RuFiDiM

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Approximation of reset thresholds with greedy algorithms

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Definitions

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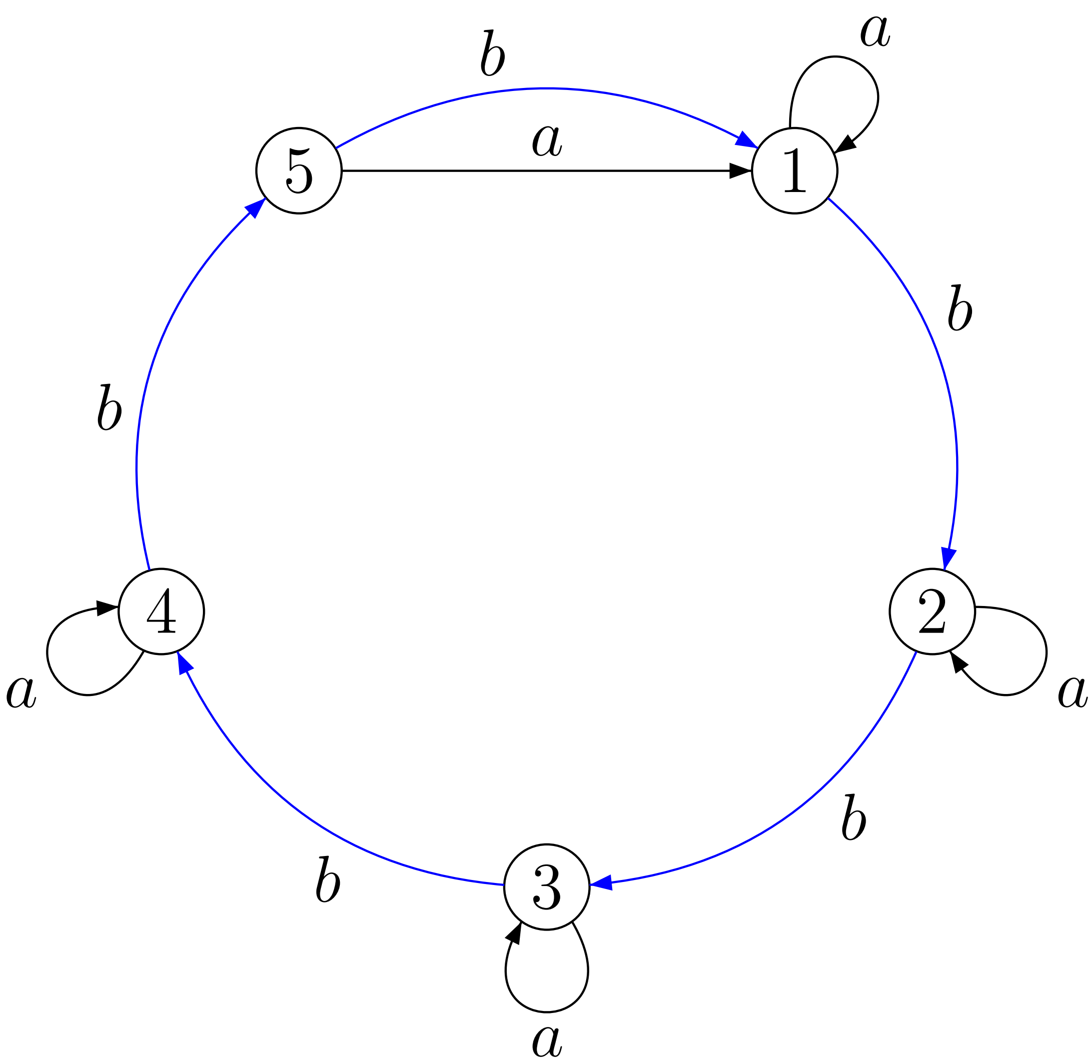
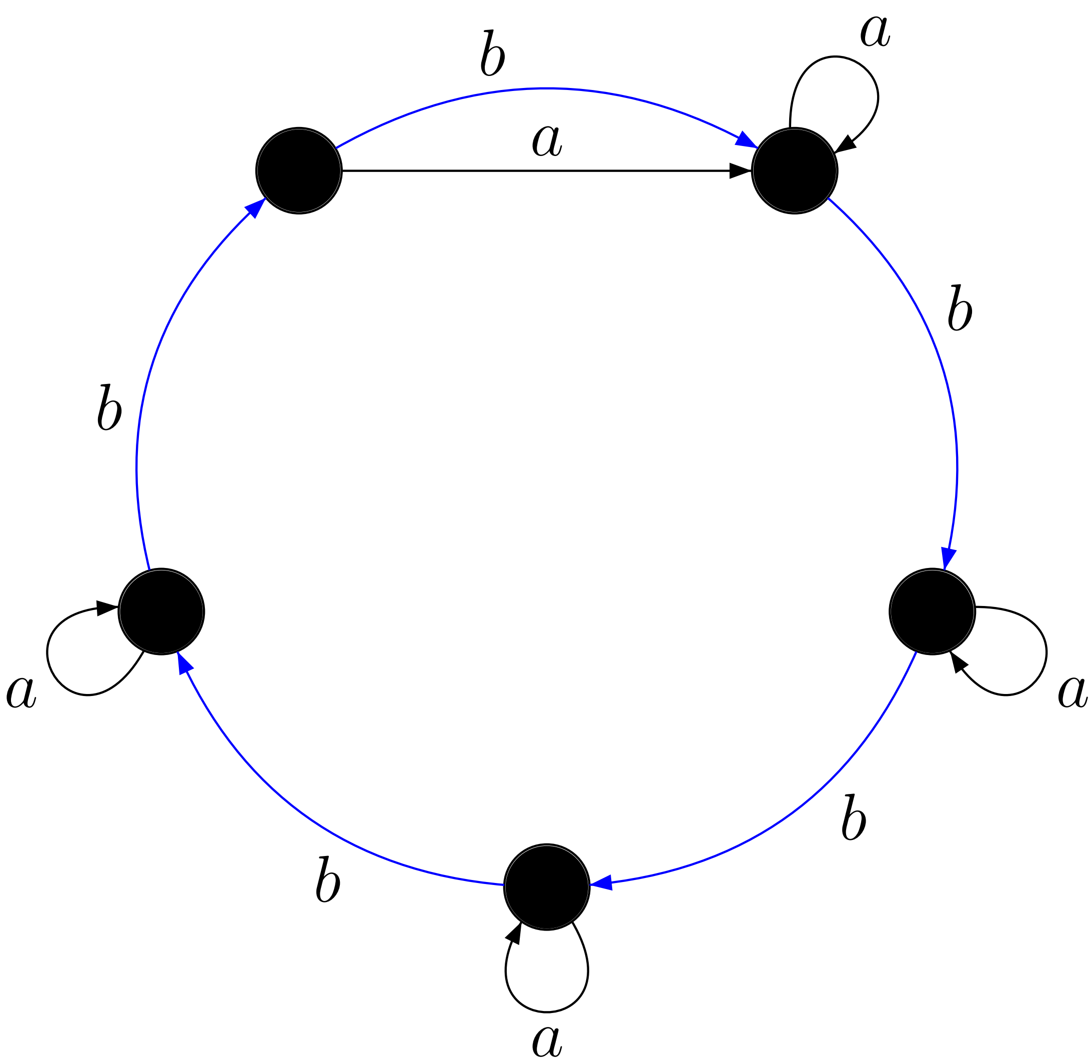
The length of the shortest synchronizing word is called *reset threshold* of $\mathscr{A}$.

reset word



$abbbbabbbbabbbba$

reset word



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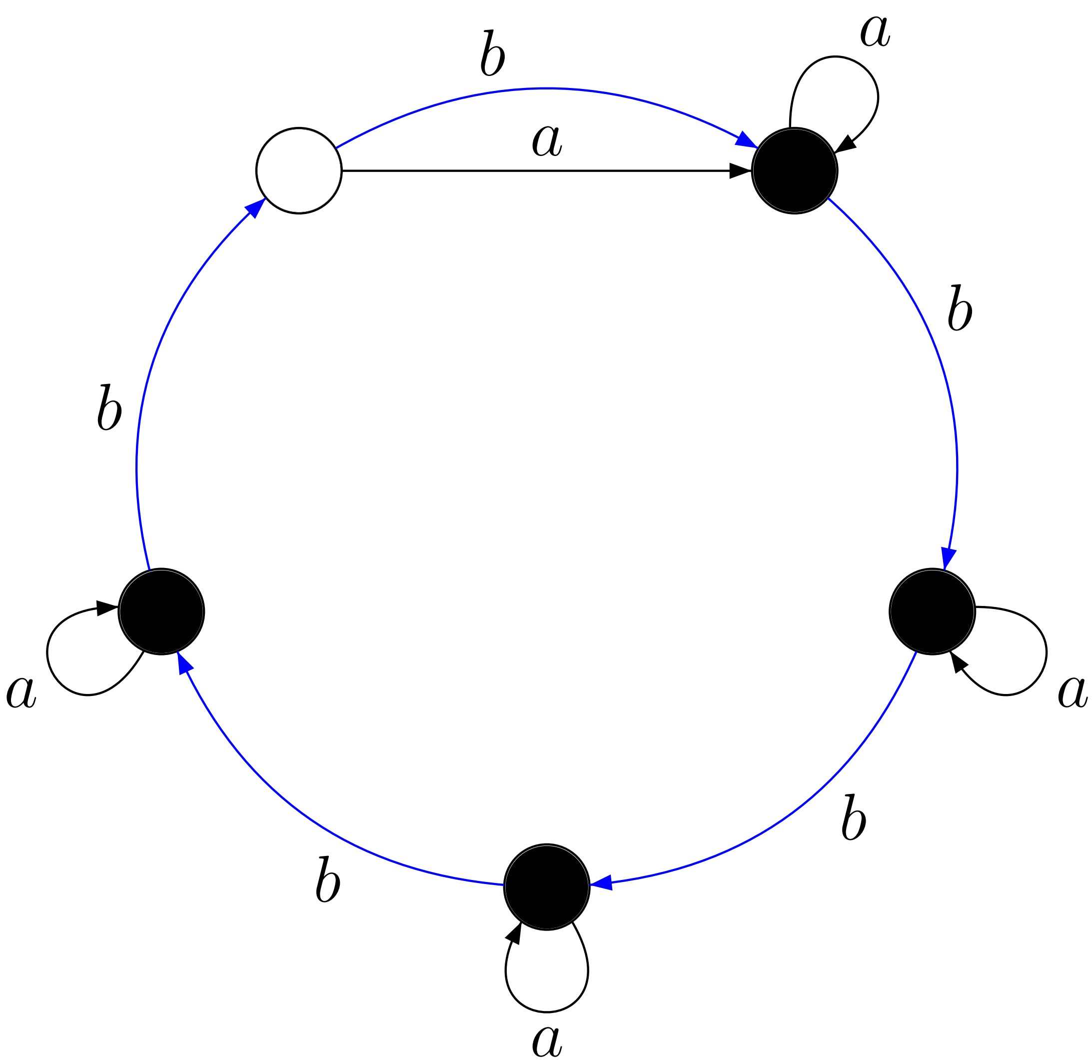

$\color{red}{a}bbbbabbbbabbbba$

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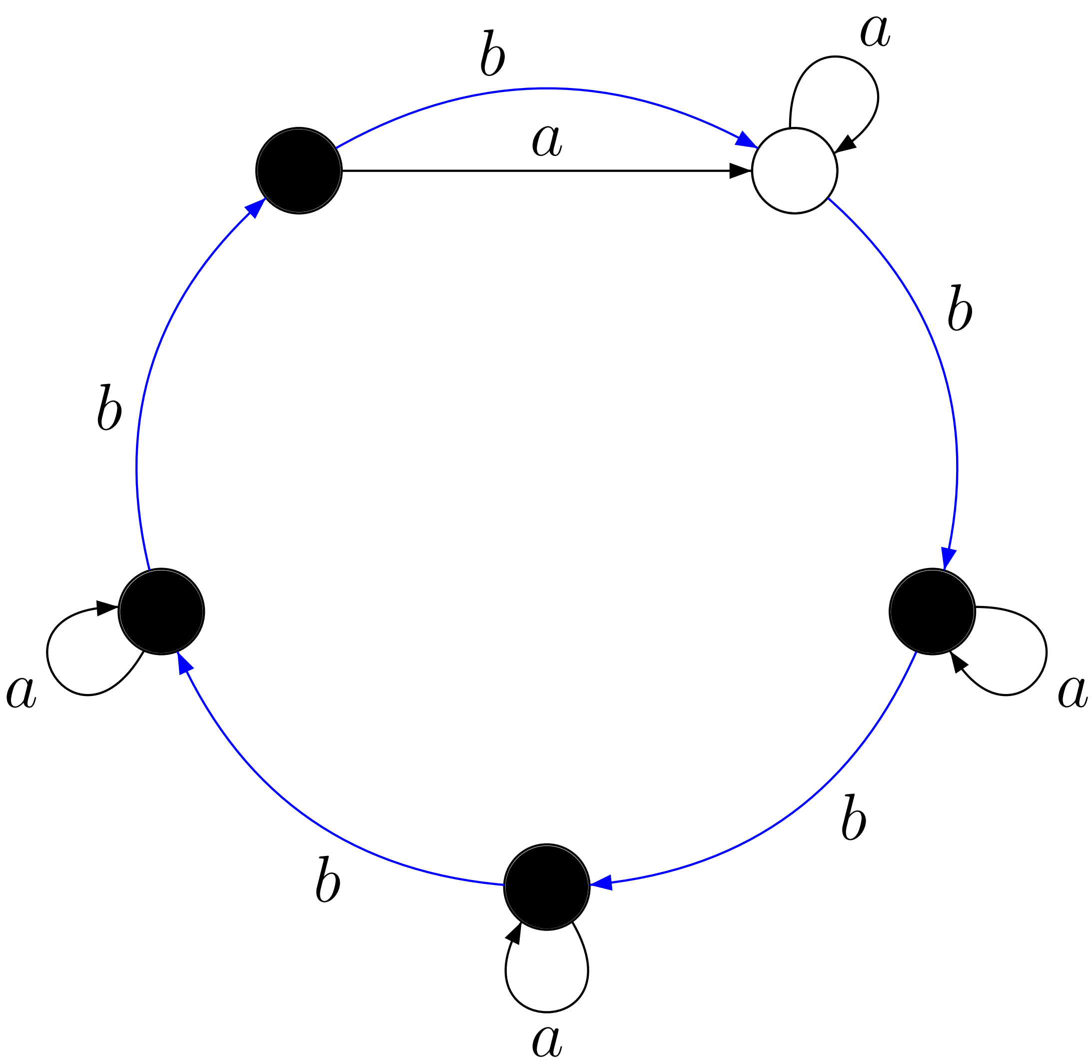

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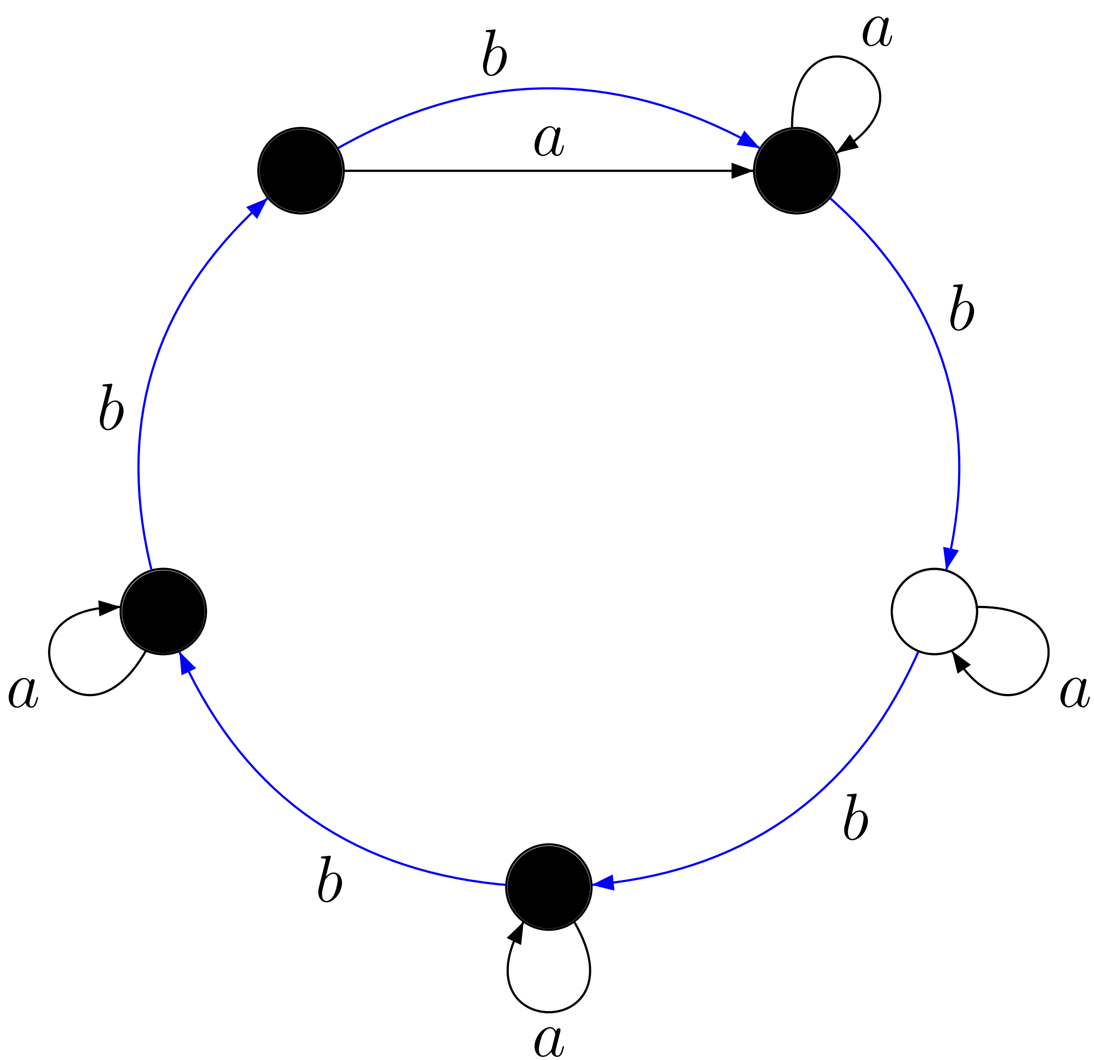

