c) = p(a,c)s_i(a,b), \ i \in \{1,2\}. \qquad (11)$$

Здесь $p(a,c)$ – вероятность обнаружения факта коррупции, $s_i(a,b)$ – величина штрафа, $s_i(a_0, 0) = 0, \ p(a,0) = 0$.

Были исследованы различные примеры этих функций, в том числе на основании функций штрафа, описанных в ст. 290, 291 УК РФ. Одним из результатов является тот факт, что в данной модели при некоторых значениях параметров центр экономически не заинтересован в полной победе над коррупцией, поскольку она влечет увеличение затрат на обнаружение нарушений и отсутствие штрафов.

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Corruption And Inspectors' Mistakes in Tax Control

S. Kumacheva

St.Petersburg State University, Saint-Petersburg, Russia

E-mail: s_kumach@mail.ru

A generalization of the game-theoretical model of tax control adjusted for possible corruption and inspectors' mistake is considered.

In the basis of this model there is a hierarchical game, constructed in [5]. The mentioned game has a three-level structure: at the highest level of the hierarchy is the administration of the tax authority, in the middle is an inspector, subordinated to the tax administration, who may turn out a bribetaker or make ineffective tax audit, and at the lowest level are n taxpayers. As in [1–7], it is supposed, that an interaction between risk-neutral players of different levels of a hierarchy corresponds to scheme "principal-to-agent".

The model is studied for the case when the penalty is proportional to the level of evasion, i.e., when the evasion is revealed, the k-th taxpayer must pay $(t+\pi)(i_k-r_k)$, where i_k and r_k are his true and declared incomes, $k=\overline{1,n}$, t and π are the tax and the penalty rates correspondingly.

The tax authority sends an inspector for the tax audit with the probability p_k, which costs c_k. For the bribe b_k audit inspector can agree not to inform his administration about the evasion revealed. With the probability $\tilde{p_k}$ the tax administration makes re-auditing of the taxpayer, which costs $\tilde{c_k}$. Both of the audits are supposed to be effective. If a result of re-auditing is the revelation of the tax evasion concealed by the inspector, the taxpayer must pay $(t+\pi)(i_k-r_k)$ (as earlier) and the inspector must pay a fine $F=f\cdot(i_k-r_k)$, where f is an inspecor's penalty coefficient. As in [4], it is supposed, that the fact of corruption is very difficult to reveal and an inspector is punished only for negligent audit.

For search of optimal strategies the condition of the evasion of the k-th taxpayer

$$p_k(t+\pi)(i_k - r_k) < t(i_k - r_k) \tag{1}$$

and the existence condition of mutually beneficial bribe

$$\tilde{p_k} f(i_k - r_k) < b_k < (1 - \tilde{p_k})(t+\pi)(i_k - r_k) \tag{2}$$

were obtained.

While analyzing (1) and (2), different situations, combining the facts of tax evasions and corruption or their absence (compliance or non-compliance with (1) and (2)) are considered.

In the case of ineffective auditing it is assumed that the tax inspector can mistake and miss an existing evasion with the probability μ. The variable μ can be considered as a part of negligent inspectors of their total number. As in previous case, the tax administration makes re-auditing with probability $\tilde{p_k}$, which depends on μ. The negligent inspector pays a fine F and the tax evader pays penalty.

For every case of tax control the players' profit functions and optimal strategies are found.

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Cloud computing ecosystem

Andrey Lukyanenko

Aalto University, Finland

E-mail: `andrey.lukyanenko@aalto.fi`

Abstract

The growth of the public computational clouds picks up speed recently. The operators provides new services and schemes to sell the capacity. While the market is still in a forming state, it can be already foreseen that the future market will have some equilibrium form, where all public cloud operators provide a service with given QoS/money, while private cloud operators select how many of the resources they should support locally, how many and from whom should they buy from the public infrastructure.

The future Internet is not possible without the cloud computational services. This computational factories provide economic benefits both for the clients and for the operators. One side sells the computational resources, while another consumes them. While currently the price for the quality of services (QoS) is selected individually by the cloud providers, it is reasonable to assume that the price will be formed by the market.

Here we assume that the customers are also private clouds, which means that the customers have a possibility to run own private servers to do the same task, they want to be performed on the public clouds. Though, the customers cannot predict exactly how many of the resources they will need each moment of time. As maintaining own service which is able to serve any pick in demand is rather expensive for the customers, at some moment of time they prefer to buy/rent the resources from the public clouds.