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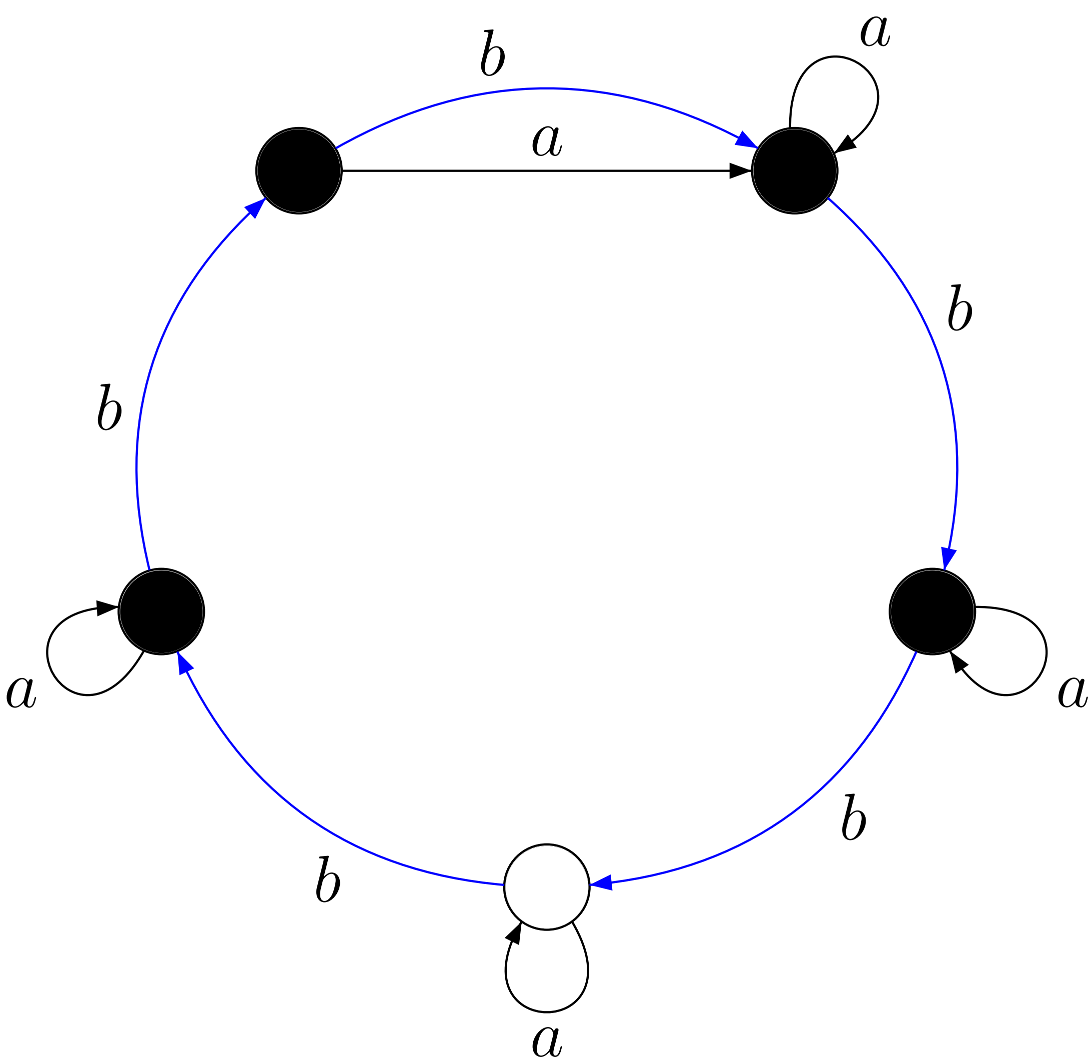

$abb\color{red}{b}\color{black}{babbbbabbbba}$

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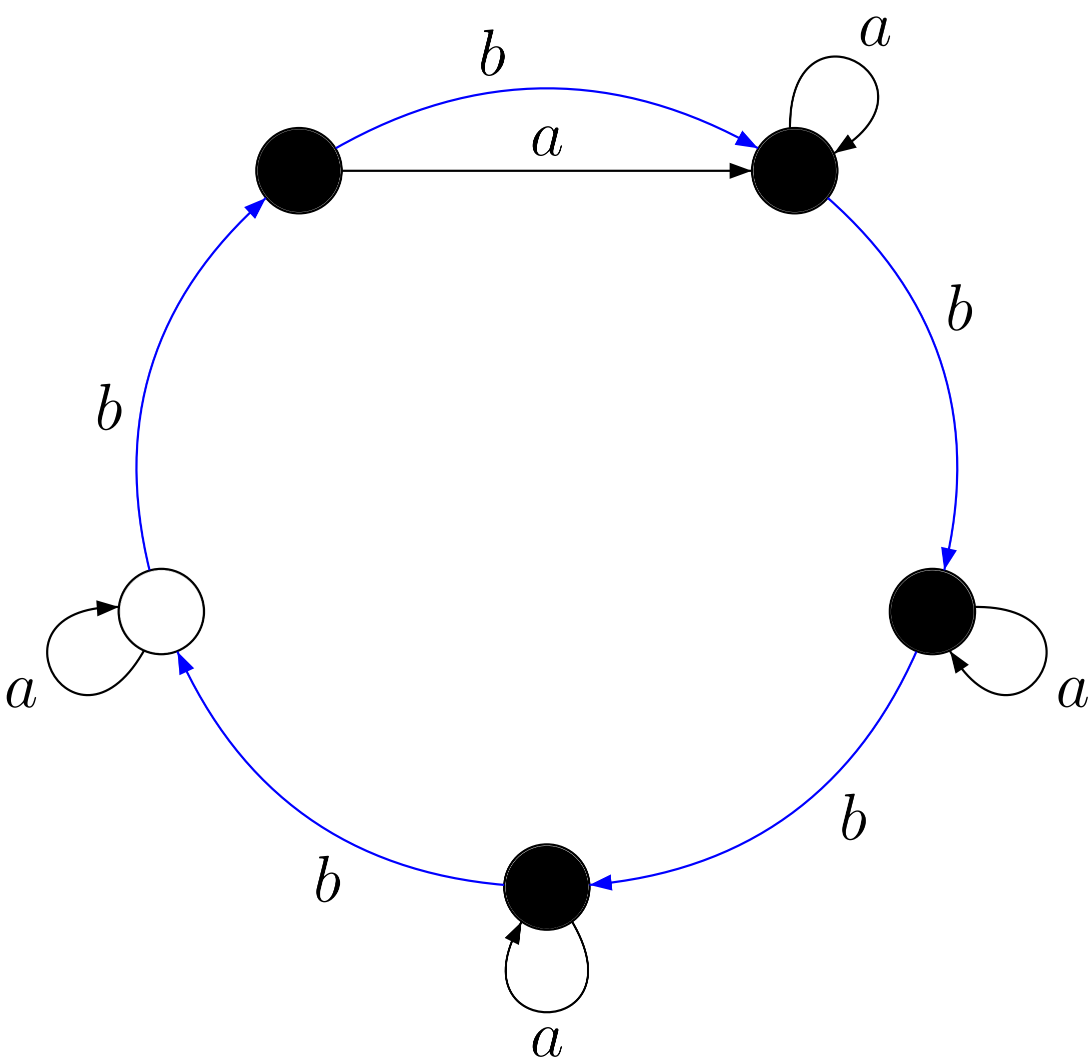

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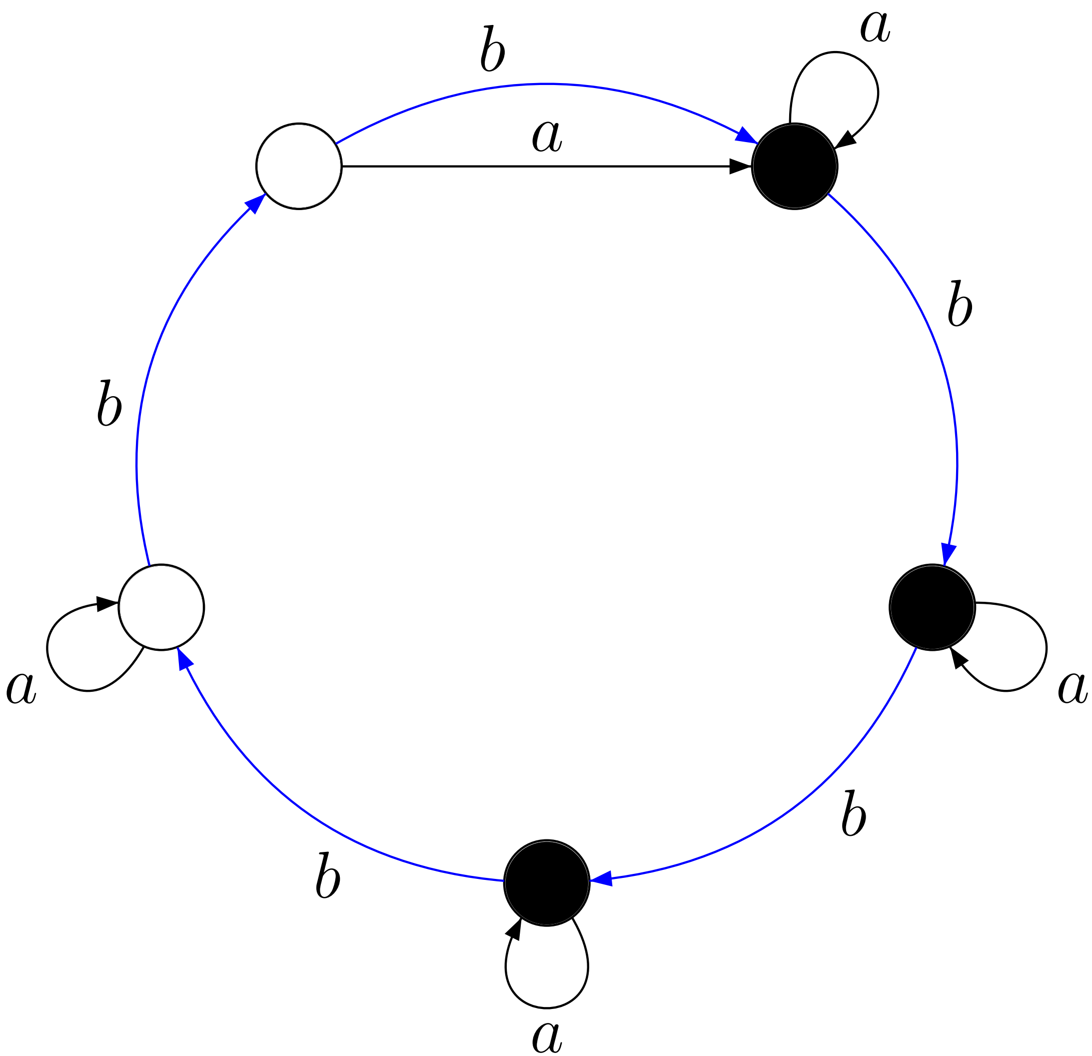

$ab bbbabbbbabbbba$

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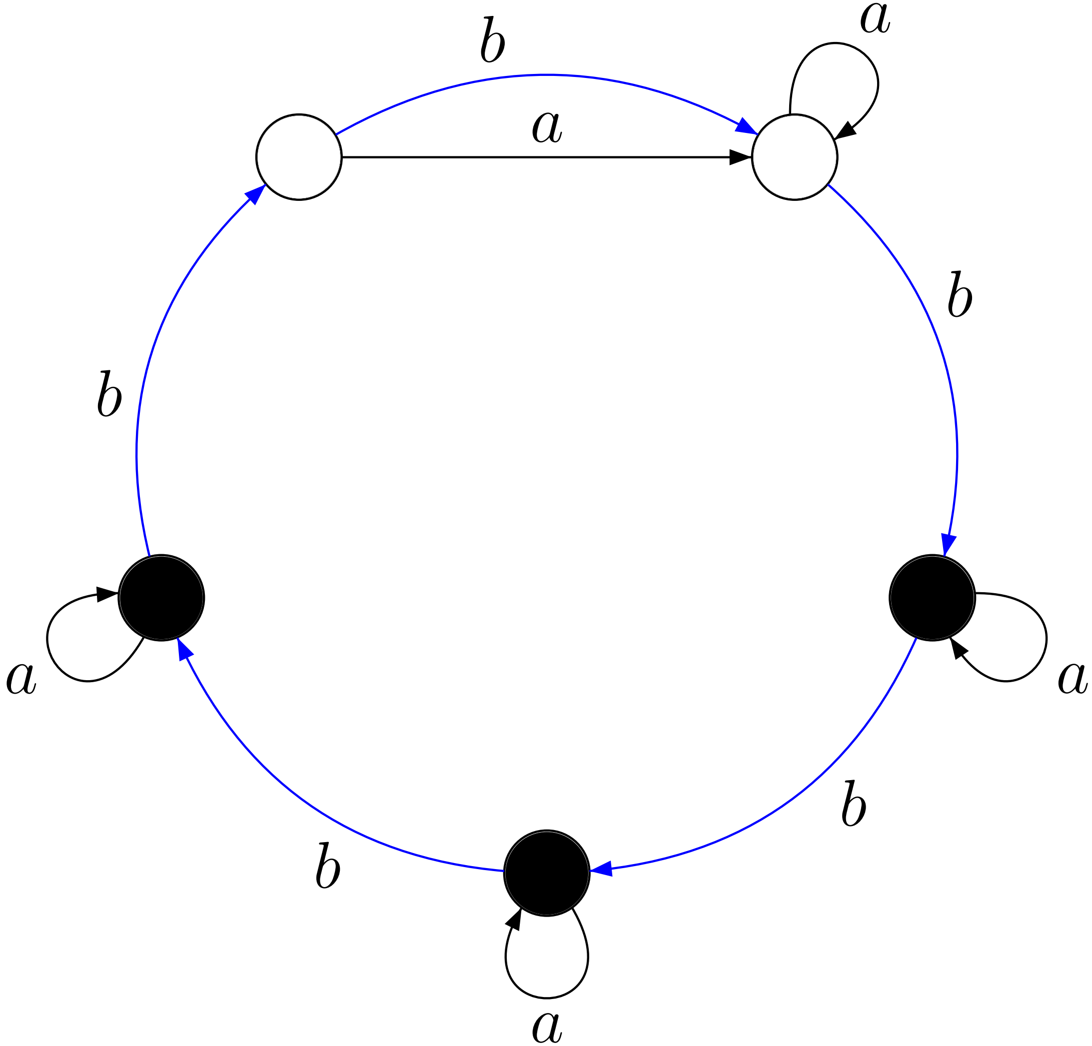

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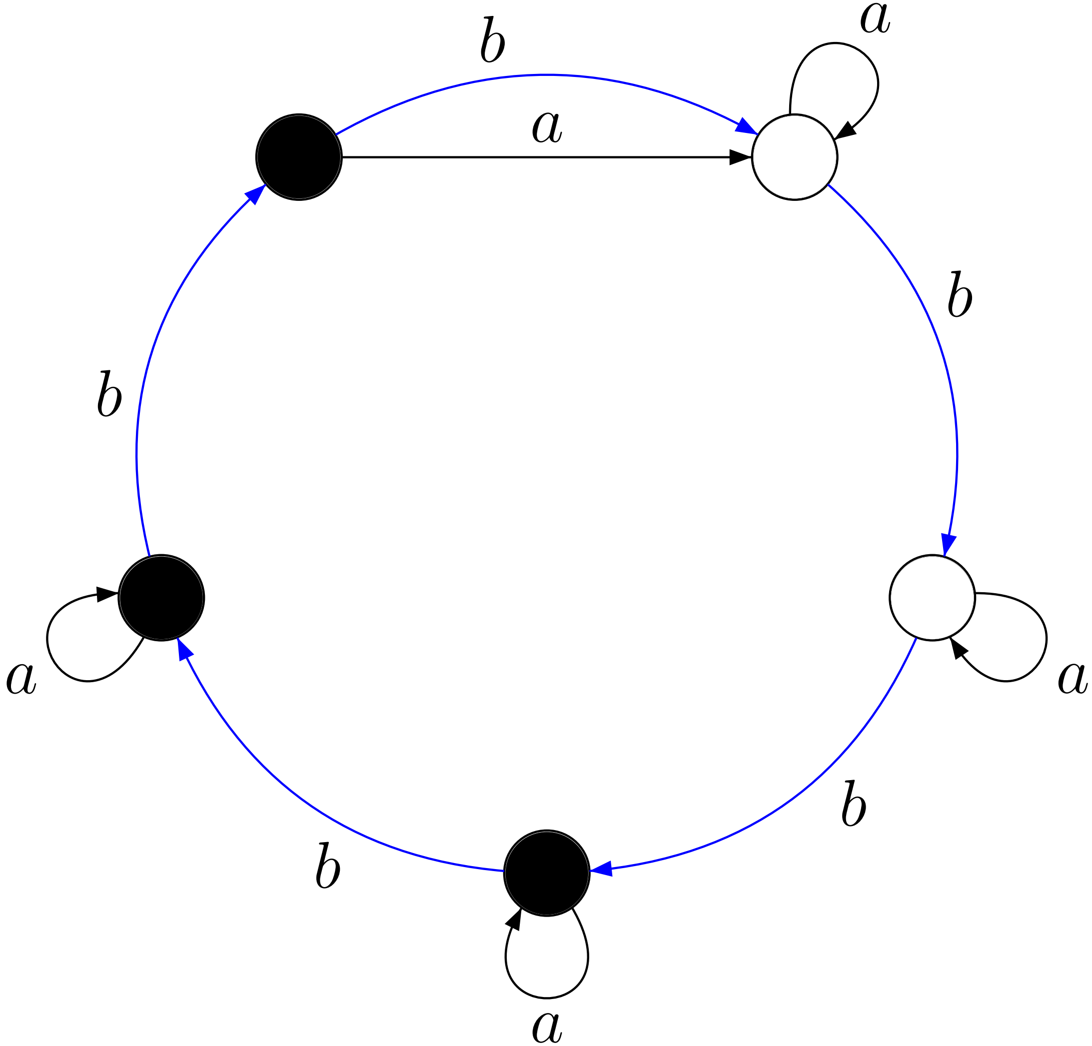

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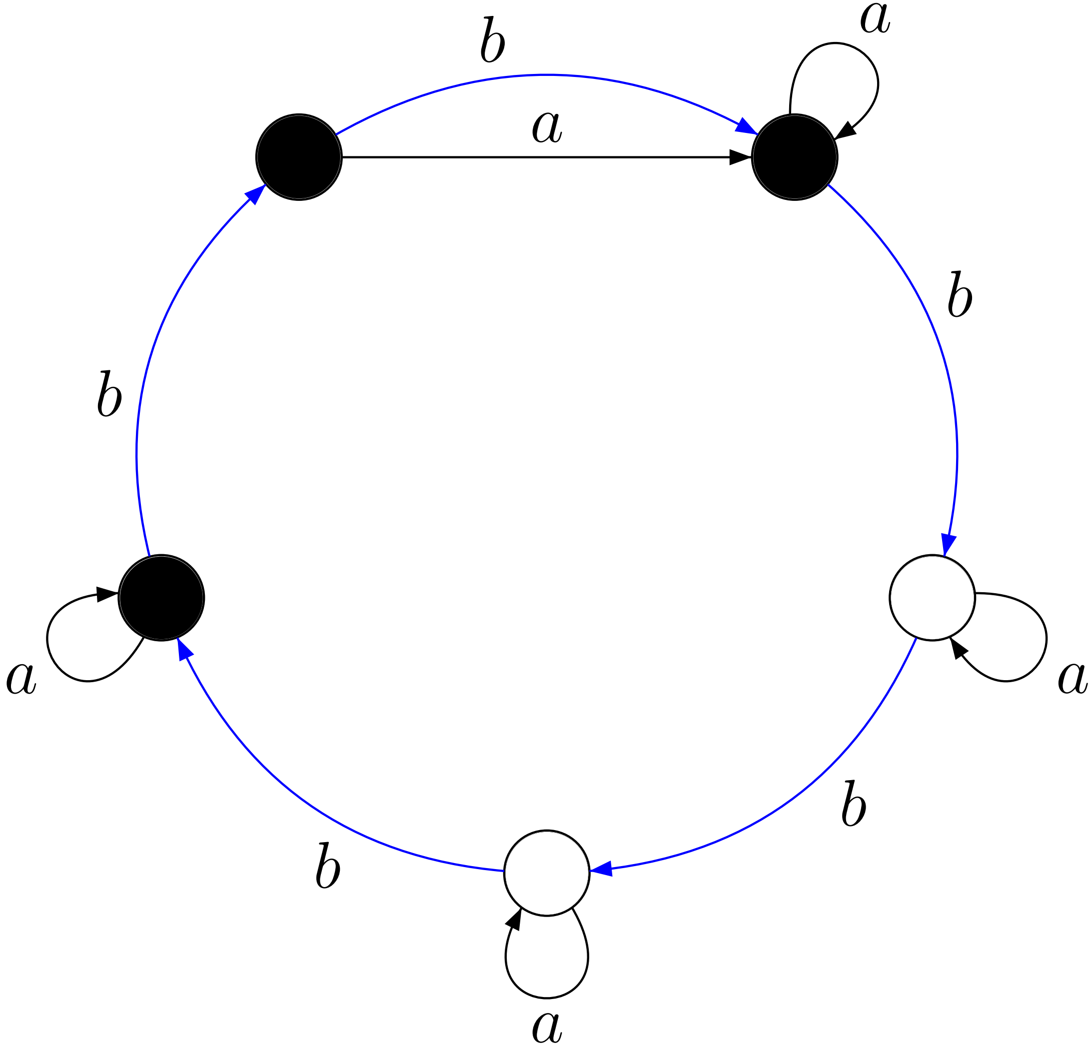

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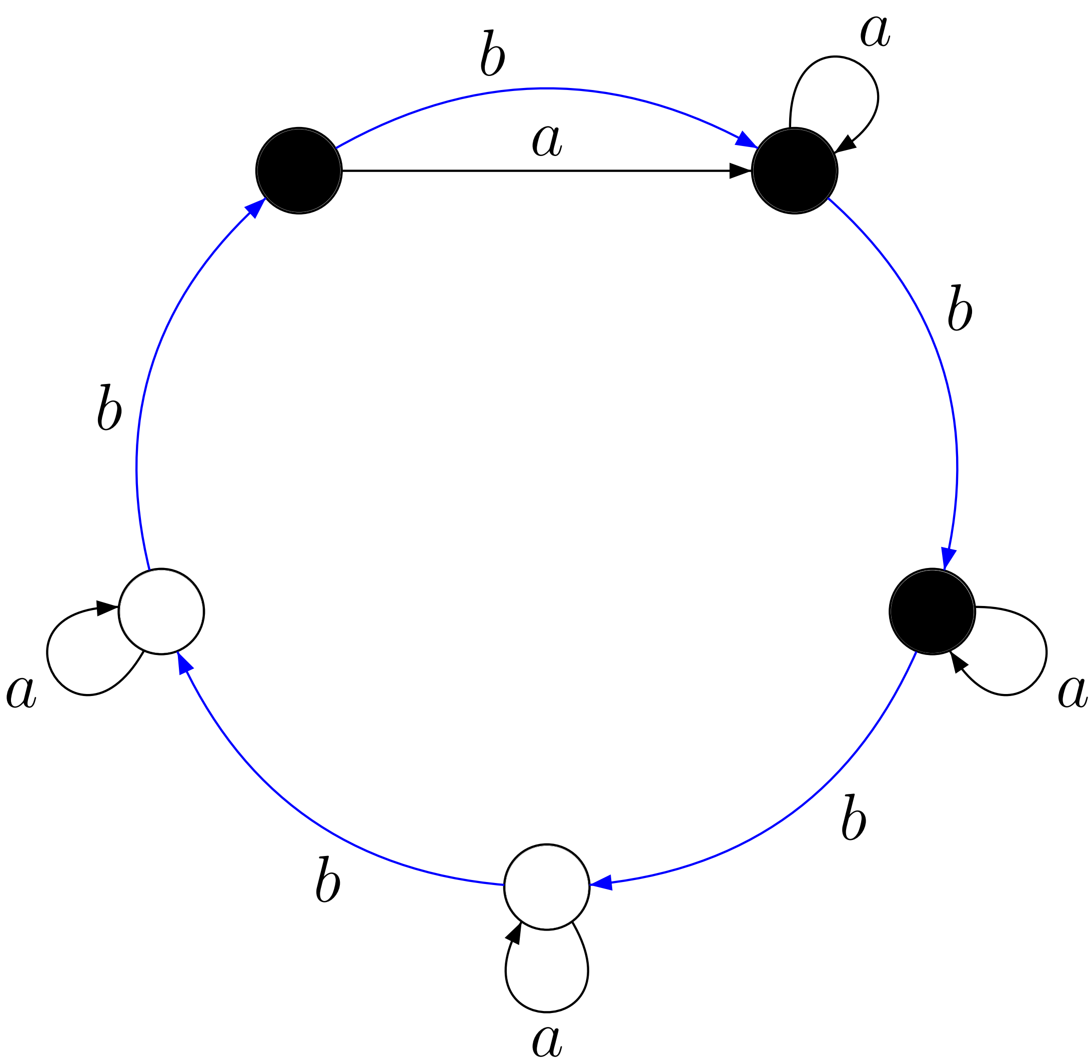

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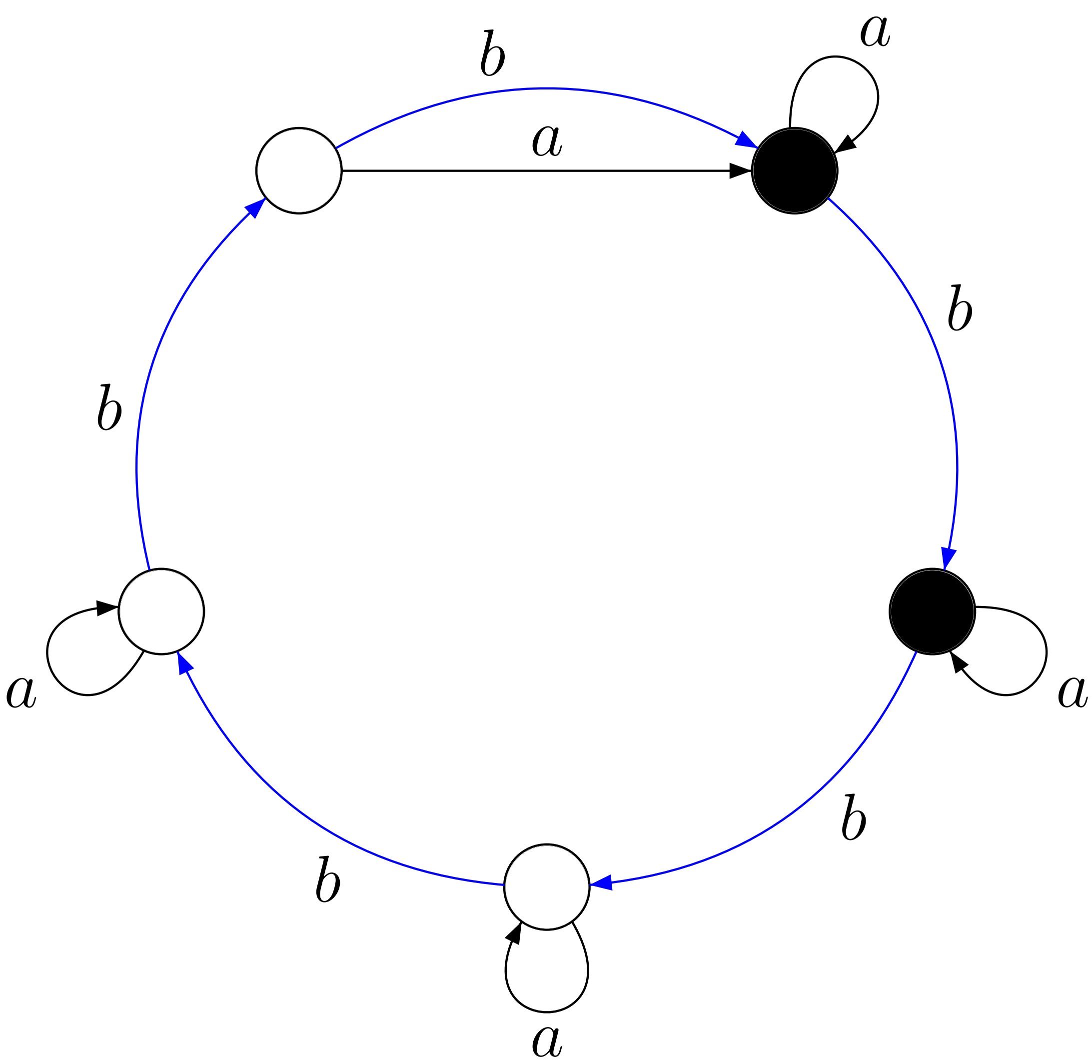

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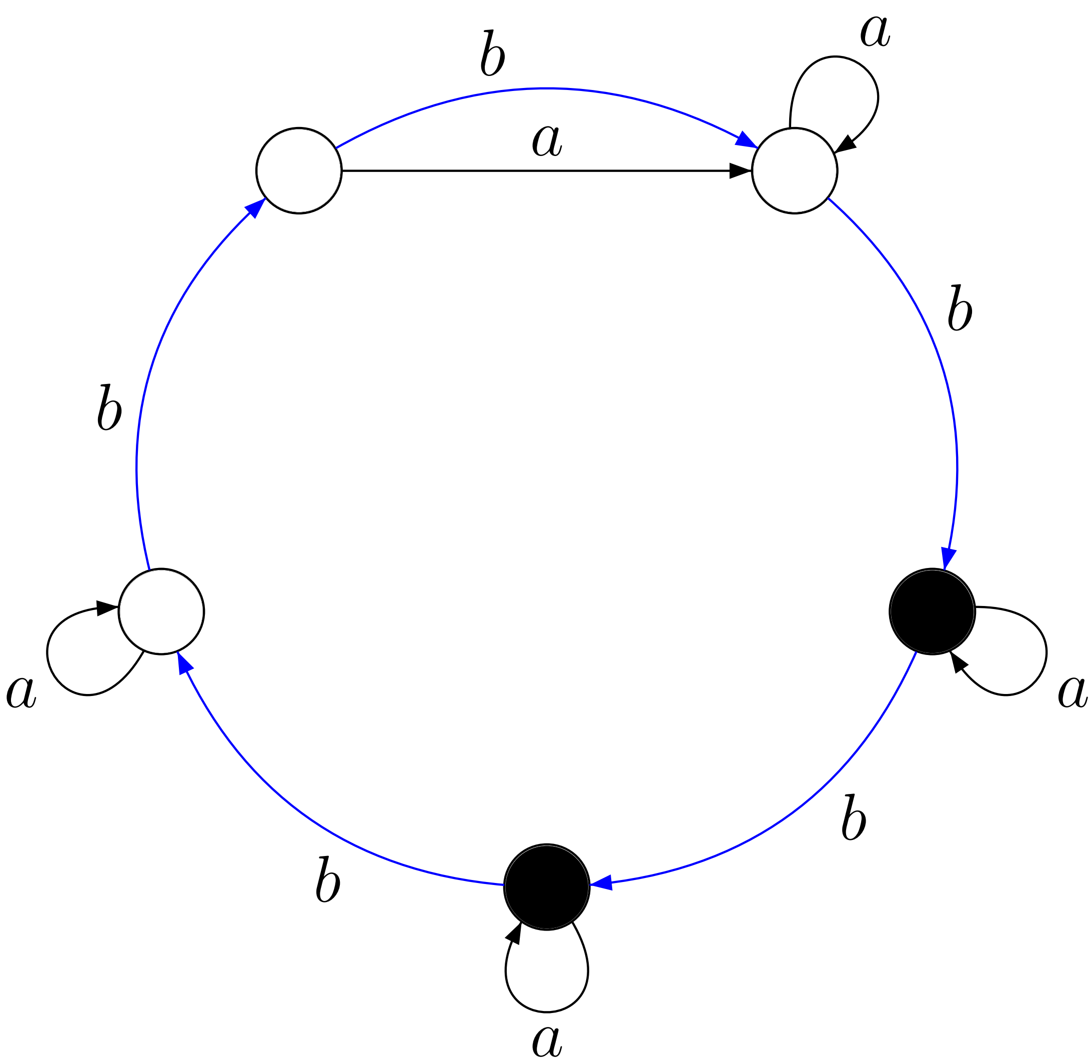