On the other hand, the public providers do not provide they same resource for sell, the public clouds may even be in different states: some of them overloaded with the requests, some of them remain idle for short period of time. Thus, the QoS and price of comparable resources may vary a lot depending on the public providers. The whole said, form an economic reason for selecting one provider over another or switching from one provider to another.

This work is dedicated to construction of model of aforementioned ecosystem of the cloud providers, the private clouds (customers) and even a third entity – brokers – agents, who may be in-between the former two entities.

Игровая модель приращения знаний

В.Д. Матвеенко, О.А. Нев

Национальный исследовательский университет
"Высшая школа экономики"
Санкт-Петербург, Россия

E-mail: nevolga@gmail.com

Работа посвящена построению и изучению модели приращения инновационных знаний в контексте влияния на долгосрочный экономический рост. В отличие от других подходов к моделированию инновационного роста [5, 6, 9, 7, 10], учитывается несовпадение интересов исследователей и государства, регулирующего инновационный сектор.

Следуя [2, 3, 8], рассматривается динамическая конечная игра с полной информацией, где игроками являются государство и исследователи. Государство делает ход первым, определяя стимулирующие политики по отношению к отдельным научным направлениям в соответствии со своими приоритетами. После того, как второй игрок – исследователи – сделал свой ход (определил траектории научных разработок), игроки получают свои выигрыши в соответствии с реализовавшейся ситуацией. Исходные научные идеи (проекты, разработки, темы) моделируются как вершины ориентированного, вполне взвешенного графа. Веса соответствующей дуги соответствуют полезностям, которые исследователь и государство ожидают получить при таком объединении идей. Под объединением идей понимается переход по дуге из одной вершины графа в другую. Рассматриваются возможные варианты поведения игроков: недальновидное поведение, когда игрок оптимизирует полезность только на данном шаге, не заботясь о том, чтобы долгосрочная полезность была максимальной; дальновидное поведение, когда игрок оптимизирует полезность на всем пути заданного горизонта; поведение, когда игрок существенно различает на данном шаге ожидаемые полезности перехода из текущей вершины.

Для изучения модели использовались методы динамического программирования и методы идемпотентной (тропической)математики (см., например, [1, 4]). Сформулированы и доказаны теоремы о различных типах поведения исследователей, а также получены численные результаты (подтверждающие теоретические) с помощью программной реализации построенной модели. В частности, доказано, что действия исследователя одни и те же, если он:

1. Совсем близорукий, т.е. использует недальновидную стратегию.

2. Использует дальновидную стратегию, но с достаточно малым коэффициентом дисконтирования.

3. Использует стратегию с сильно различающимися предпочтениями.

Получена численная оценка того, насколько малым должен быть коэффициент дисконтирования дальновидного исследователя, чтобы такие результаты сохранялись. Найдены условия, при которых различные исследователи, использующие недальновидные стратегии, будут выбирать одни и те же пути перехода из одной вершины в другую.

В качестве примера на реальных данных рассмотрена динамика такой области знаний как идемпотентая (тропическая) математика. Сбор информации основан на методике, позволяющей наиболее адекватно отобразить взаимосвязь различных подсекторов изучаемой области, а именно: использование цитирования в работах и исследованиях, посвященных выбранной тематике. На основе собранных данных построена математическая модель, отражающая данную область знаний, ее подобласти (подсектора) (Теория оптимизации (включая идемпотентный анализ), Динамические стохастические системы, Линейная алгебра, Графы и сети, Математическая экономика) и взаимосвязи между ними (силу взаимосвязи условно считаем пропорциональной цитируемости).

Модель показывает, каким образом будет вести себя исследователь для получения максимальной полезности, находясь в начальный момент в каждом из подсекторов соответственно.

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Evaluation of a probabilistic backfilling algorithm for job scheduling on a computing cluster

Natalia N. Nikitina

Institute of Applied Mathematical Research,
Karelian Research Centre RAS, Petrozavodsk, Russia

E-mail: nikitina@krc.karelia.ru

Currently there exist a variety of scheduling algorithms that are aimed to optimize utilization of computational resources by different criteria. Backfilling is a popular scheduling optimization technique that allows short jobs to run without waiting for their turn in the main queue provided that they do not delay other jobs. Such technique requires information about job runtimes which is often inexact. A possible alternation of the basic backfilling algorithm may be used when exact information is unavailable. Backfilling should be applied only if it does not delay the first queued job with high enough probability, or if error probability is low enough.