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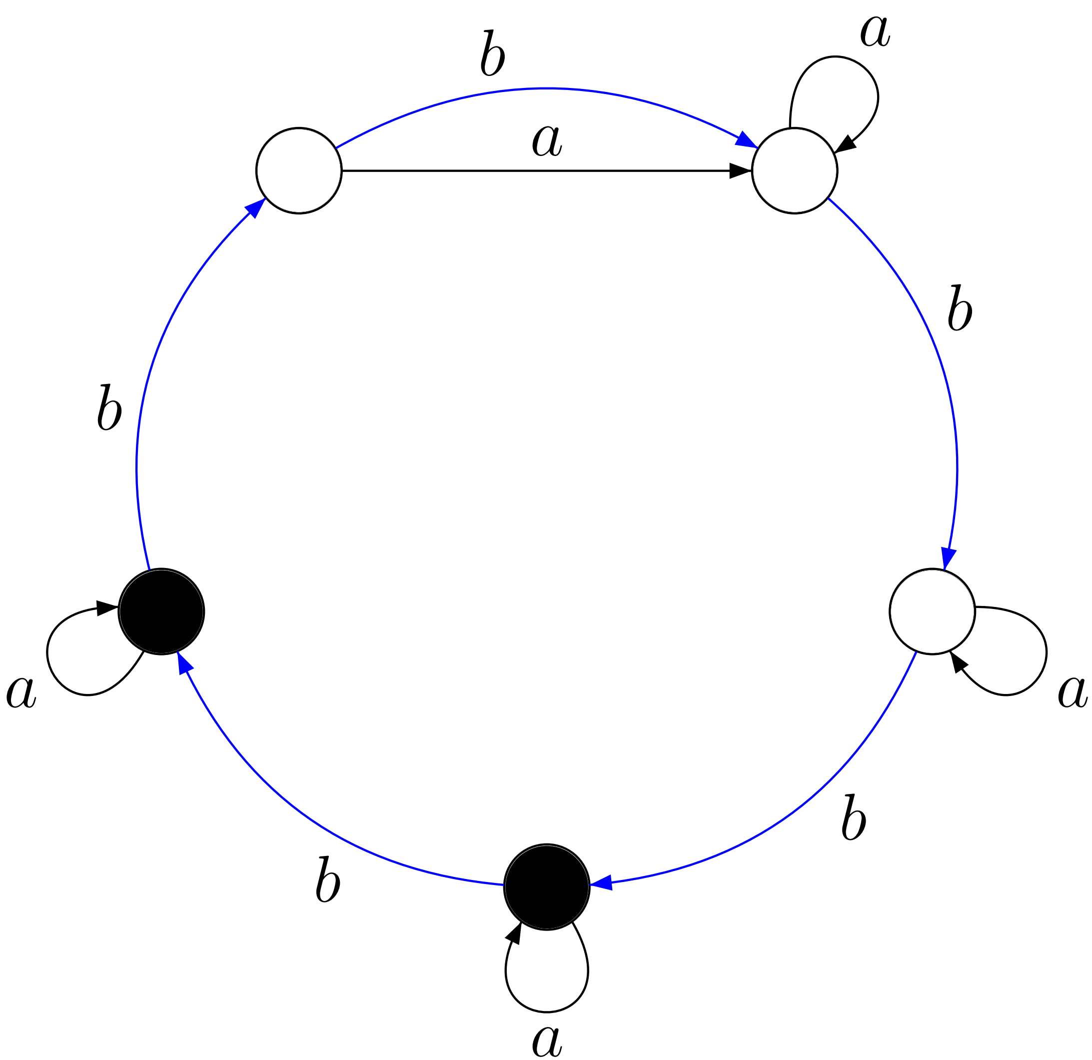

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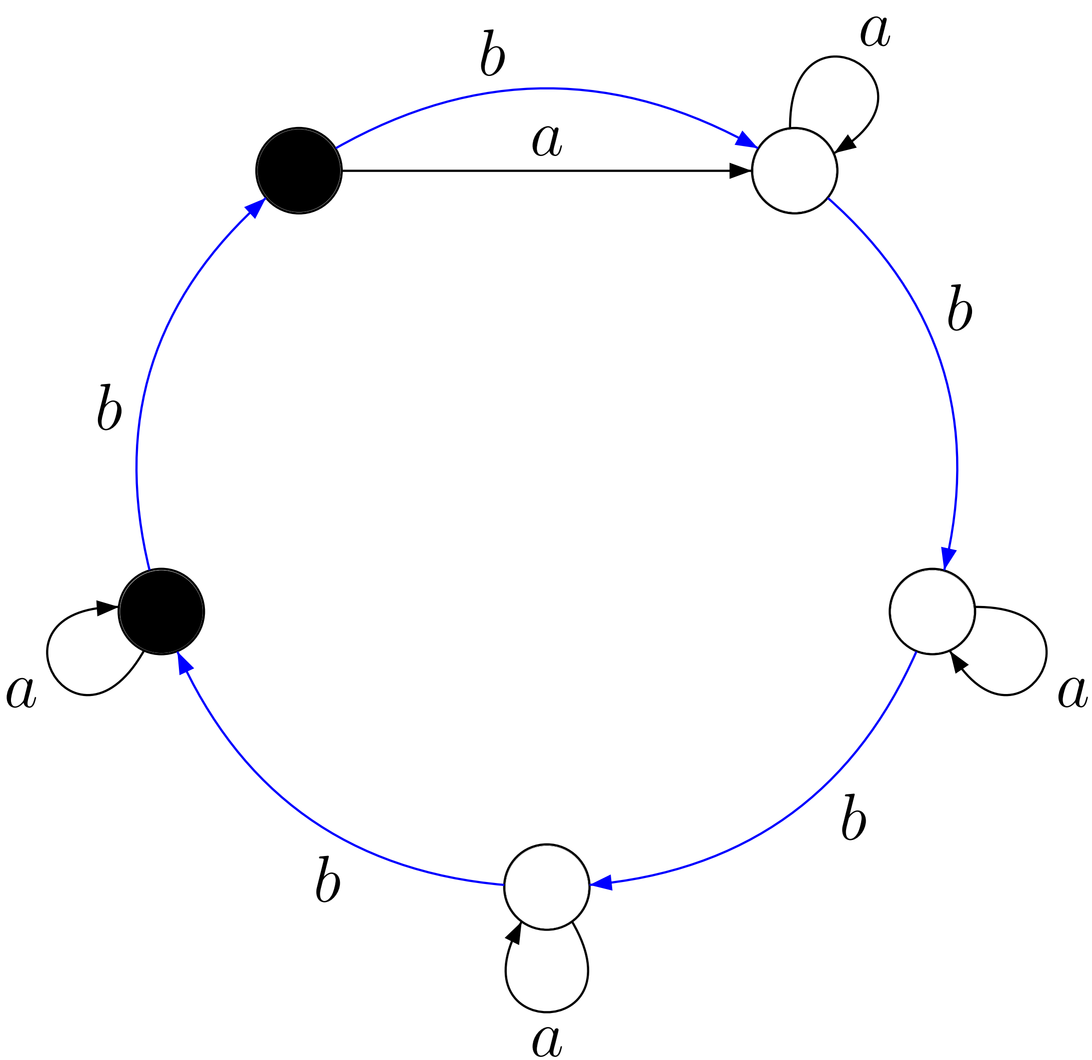

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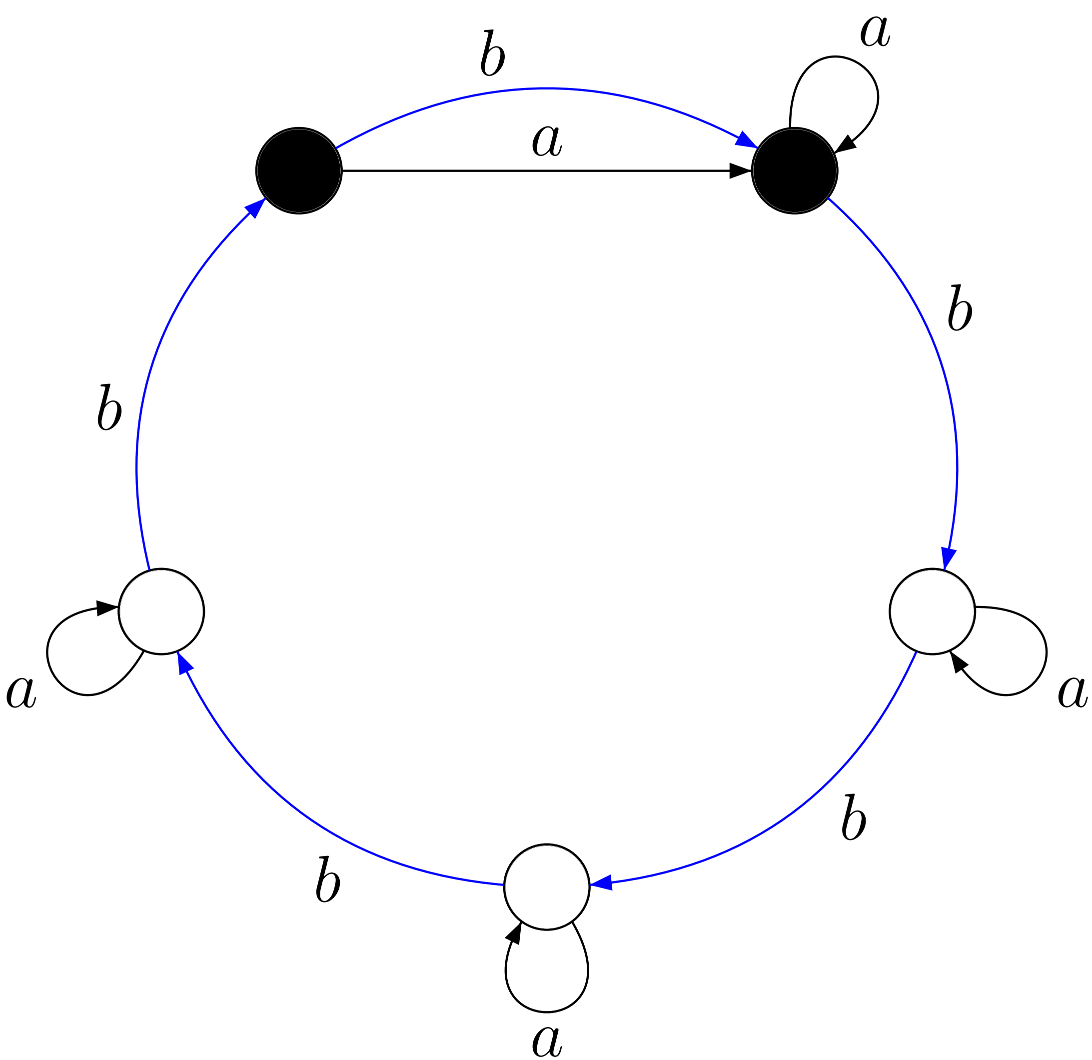

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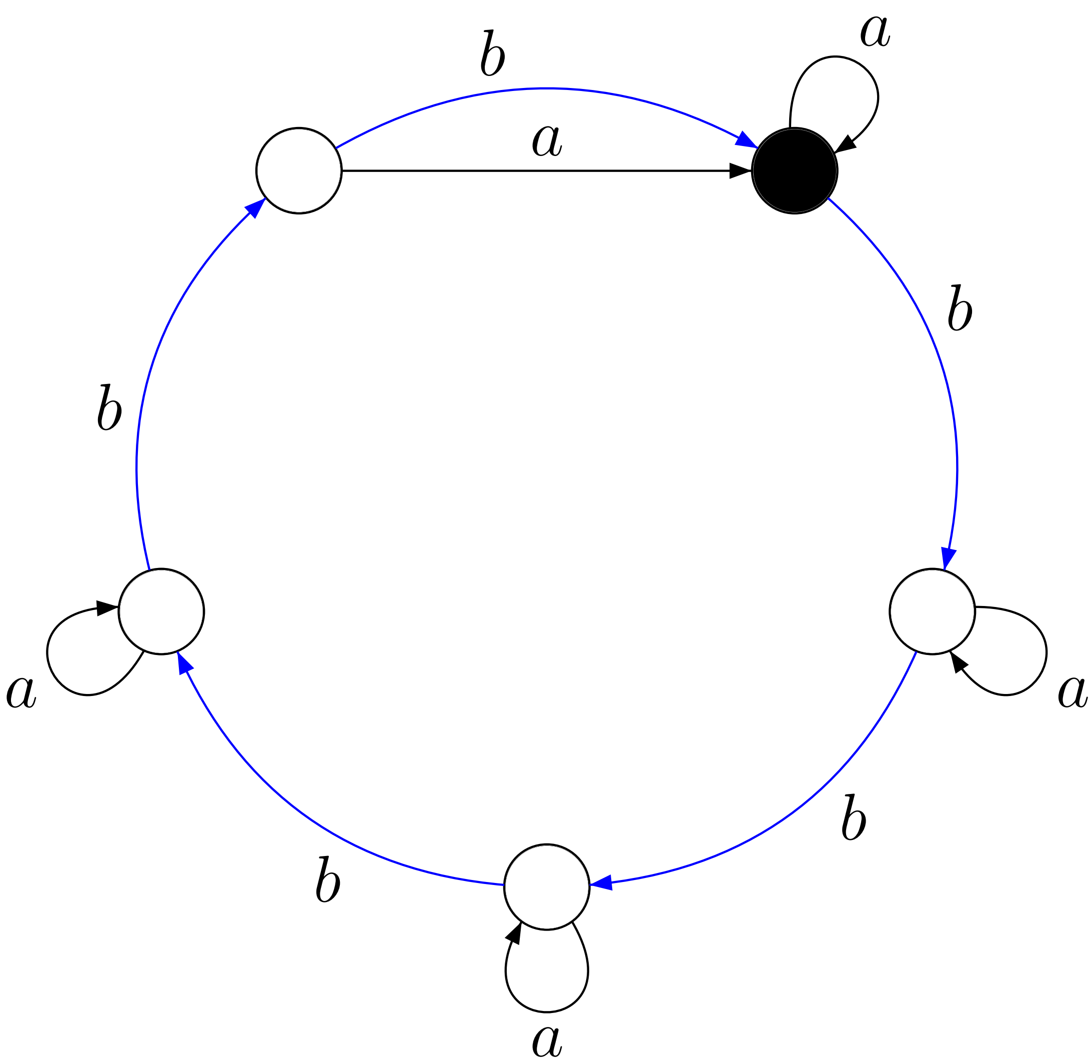

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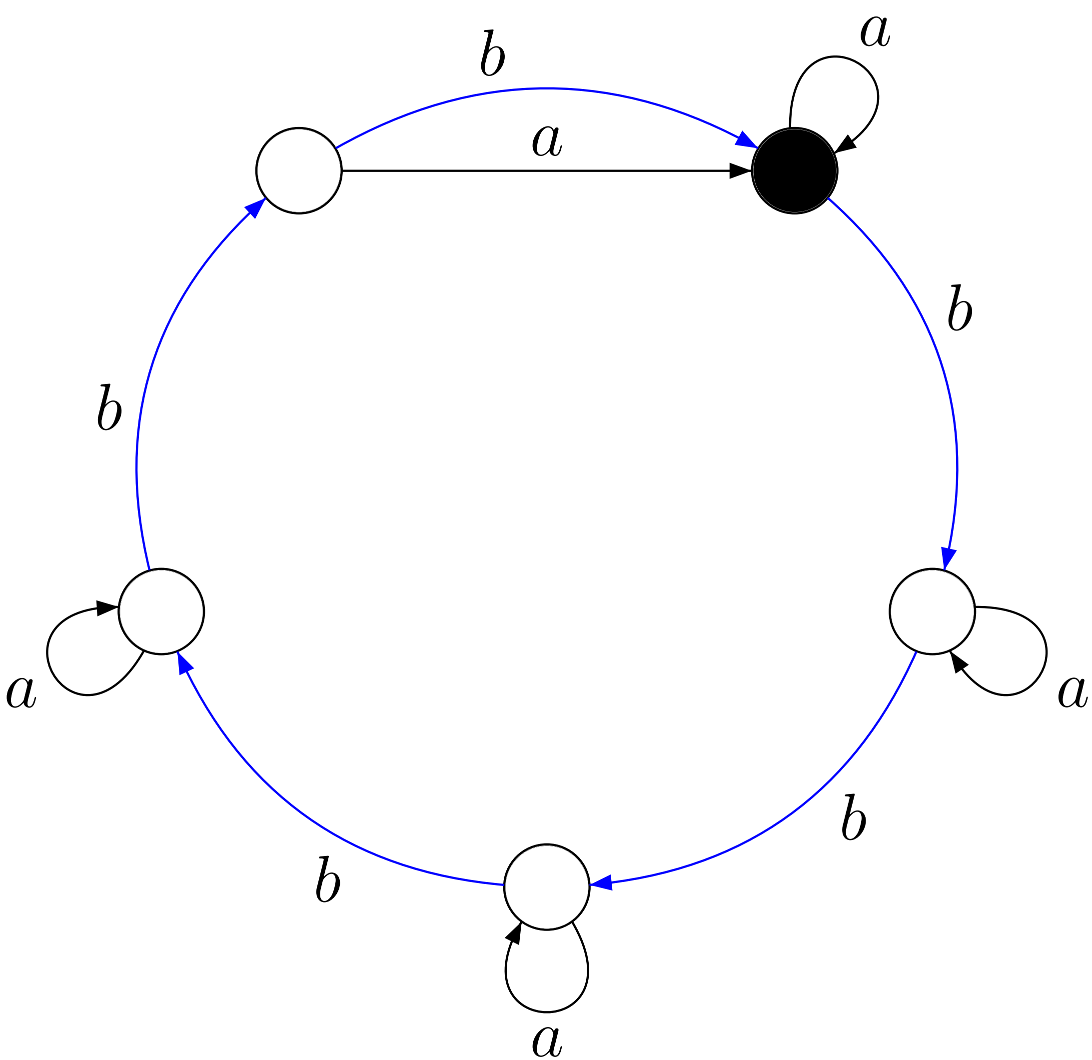

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reset threshold 16

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even if we restrict ourselves to two-letter automata [M.Berlinkov, 2013].

Class of algorithms

$SYNCH-SUBSET(k)$

Input: Synchronizing automaton $\mathscr{A} = \langle Q, \Sigma, \delta \rangle$

$S = Q$

$u = \varepsilon$

Until $|S| == 1$ do

    If $|S| \geq k$:

        Choose a set $P \subseteq S$ s.t. $|P| = k$

    Else:

        $P = S$

    Choose a word v s.t. $|P.v| = 1$

    $u = u \cdot v$

    $S = S.v$

Return length of u

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Greedy choices can be done after polynomial-time computation.

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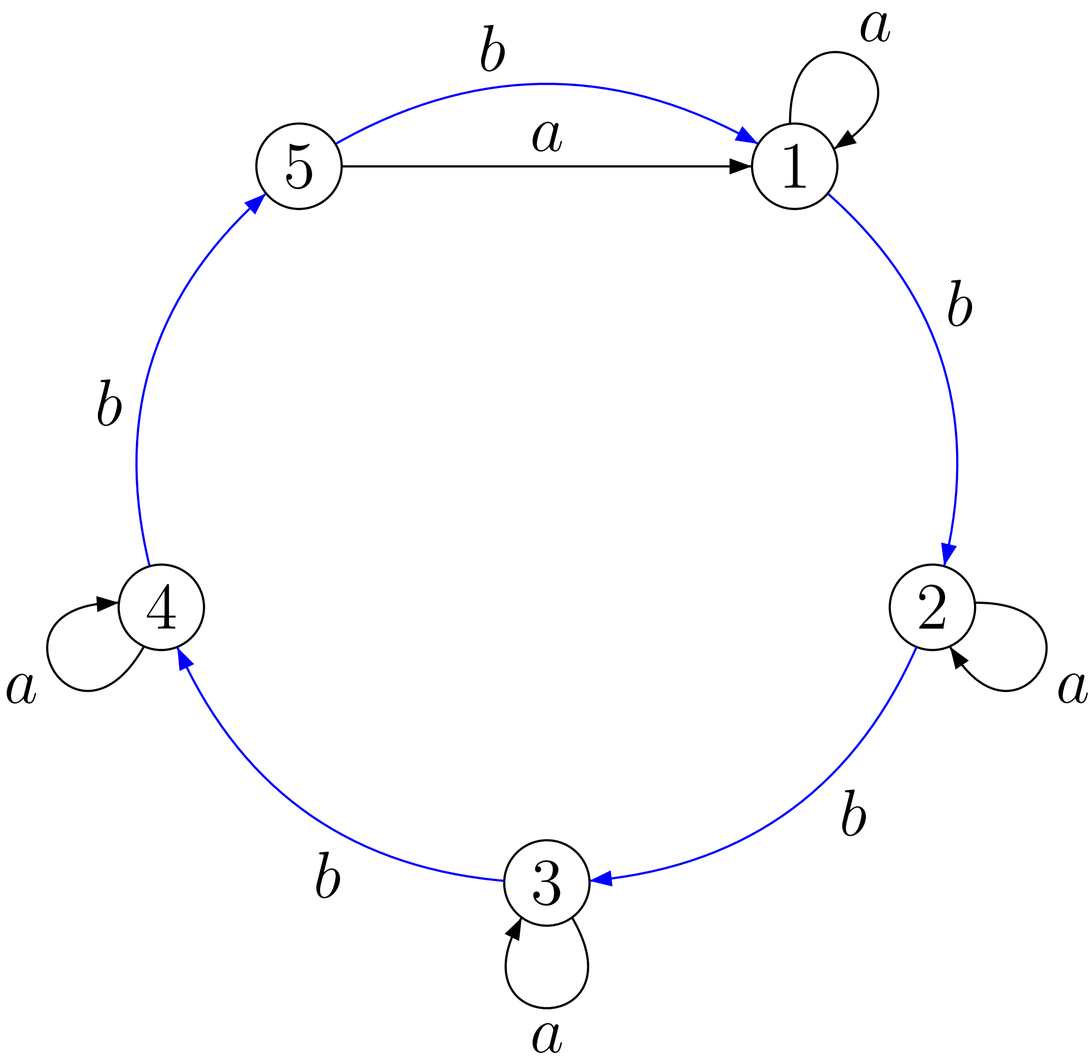

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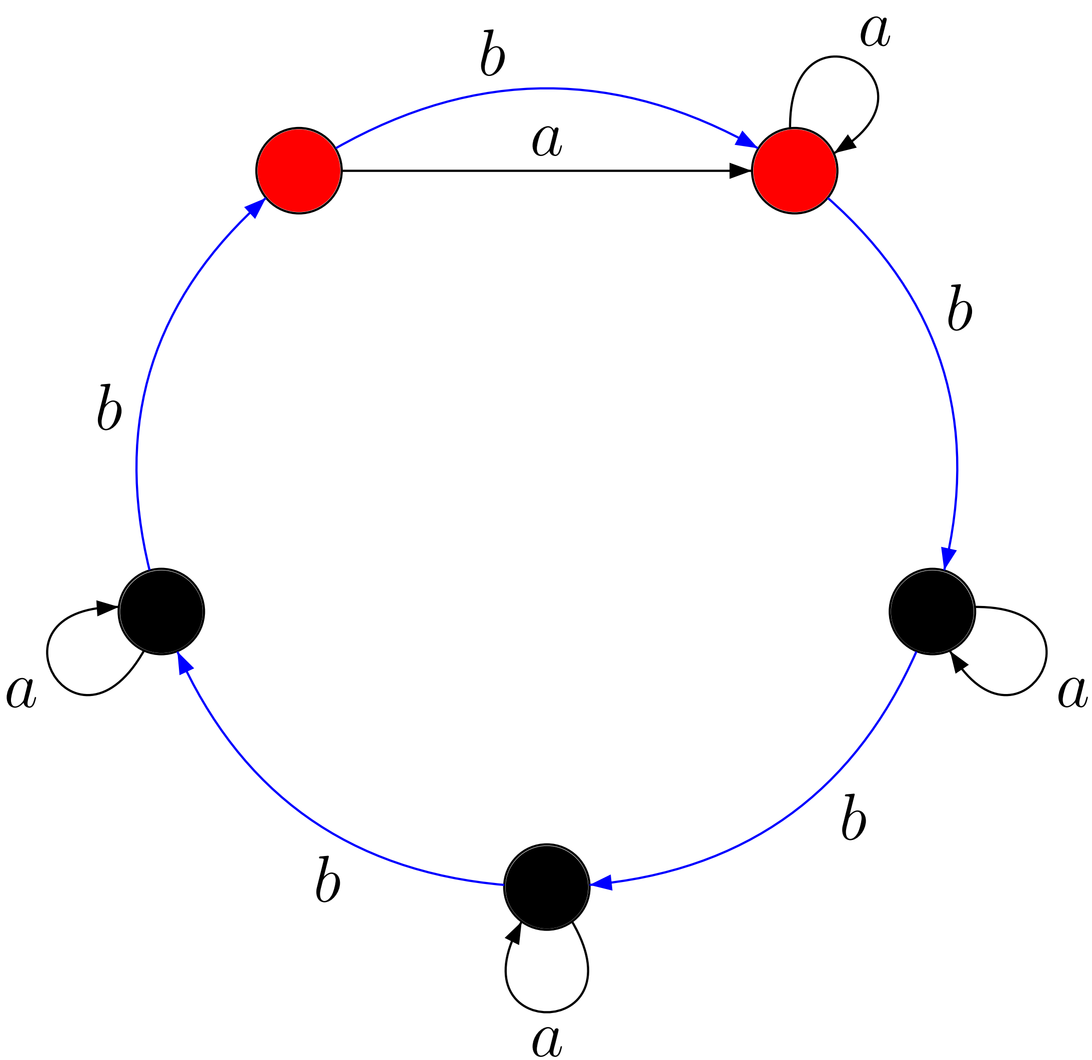

a

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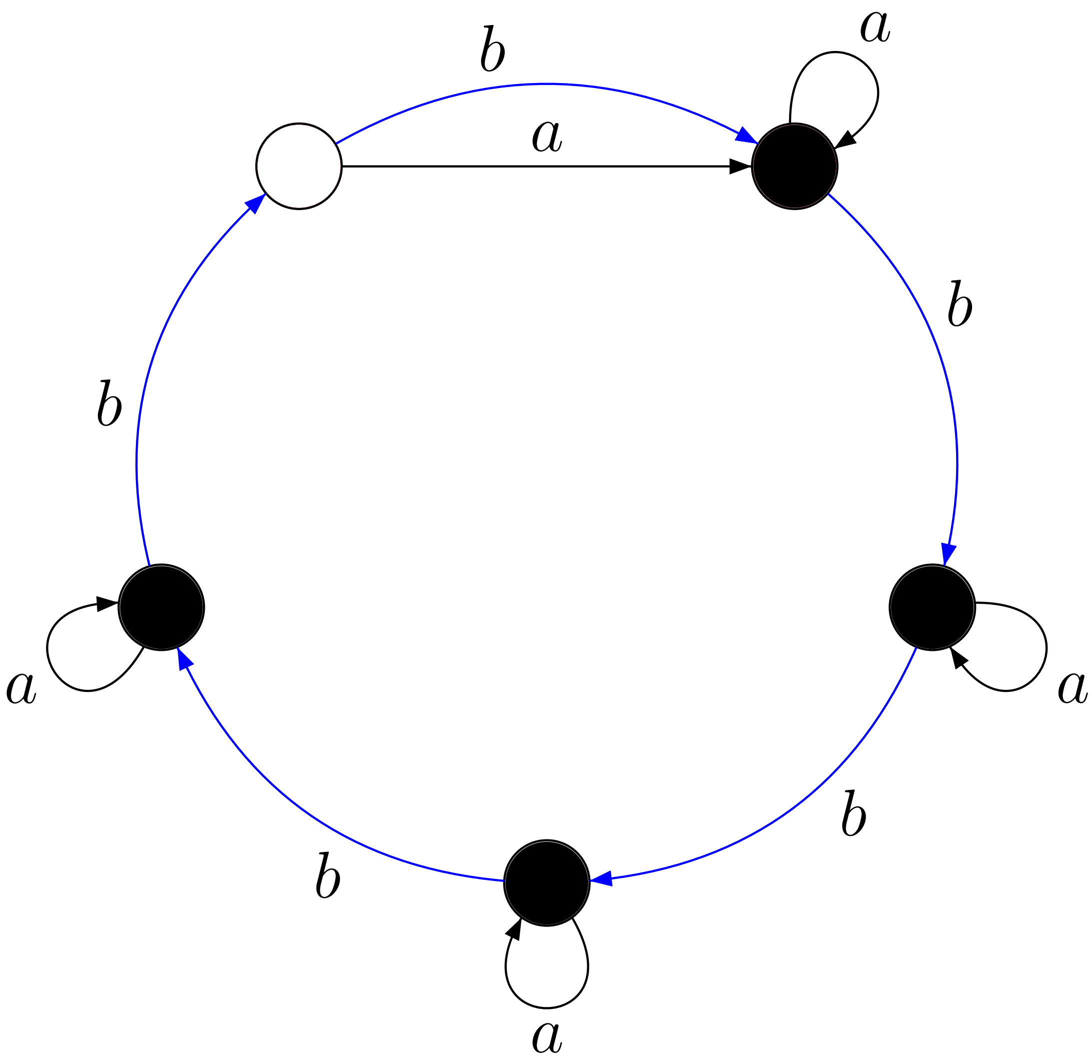

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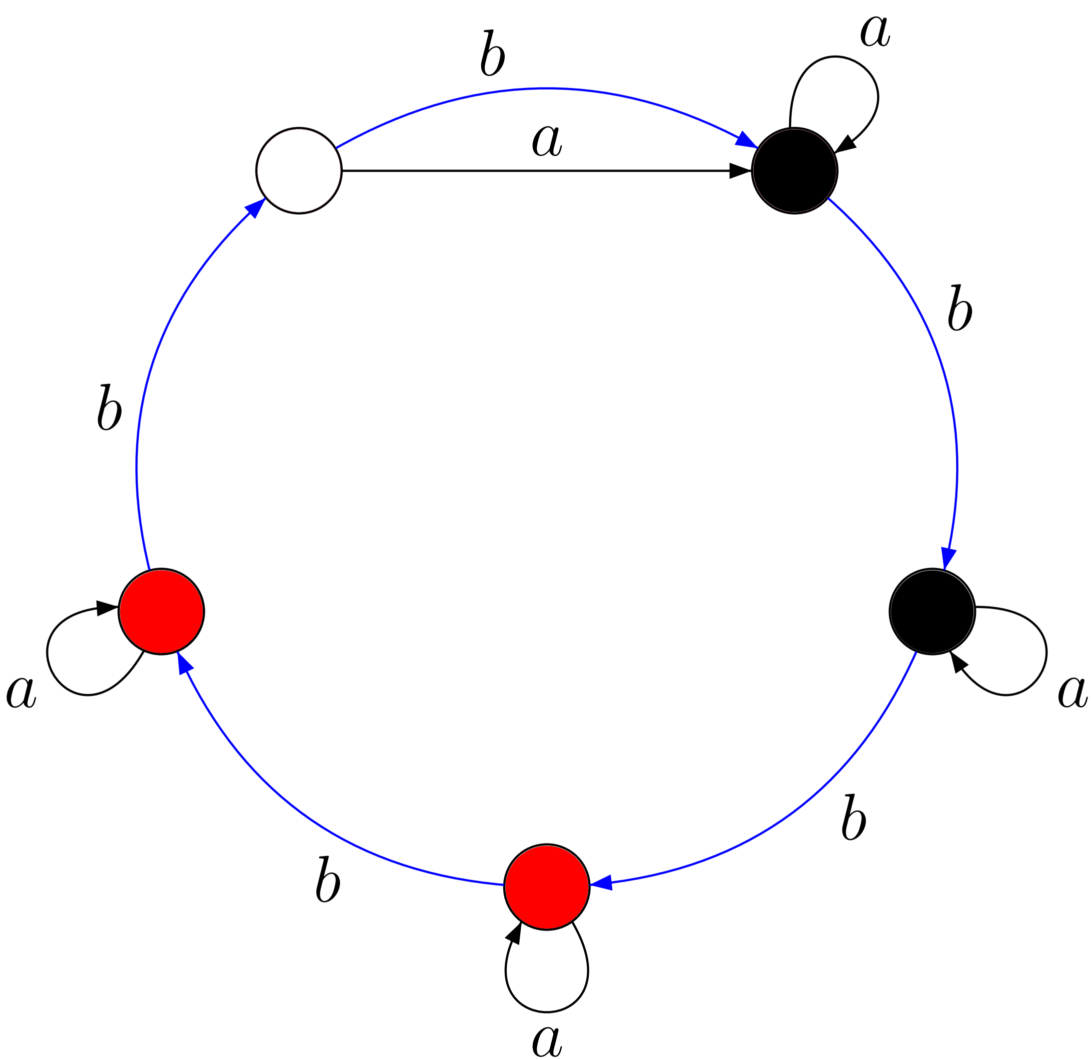

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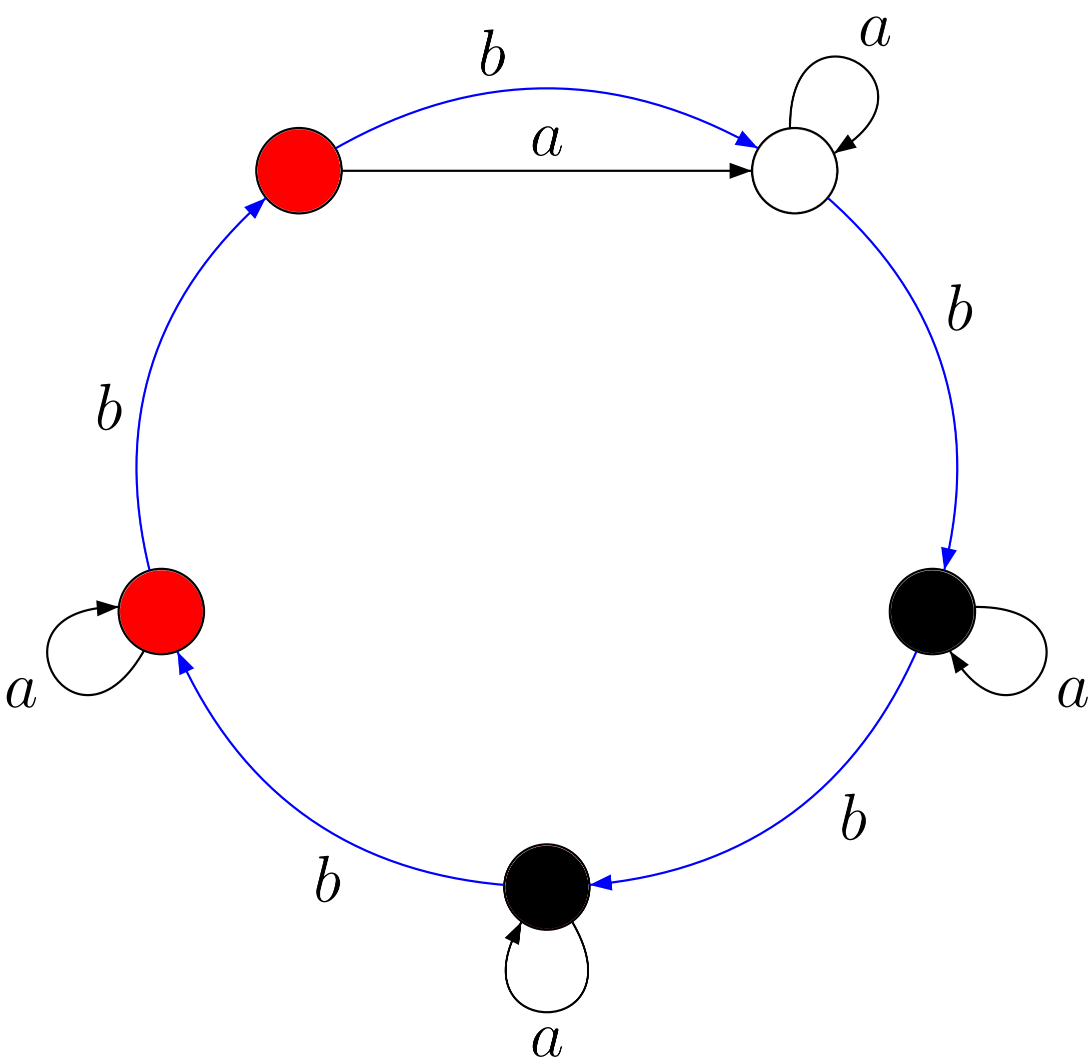