In this work we present a mathematical model of the problem and provide a closed-form expression for the error probability under condition of exponential distributions of the corresponding random variables. We describe the results of experiments on the simulation model of the computing cluster and make conclusions about the influence that job flow characteristics do have on efficiency of the algorithm. The resulting algorithm has been proved to increase efficiency of the basic scheduling algorithms

On a Discrete Model of the Cake Division Problem

Tatyana E. Nosalskaya

Zabaikalsky Institute of Railway Transport, Russia

E-mail: `tenosalskaya@gmail.com`

Keywords: *Time-sequential game, cake division, threshold strategy, discounting.*

The presented research covers a problem of some limited resource sharing between participants of negotiations. Players conclude an agreement on sharing of this resource, which is called a cake, by voting. The considered negotiation scheme is based on research of consecutive negotiations process with discrete time. Proposals are distributed under the law of Dirichlet with fixed parameters. At each stage of negotiations some cake division is offered to players, and each of players is informed only on a size of his piece, that is a payoff. The decision about acceptance of such division is made by a majority rule, considering the views of each participant of negotiations. In case of a negative decision there is discounting, and game moves to the following stage. In the given model recurrence relations for an evaluation of payoffs, which each player aspires maximize for itself, are received and the Nash equilibrium in a class of threshold strategy is found.

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Теория игр и мультиагентные системы

Д. А. Новиков

Институт проблем управления РАН, Москва, Россия

E-mail: novikov@ipu.ru

Традиционно в классических теоретико-игровых моделях [11] и/или в моделях принятия коллективных решений (группового, коллективного поведения [1, 8]) используется одно из двух предположений об «интеллектуальности» агентов. Либо считается, что агенты «бесконечно интеллектуальны», то есть вся существенная информация и принципы принятия другими агентами решений всем им известны, всем известно, что всем это известно и т. д. до бесконечности (так называемая концепция общего знания). Либо предполагается, что агенты «примитивны» – каждый агент в рамках своей информированности следует некоторой процедуре принятия индивидуальных решений и почти «не задумывается» над тем, что знают и как ведут себя остальные агенты. Первый подход является каноническим для теории игр, второй – для моделей коллективного поведения (см. обзор [5]). Но между двумя этими «крайностями» существует достаточно большое разнообразие возможных ситуаций. Например, агенты могут осуществлять рефлексию [7].

Начиная с момента зарождения и интенсивного развития искусственного интеллекта, считается, что увеличение когнитивных, вычислительных и других ресурсов кибернетических систем расширяет их возможности и повышает эффективность решения ими тех задач, для которых они создаются. Более того, исходя из вышесказанного, под интеллектуализацией («увеличением интеллектуальности») мультиагентных систем (МАС) можно, помимо приращения перечисленных видов ресурсов, условно понимать наделение первых такими свойствами, как способность к:

- адаптации;
- прогнозированию изменения состояний окружающей среды и поведения других агентов;
- дальновидности (учету будущих последствий принимаемых решений);
- не только целенаправленному поведению, но и самостоятельному целеполаганию;
- рефлексии;
- кооперативному и/или конкурентному взаимодействию

и т.д. То есть, имеет место тенденция к максимальной интеллектуализации в рамках имеющихся ресурсов – массо-габаритных, стоимостных, функциональных (например, требование функционирования в режиме реального времени) и других ограничений.

Однако существует ряд примеров, свидетельствующих о том, что не всегда рост «интеллектуальности» приводит к повышению эффективности функционирования МАС. В докладе рассматривается ряд примеров, в которых:
- увеличение рангов стратегической рефлексии агентов приводит к их «проигрышу» менее интеллектуальным агентам [2, 3, 7];
- увеличение рангов информационной рефлексии агентов приводит к хаотизации поведения МАС [6],
- наделение агентов способностью к адаптации не изменяет поведения МАС [4];
- увеличение дальновидности агентов приводит к снижению эффективности их функционирования и др.

Например, применительно к рефлексии оказывается, что существует (для каждой задачи в общем случае свой) так называемый максимальный целесообразный ранг рефлексии – такой, увеличение которого не дает агенту никакого «выигрыша» [3, 7]. Поэтому можно предположить, что интеллектуализация должна быть не максимальной, а рациональной, то есть адекватной тем задачам и ситуациям, которые решают и в которых функционируют агенты. Универсальных рецептов на сегодняшний день, к сожалению, не известно – в каждом конкретном случае приходится строить и исследовать соответствующую аналитическую (в лучшем случае) или имитационную (в худшем случае) модель коллективного поведения.

На сегодняшний день наблюдаются две «параллельные» но несовпадающие тенденции. С одной стороны, МАС [12] «движутся» в сторону усложнения алгоритмов взаимодействия агентов не только за счет решения ими задач распределенной оптимизации, но и наделения их собственными целями и стратегическим поведением (обучение, адаптация, рефлексия и т.п.). То есть, МАС стремятся использовать подходы и аппарат теории игр. С другой стороны в теории игр наблюдается тенденция к «децентрализации» – декмопозиции взаимодействия игроков, исследованию возможностей распределенной реализации тех или иных решений (не случайно за последнее десятилетие сформировалась «вычислительная теория игр» [10], «алгоритмическая теория игр» [9]). Но, к сожалению, движение МАС и теории игр «навстречу» происходит по параллельным путям – каждое научное направление развивается независимо, и ссылки на взаимные результаты очень редки.

Современный уровень исследований аналитических моделей поведения группы взаимодействующих интеллектуальных агентов (технических или программных) таков, что пока не существует универсального «аппарата» их описания и исследования – все успехи ограничены набором частных и достаточно простых моделей. Видятся два направления возможных будущих прорывов. Первое – экспериментальные исследования принятия людьми решений и поиск общих закономерностей на основе анализа результатов экспериментов (с последующим переносом на группы искусственных агентов). Второе направление (теоретическое) – разработка языка описания моделей, позволяющего достаточно просто и единообразно ставить и решать различные задачи группового управления игровым взаимодействием интеллектуальных агентов.

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Dynamic Scenarios of Data Transmission in Wireless Networks

Elena M. Parilina

Saint-Petersburg State University
Saint-Petersburg, Russia

E-mail: elena.parilina@gmail.com

Theoretical game methods are often used in the modeling of joint behavior of telecommunication devices in wireless network data transmission. Devices in wireless networks affect each other and can conflict. Arising conflicts can be described as games.

We consider several types of simple interactions between devices in wireless networks in static form and then extend these models to the dynamic case. So, we consider repeated games arising in the data transmission modeling.

The first game is the so-called Forwarder's Dilemma (Prisoner's Dilemma) in which two devices (players) want their packets to be sent to destinations 1 and 2 respectively. It is assumed that communication between a player and his destination is possible only through the other player. The game is symmetric. A player receives a payoff of 1 minus the transportation cost if he forwards the packet of the other player. The dilemma is that the player receives more if he does not forward the other player's packet and at the same time the other player forwards the first player's packet. But if both players decide not to forward packets of each other, they receive less than they could if they forward packets of each other.

The second game which can arise in wireless data transmission process is called Joint Packet Forwarding Game. Here a source wants a packet to be sent to the destination but it is possible only if two devices (players) forward the packet. If they forward, then each of them receives payoff of 1 minus transportation cost.

The third scenario supposes that there are two devices (players) who want to access the same communication channel to send their packets to their receivers. Then the channel can send the received packet to the destination. The problem is that the channel can accept only one packet at each time slot. So, the transmission is possible if one player sends a packet and the other one waits at this time slot. This scenario can be modeled as a game which is called Multiple Access Game. There are other types of scenarios of telecommunication device interactions. Some of them model the antagonistic players' interests (or attacks) in data transmission.

We consider the dynamic scheme of these games. The dynamics means the repetition of one of the games mentioned above. Non-cooperative and cooperative strategies are found for these games. Some numerical examples illustrate the results of the work.

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Пространственная конкуренция на рынке товаров двух видов

А. В. Щипцова

ИПМИ КарНЦ РАН,
Петрозаводск, Россия

E-mail: anna.shchiptsova1@gmail.com

Классическая модель дуополии Хотеллинга описывает конкурентное поведение участников рынка – продавцов одного товара. Работа Хотеллинга послужила началом для целого ряда исследований, посвященных анализу конкурентного поведения в условиях, когда на величину потребительского спроса влияет цена товара и транспортные издержки.

В данной работе рассматривается рынок двух товаров на плоскости. Спрос зависит от цены и квадратичных транспортных затрат потребителя. Каждый покупатель заинтересован в приобретении двух различных товаров. Исследуется конкурентное поведение игроков, продающих товар одного вида. На рынке также присутствует продавец второго товара.