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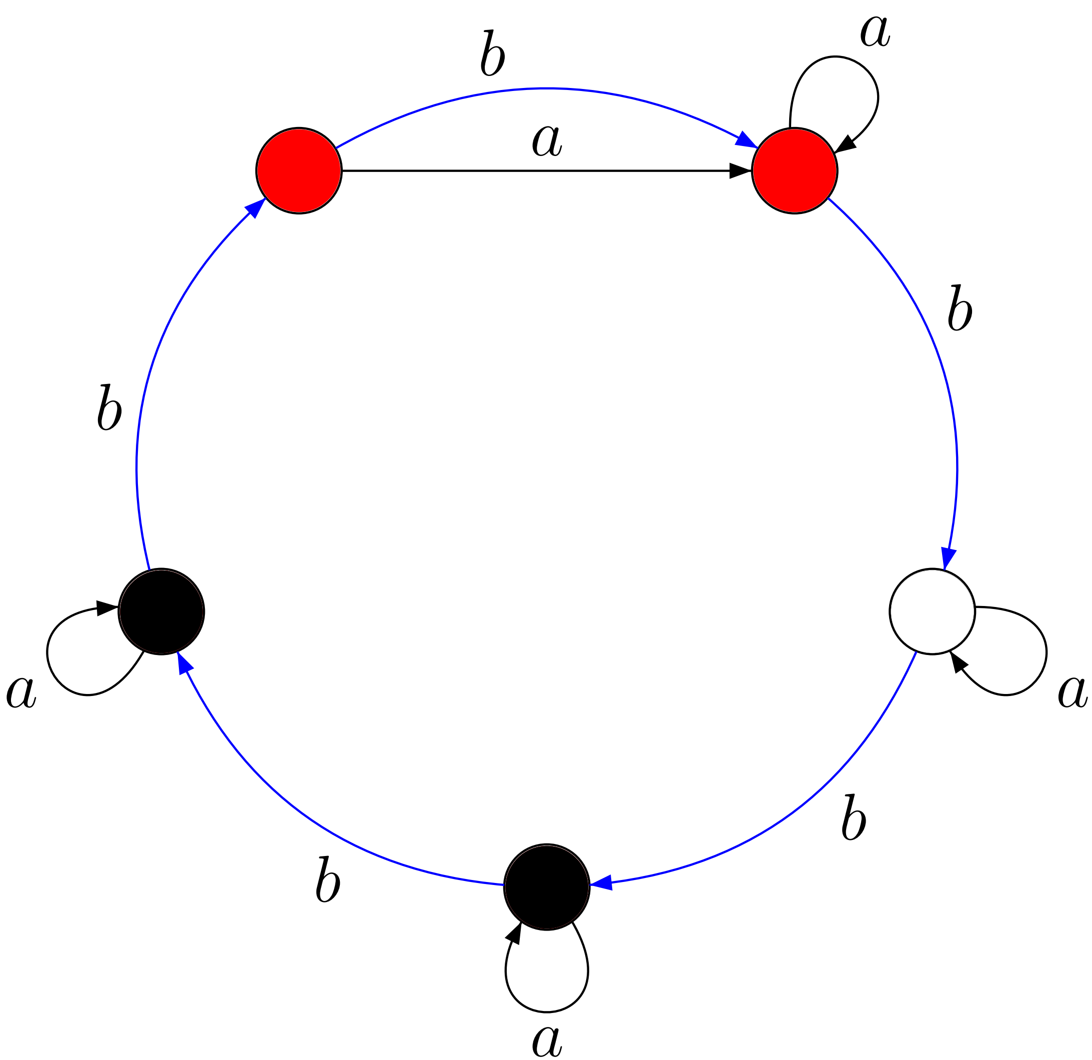

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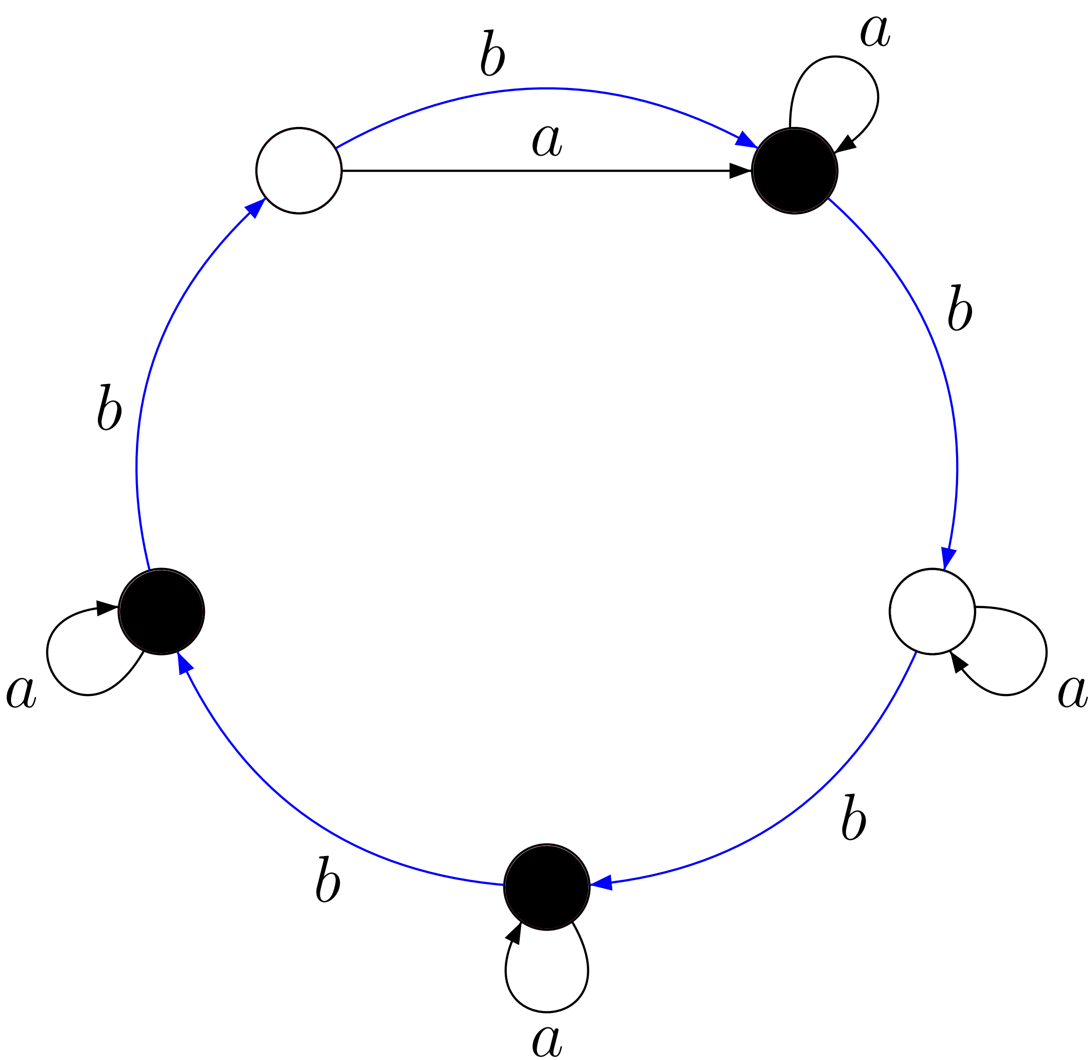

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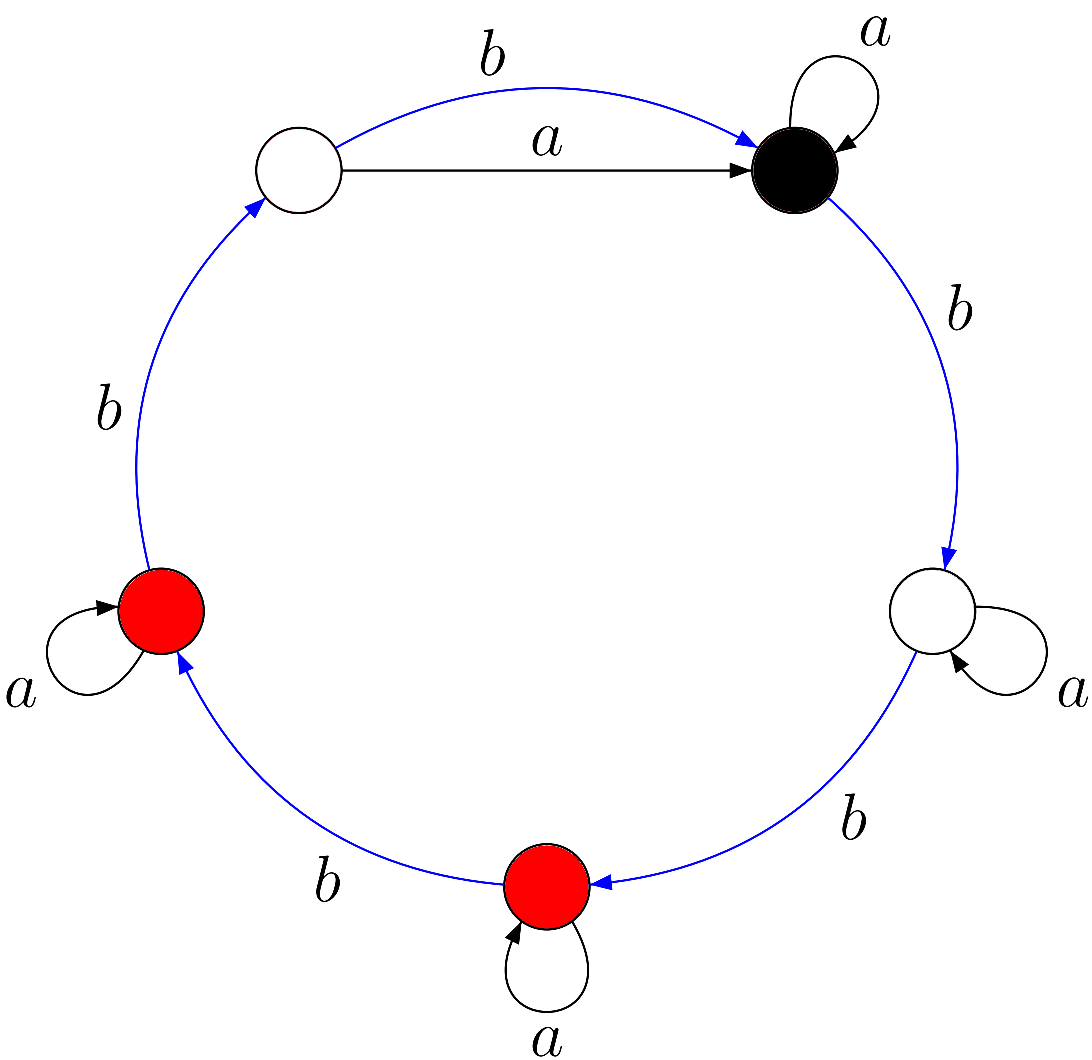

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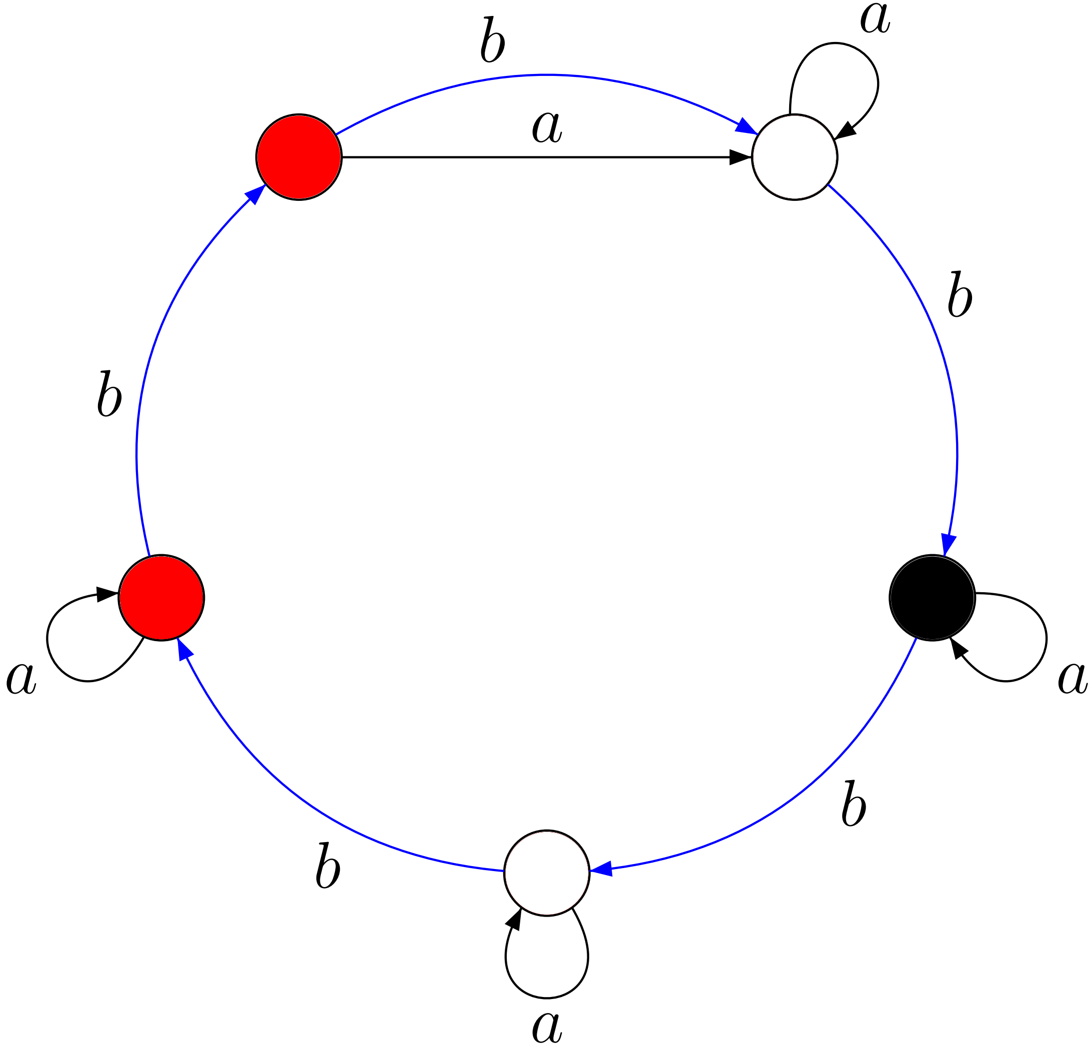