Найдены условия, которым удовлетворяет равновесие по Нэшу в задаче о размещении при плотности потребителей, заданной произвольной функцией. При различных заданных видах функции плотности найдены равновесные местоположения и цены, а также значения для выигрышей игроков.

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Cooperation in Stochastic Network Games

Artem Sedakov, Leon Petrosyan

Saint-Petersburg State University,
Saint-Petersburg, Russia

E-mail: a.sedakov@yahoo.com, spbuoasis7@peterlink.ru

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An n-person finite multistage stochastic network game is considered. Coordination of players is defined in terms of networks, in which players are identified as nodes, and mutual agreements represent links. Network formation mechanism is proposed.

Let $N = \{1, \dots, n\}$ be a finite set of players, and g be a network consisting from n nodes, and connecting players from the set N. By g^N we denote a set of all possible networks.

An n-person stage game is denoted by $G = \langle N, \{N_i\}_{i \in N}, \{G_i\}_{i \in N}, \{u_i\}_{i \in N} \rangle$, and a set of all such games is denoted by Γ. Players $i \in N$ form a network in this stage game, and a set N_i consists of players to whom i may offer a link. Let $g_i = (g_{i1}, \dots, g_{in})$ be a strategy of $i \in N$. Here

$$g_{ij} = \begin{cases} 1, & \text{iff } i \text{ wants to form a mutual link with } j \in N_i \setminus i, \\ 0, & \text{otherwise or if } j = i. \end{cases}$$

A set of all strategies of i in stage game G is denoted by G_i. Suppose that link (i, j) is formed by mutual agreement of both players i and j (when $g_{ij} = g_{ji} = 1$).

In stage game G players $i \in N$ simultaneously choose their strategies $g_i \in G_i$, then a network g is formed, and players payoffs in stage game G are defined as $u_i : g^N \to R$, $i \in N$, where

$$u_i(g) = \sum_{j \in N \setminus i : (i,j) \in g} f_i(i, j).$$

Here $f_i(i, j)$ represents utility of player i from link $(i, j) \in g$. After that the game process moves to the next stage game in accordance with an a priori given transition probabilities, i. e. stage game G moves to another stage game $\Phi(g) \in \Gamma$ subject to a probability distribution $\{p(g) : p(g) \geq 0, \sum_{\Phi(g) \in \Gamma} p(g) = 1\}$, where g is the network which is realized in stage game G, and a single-valued mapping $\Phi : g^N \to \Gamma$ is given. After a finite number of stages, the game process stops.

Let a sequence of networks $\{g\}$ in stage games $\{G\} \in \Gamma$ be realized which we call game trajectory. Players (expected) payoffs $\mathrm{E}_i(g)$ along this trajectory are defined by the following recurrent equality:

$$\mathrm{E}_i(g) = u_i(g) + \sum_{\Phi(g) \in \Gamma} p(g) \mathrm{E}_i(g'), \quad i \in N,$$

where g is the network which is realized in stage game $G \in \{G\}$, and g' is the network which is realized in stage game $\Phi(g) \in \Gamma$.

A method of finding an optimal and time-consistent solution in the multistage stochastic network game is proposed. Results are illustrated by example.

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The problem of big delay service under choice of different pricing schemes

Anna Sergeeva

Saint-Petersburg State University,
Saint-Petersburg, Russia

E-mail: vlb310154@gmail.com

We consider the cases of different policies of customer order fulfillment schemes in company which provide some kind service for customers. The game-theoretic model of choosing order service is constructed.

We consider company which services to build customer orders and provides various ways to make orders. Customers, in turn, refer to the company for the service, while trying to minimize the total cost of implementing the order. Each ordering device has its own scheme of service: the first device serves all customers in a queue and takes a fixed cost for customer order fulfillment, the second device serves all clients at ones but it takes a fixed cost for customer order fulfillment and also a cost for unit service time and the third device serves all customers in a queue and takes only cost for unit service time.

Except cost of order fulfillment customers also bear costs for waiting time as a costs for missed opportunities and penalty for late. We assume that after a certain amount of time T customers pay heavy fine R for having to delay. Customers choose the ordering scheme in company trying to minimize its operational costs.

The model of n-person game with perfect information is suggested.

Define the non-antagonistic game in normal form: $\Gamma =< N, \{p_i^j\}_{i\in N}, \{H_i\}_{i\in N} >$, where

$N = \{1, \dots, n\}$ - set of players,
$\{p_i^{(j)}\}_{i \in N}$ - set of strategies, $p_i^{(j)} \in [0, 1]$, $j = 1, 2, 3$,
$\{H_i\}_{i \in N}$ - set of payoff functions.

$$H_i = -(p_i^{(1)} Q_{1i} + (1 - p_i^{(1)} - p_i^{(3)}) Q_{2i} + p_i^{(3)} Q_{3i}) =$$

$$= -(p_i^{(1)}(Q_{1i} - Q_{2i}) + p_i^{(3)}(Q_{3i} - Q_{2i}) + Q_{2i}),$$

where $p_i^{(1)}$ is the probability of player i choose service scheme 1, $p_i^{(3)}$ - is the probability of player i choose service scheme 3, $p_i^{(2)} = 1 - p_i^{(1)} - p_i^{(3)}$ - is the probability of player i choose service scheme 2.

Let c_{j1} - fixed cost of customer order fulfillment and c_{j2} - cost of - unit service time, $\tau_i^{(j1)}$ - time of waiting service, $\tau_i^{(j2)}$ - time of service for the device j, $j = 1, 2, 3$ and player i, $i = 1, \dots, n$. For the first and second devices we have $\tau_i^{(1)} = \tau_i^{(11)} + \tau_i^{(12)}$ and $\tau_i^{(3)} = \tau_i^{(31)} + \tau_i^{(32)}$ respectively, but for the second device $\tau_i^{(22)} = 0$ because we don't have a queue in the second device, so we have $\tau_i^{(2)} = \tau_i^{(22)}$. Duration of the customer service by the device 1 , 2 and 3 are independent random variables with densities functions:

$$f_1(t) = \frac{1}{\mu_1} e^{-\frac{1}{\mu_1} t}, \quad t > 0,$$

$$f_2(t) = \frac{1}{\mu_2} e^{-\frac{1}{\mu_2} t}, \quad t > 0,$$

$$f_3(t) = \frac{1}{\mu_3} e^{-\frac{1}{\mu_3} t}, \quad t > 0.$$

Also define customer specific loss of waiting service r_i for player i and indicator

$$I\{t_i^{(j)}, T_j\} = \begin{cases} 1, & \text{if } t_i^{(j)} \leq T_j, \\ 0, & \text{if } t_i^{(j)} < T_j, \end{cases}$$

which define the time when customer begin to loose an amount R_j by waiting service, $i = 1, \dots, n$ $j = 1, 2, 3$. (For some period of time customer prefer to wait the service and after this it begin to loose.)

Q_{1i}, Q_{2i}, Q_{3i} - player i expected loss for 1'st, 2'nd and 3'rd service scheme respectively where $Q_{1i} = E(\tilde{Q}_{1i}), Q_{2i} = E(\tilde{Q}_{2i}), Q_{3i} = E(\tilde{Q}_{3i})$. So we can define

$$Q_{1i} = E(\tilde{Q}_{1i}) = E(r_i(\tau_i^{(11)} + \tau_i^{(12)}) + R_1 I\{t_i^{(1)}, T_1\} + c_1),$$

$$Q_{2i} = E(\tilde{Q}_{2i}) = E((r_i + c_2 2)\tau_i^{(22)} + R_2 I\{t_i^{(2)}, T_2\} + c_{21}),$$

$$Q_{3i} = E(\tilde{Q}_{3i}) = E((r_i + c_{32})\tau_i^{(32)} + R_3 I\{t_i^{(3)}, T_3\}), i = 1, \ldots, n.$$

We consider the casualty functions below: $h_i = -H_i$, $i = 1, \ldots, n$.

Customers choose order fulfillment schemes trying to minimize expected losses. So to find the optimal behavior of customers we should find mean value of $Q_{ji}, i = 1, \ldots, n$ $j = 1, 2, 3$.

The equilibrium strategies for clients of company with different cases of ordering schemes is found. The existence of these equilibria is proved.

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Теоретико игровая транспортная задача на сети с заданными пропускными способностями

Илья Алексеевич Серяков

Санкт-Петербургский государственный университет,
Санкт-Петербург, Россия

E-mail: ilya.seryakov@gmail.com

Нечетный вариант.

Для производства трёх видов продукции используются три вида сырья. Нормы затрат каждого из видов сырья на единицу продукции данного вида, запасы сырья, а также прибыль с единицы продукции приведены в таблицах вариантов. Определить план выпуска продукции для получения максимальной прибыли при заданном дополнительном ограничении

Требуется:

1. Построить математическую модель задачи;
2. Привести задачу к канонической форме;
3. Решить задачу симплекс-методом.