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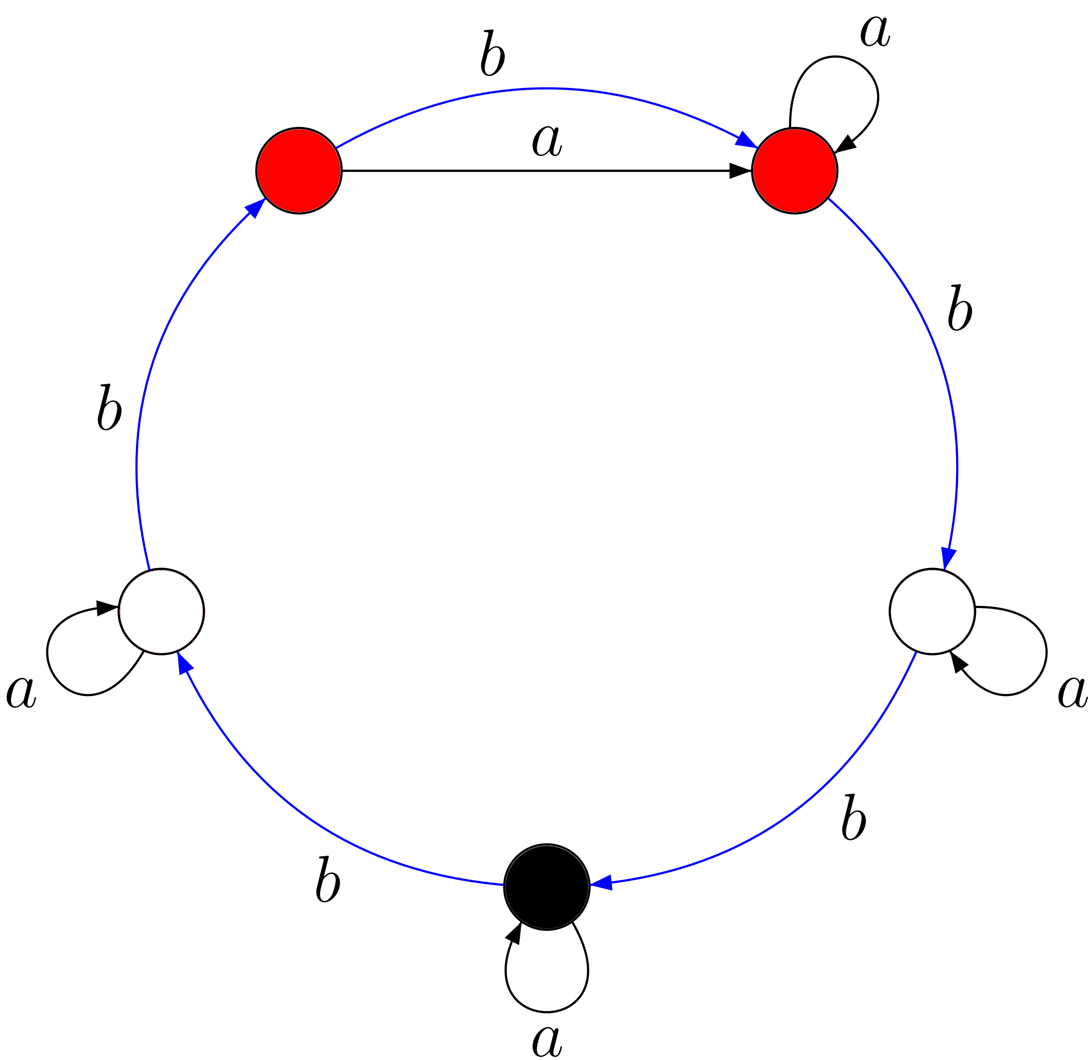

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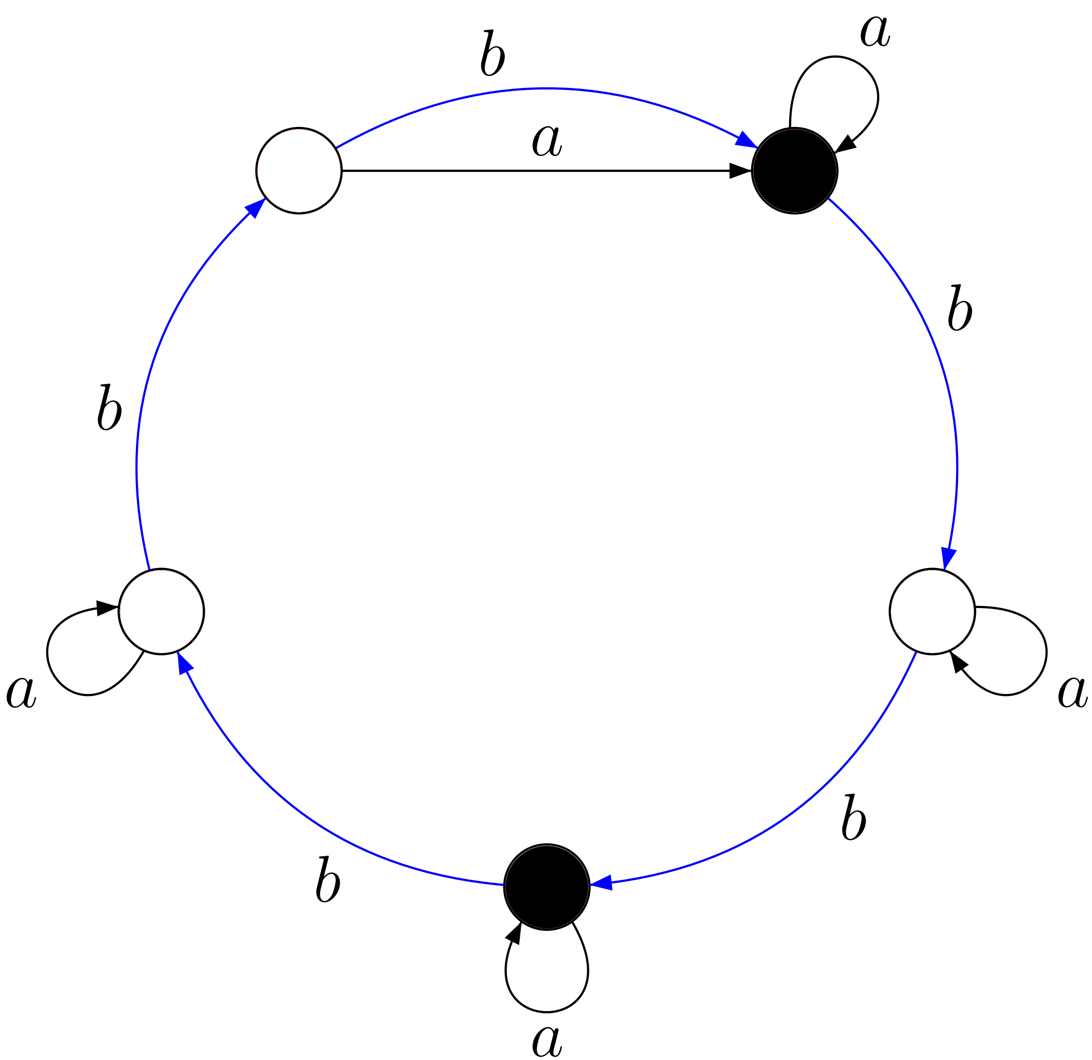

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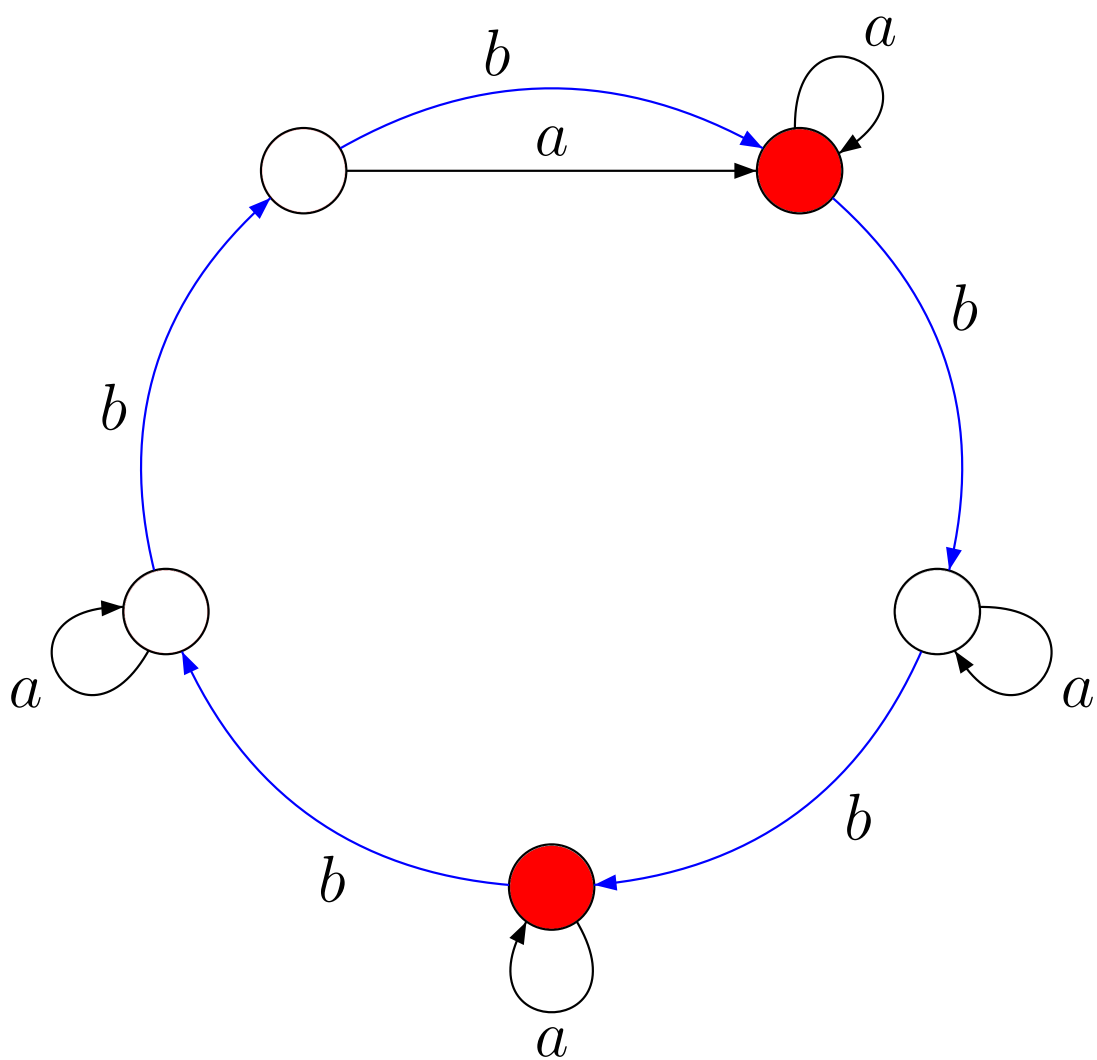

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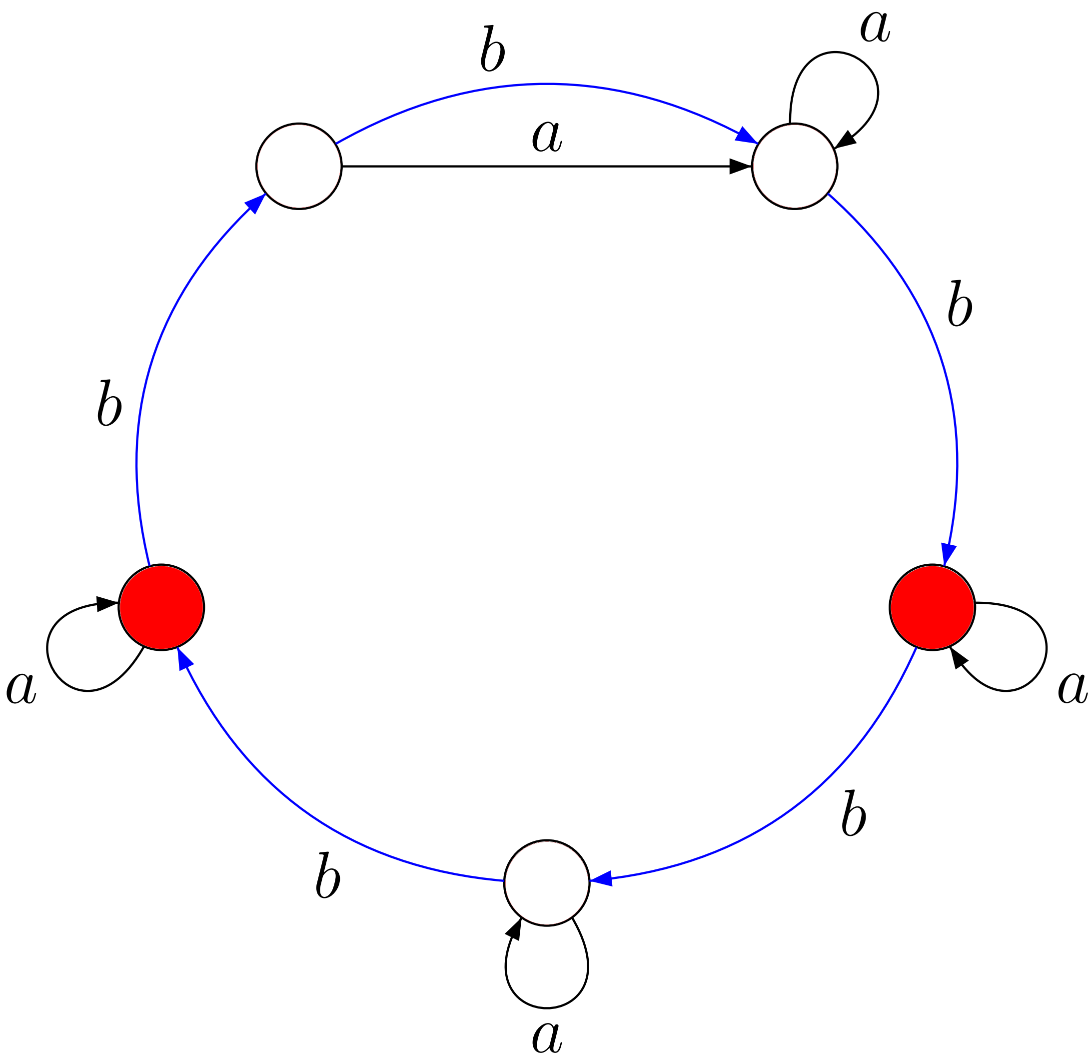

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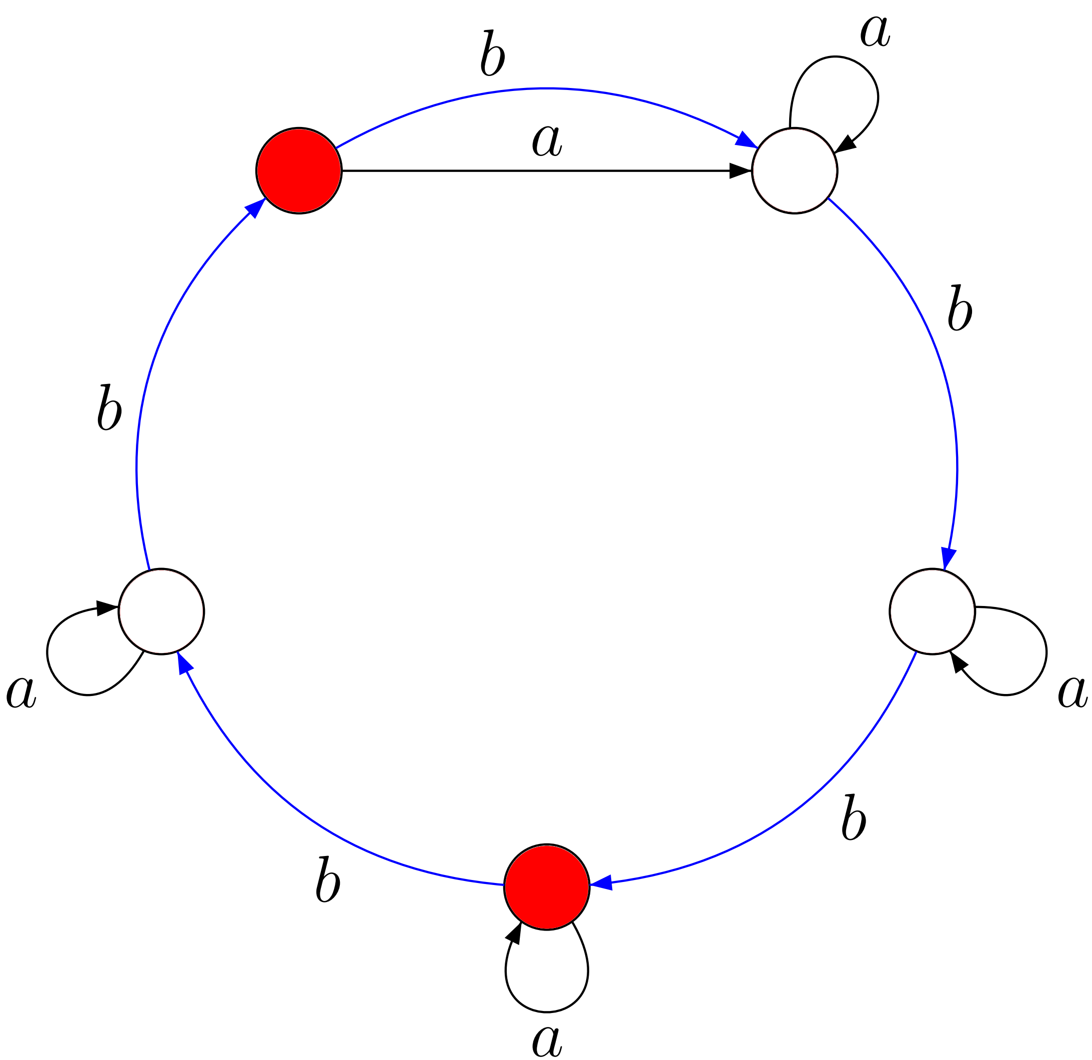