Четный вариант.

Из двух видов сырья необходимо составить смесь, в состав которой должно входить не менее указанных единиц химического вещества A, B и C соответственно. Цена 1 кг сырья каждого вида, а также количество едениц химического вещества, содержащегося в 1 кг сырья каждого вида, указаны в таблицах вариантов. Составить смесь, имеющую минимальную стоимость.

Требуется:

1. Построить математическую модель задачи;
2. Привести задачу к канонической форме;
3. Решить задачу **двойственным** симплекс-методом.

Coalitional Stability in Management Problems

Anna N. Rettieva

Institute of Applied Mathematical Research,
Karelian Research Centre RAS, Petrozavodsk, Russia

E-mail: annaret@krc.karelia.ru

A discrete-time game model related to a bioresource management problem is considered. The reservoir is divided into regions, where the players of two types harvest the fish stock. There are migratory exchanges between the regions of the reservoir.

We consider the coalition structure where players of each type can form a coalition, and show that it is more profitable for them than to join into one mixed coalition. Two forms of the coalition formation process are investigated: Nash-Cournot and Stackelberg. It is demonstrated that the first model is better for free-riding and the second one – for coalition formation.

The main goal of this work is to investigate the stability of coalition structure. We consider well-known concepts of external and internal stability and introduce another one – coalitional stability, which gives the possibility to form coalitions of larger sizes.

We discovered that only a small sized coalition can be internally stable (Nash-Cournot strategies) and there can be no externally stable coalitions at all (Stackelberg strategies). We introduce the concept of coalition stability under which the coalition structure of large size can be stable. This concept is an extension of the intercoalition stability (Carraro, 1997) to the models with more than one coalition and possible moves of a set of coalition members.

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Evaluation of Market Power in Local and Two-node Markets[1]

Alexander A. Vasin

Lomonosov Moscow State University, Moscow, Russia

E-mail: vasin@cs.msu.su

Yulia Sosina

Lomonosov Moscow State University, Moscow, Russia

Gerhard-Wilhelm Weber

Middle East Technical University, Turkey

E-mail: gweber@metu.edu.tr

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This study obtains an evaluation of Cournot price deviation from Walrasian price for a market of a homogenous good. The deviation characterizes the market power of producers. The evaluation proceeds from available information on the market structure and the market demand. It relates to such markets where Nash equilibrium (NE) behavior of agents corresponds to the Cournot equilibrium outcome (see Kreps and Scheinkman (1983)). Vasin and Vasina (2005) show that this is a typical case for uniform price auctions. We also discuss a generalization of the result for a two-node network market.

Consider a market with a finite set A of producers. Each producer a is characterized by his cost function $C^a(q)$ with non-decreasing marginal

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costs for $q \in [0, Q^a]$, where Q^a is his production capacity. The precise form of $C^a(q)$ is his private information. Consumers have no market power, and their behavior is characterized by a demand function $D(p)$ with standard properties.

The combination $(\tilde{q}^a, a \in A)$ of production volumes is a *Walrasian equilibrium* (WE) and $\tilde{p}$ is a *Walrasian price* of the market if, for any a, $\tilde{q}^a \in S^a(\tilde{p}) \stackrel{\text{def}}{=} \text{Argmax}_{q^a}(q^a\tilde{p} - C^a(q^a))$, $\sum_{a\in A} \tilde{q}^a = D(\tilde{p})$. Consider a model of Cournot competition for the given market. Then a strategy of each producer a is his production volume $q^a \in [0, Q^a]$. Producers set these values simultaneously. Let $\vec{q} = (q^a, a \in A)$ denote a strategy combination. The market price $p(\vec{q})$ equalizes the demand with the actual supply: $p(\vec{q}) = D^{-1}(\sum_{a\in A} q^a)$. The payoff function of producer a determines his profit $f^a(\vec{q}) = q^a p(\vec{q}) - C^a(q^a)$.

Let $(q^{a*}, a \in A)$ denote NE production volumes and $p^* = D^{-1}(\sum_{a\in A} q^{a*})$ be the corresponding price for the Cournot competition. They meet the following first-order condition:

$$q^{a*} \in (p^* - C^{a\prime}(q^{a*}))|D'(p^*)|, \text{ for any } a \text{ s.t. } C^{a\prime}(0) < p^*,$$

$$q^{a*} = 0 \text{ if } C^{a\prime}(0) \geqslant p^*,$$

where A is the set of producers and $C^{a\prime}(q) = [C^{a\prime}_{-}(q), C^{a\prime}_{+}(q)]$ at the break points (discontinuities) of the marginal cost function.

The well-known Lerner Index characterizing the price-cost mark-up for firm a is then given by $L^a = (p^* - C^{a\prime}(q^{a*}))/p^* = s^{a*}/e(p^*)$, where $s^{a*} = q^{a*}/D(p^*)$ is the equilibrium market share of firm a, and $e(p^*)$ is the elasticity of market demand at p^*.

Proposition 1 *Deviation of the Cournot price from the Walrasian price meets inequality*

$$\frac{p^* - \tilde{p}}{p^*} \leqslant \max_{a\in A} \frac{s^{a*}}{e(p^*)}. \tag{1}$$

This estimate becomes precise if the marginal cost of the largest company at the Cournot equilibrium is equal to the competitive equilibrium price $\tilde{p}$.

In particular, this is true for a symmetric oligopoly with a fixed marginal cost. Another example that provides the same ratio between Cournot and Walrasian prices is a market where a large firm with a fixed marginal cost interacts with the competitive fringe with lower costs and a limited capacity (Vasin and Vasina (2006)).

Consider two local markets connected by a transmitting line. Every local market $l = 1, 2$ is characterized by the finite set A_l of producers, $|A_l| = n_l$, the cost functions $C^a(q), a \in A_l$, and demand function $D_l(p)$, in the same way as the local market above: each cost function is a private information of agent a, the demand function and other market parameters are a common knowledge. There are no losses or costs under transmission from one market to the other, and transmission capacity Q determines the maximal amount of the transmitted good. Consider Cournot competition in this model.

Vasin and Vasina (2006) describe three types of NE for this model: type b with unbinding transmission capacity constraint, type c_{12} with the binding constraint and the flow of the good from market 1 to market 2, and a symmetric type c_{21}. For type b, the first-order conditions of Cournot equilibrium and an estimate of the Cournot price deviation from the Walrasian price are quite similar to the given above for a local market. For type c_{12}, the first order conditions and Lerner indeces are as follows: $q^{a*} \in (p_l^* - C^{a\prime}(q^{a*}))\,|D_l'(p_l^*)|$, $L^a = (p_l^* - C^{a\prime}(q^{a*}))/p_l^* = s^{a*}/e_l(p_l^*)$ for any $a \in A_l$ s.t. $C^{a\prime}(0) < p_l^*$, $l = 1, 2$, where $s^{a*} = q^{a*}/D_l(p_l^*)$ is the equilibrium market share of firm a in the demand at market l. Of course, the relationships hold only if the ratio is less than 1. In this case the estimate similar to (1) holds for each submarket. Otherwise the Cournot equilibrium with such characteristics does not exist.

There exists one more type of NE that usually realizes in the latter case. The type d_{12} meets the following conditions: the flow from node 1 to node 2 is is equal to transmission capacity Q, $p_1^* = p_2^* = p^*$. For every $a \in A_1$ (the exporting market)

$$(p^* - {C^{a\prime}}_-(q^{a*}))\,|D_1'(p^*) + D_2'(p^*)| \geqslant q^{a*} \geqslant (p^* - {C^{a\prime}}_+)\,|D_1'(p^*)|\,.$$

So, if $q^a \uparrow$, than $p_1 \downarrow$, the constraint becomes binding, and this is unprofitable since the latter inequality holds; if $q^a \downarrow$, than $p_1 \uparrow$, the markets join, and the relation is unprofitable since the former inequality holds. For second (importing) market the first order conditions are similar:

$$(p^* - {C^{a\prime}}_-(q^{a*}))\,|D_2'(p^*)| > q^{a*} > (p^* - {C^{a\prime}}_+)\,|D_1'(p^*) + D_2'(p^*)|\,, \forall a \in A_2.$$

A typical example where such NE exists is where there is a limited efficient capacity and perfect competition at the second market and a monopoly with unbounded efficient capacity at the first market. The Lerner index for the monopoly is $(\tilde{p}_2 - c_1)/\tilde{p}_2$, where $\tilde{p}_2$ – is the Walrasian price at the second market. The welfare loss may be the whole consumer surplus at the first market.

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