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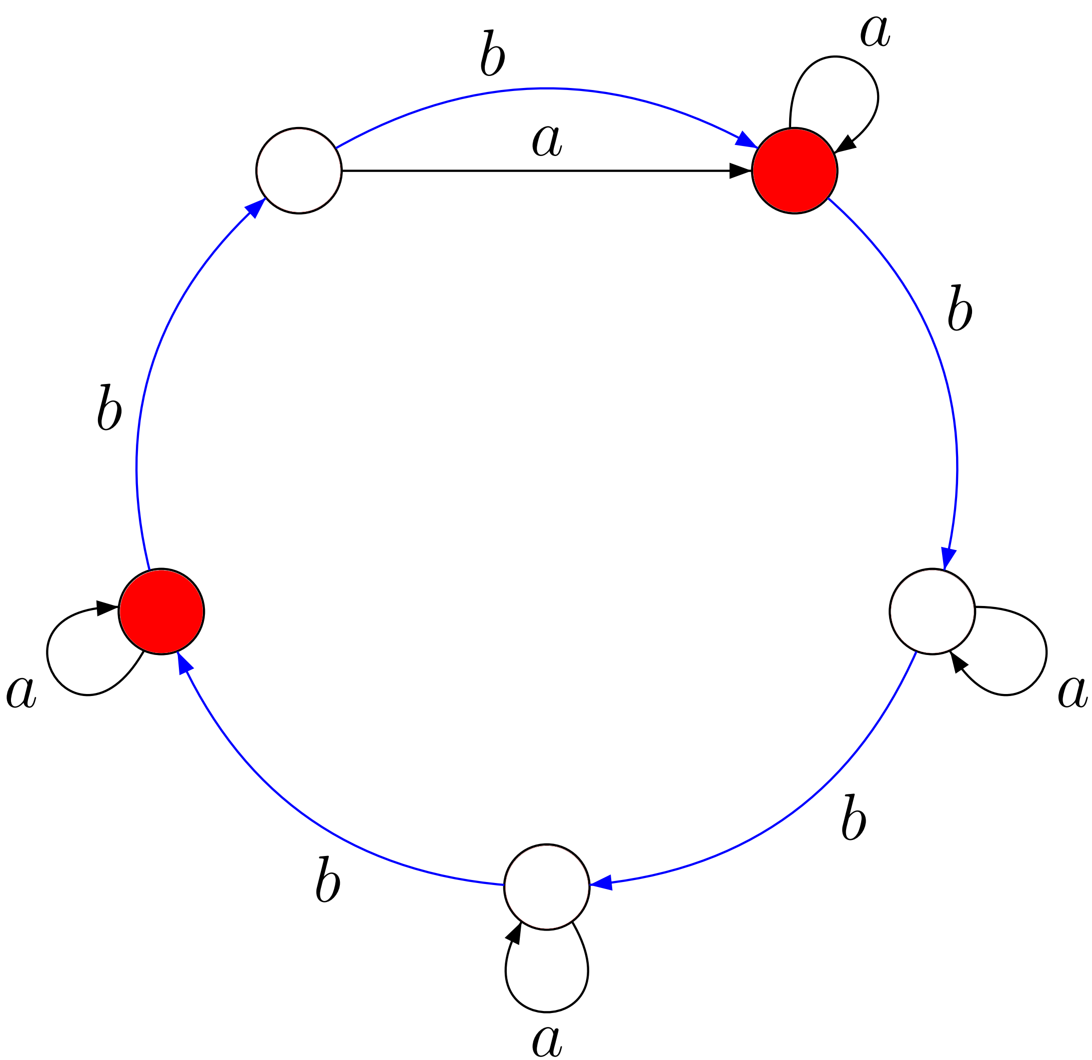

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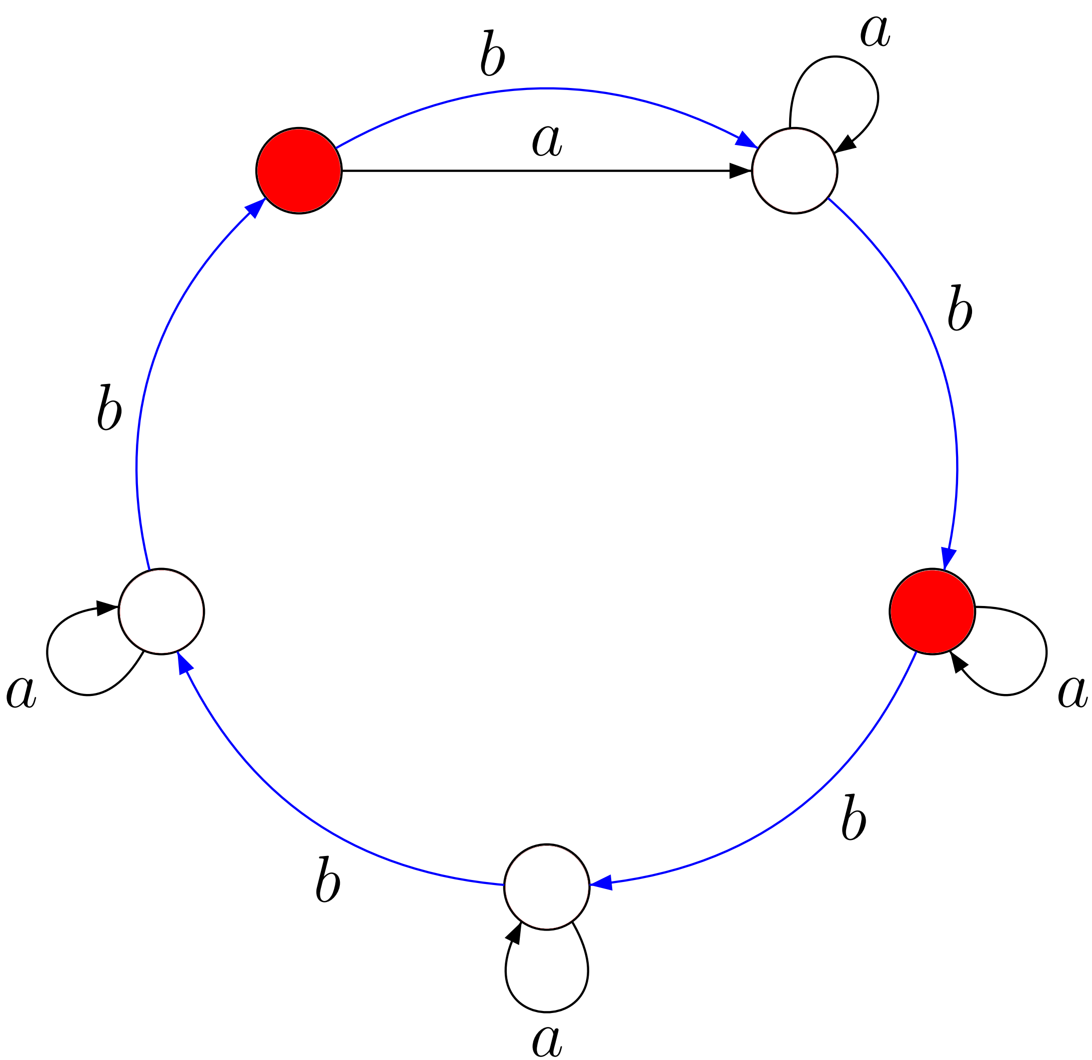

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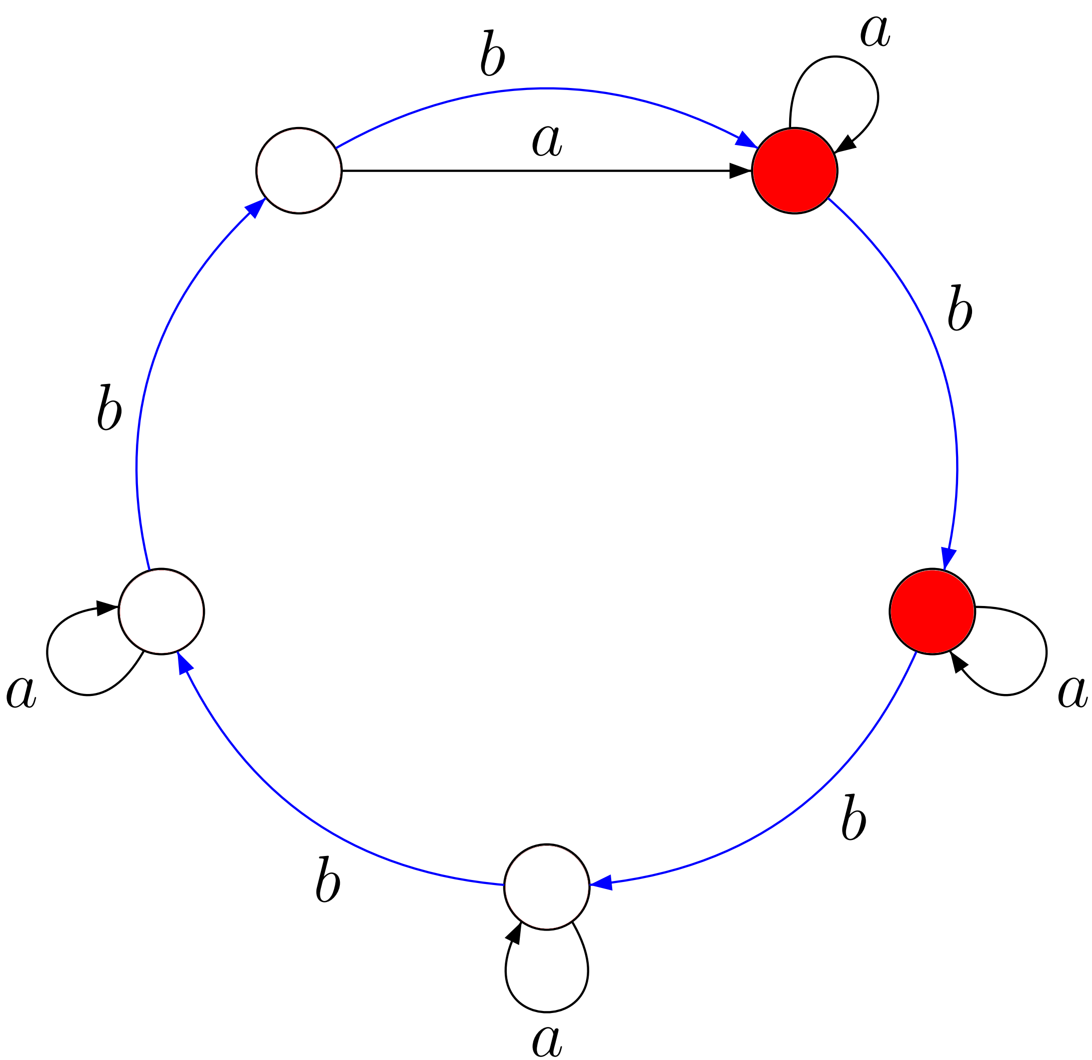

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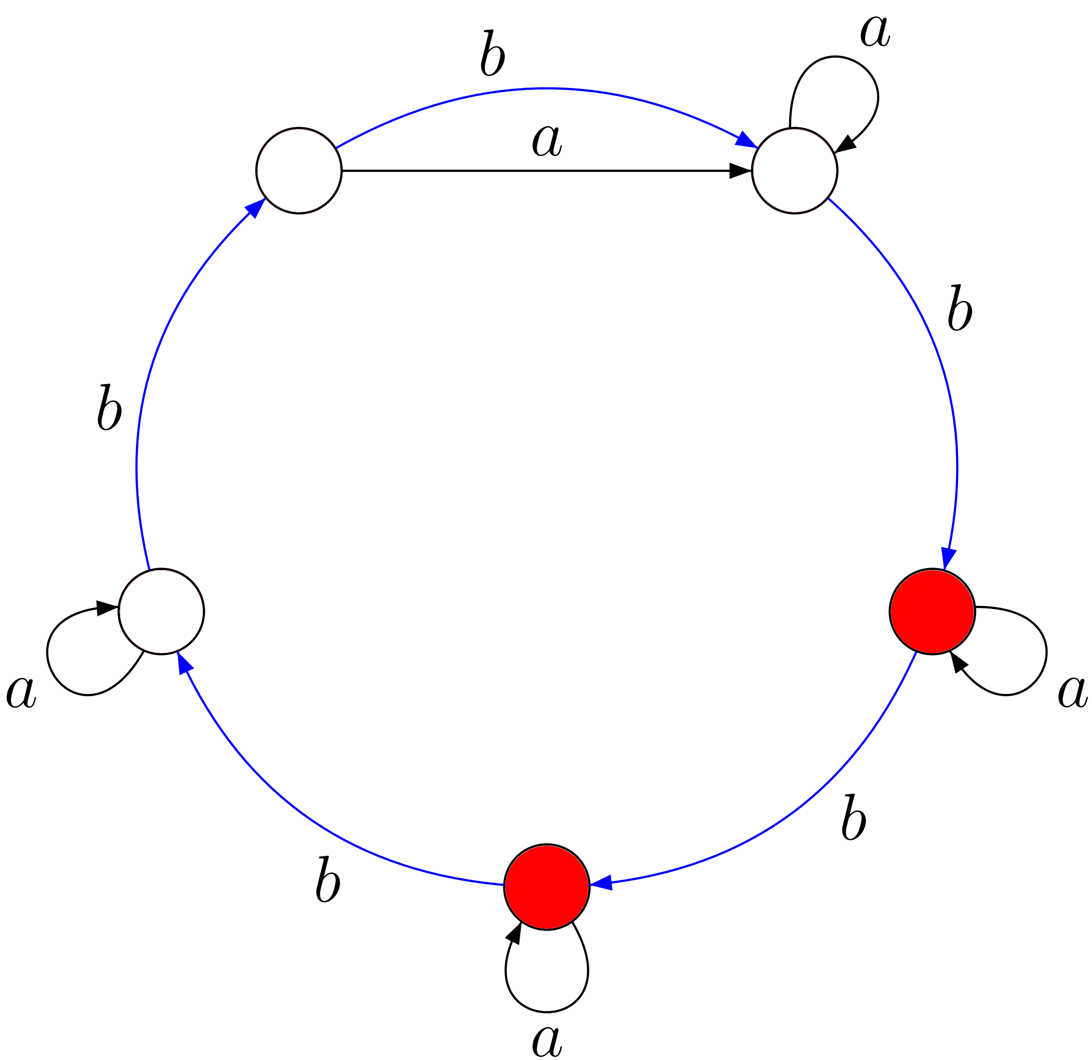

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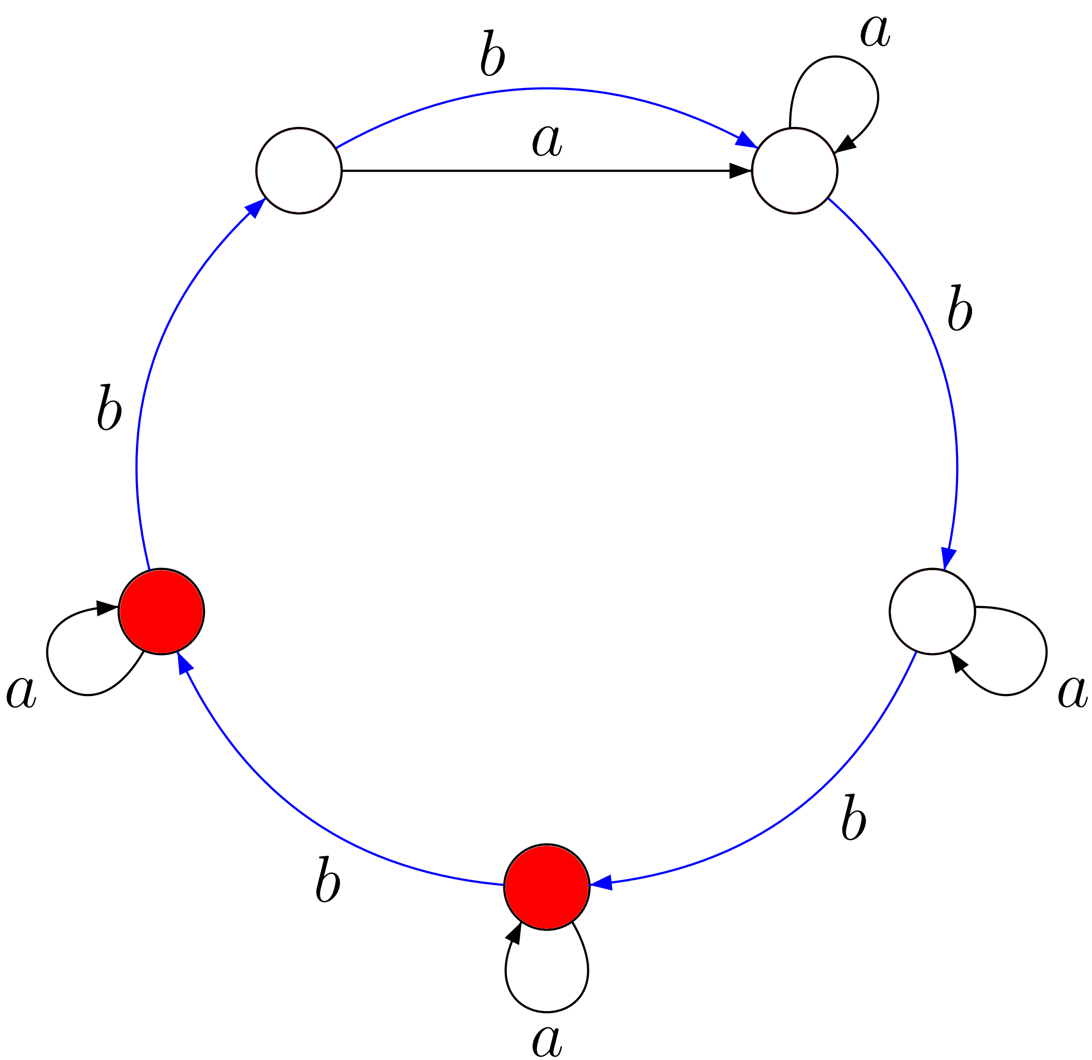

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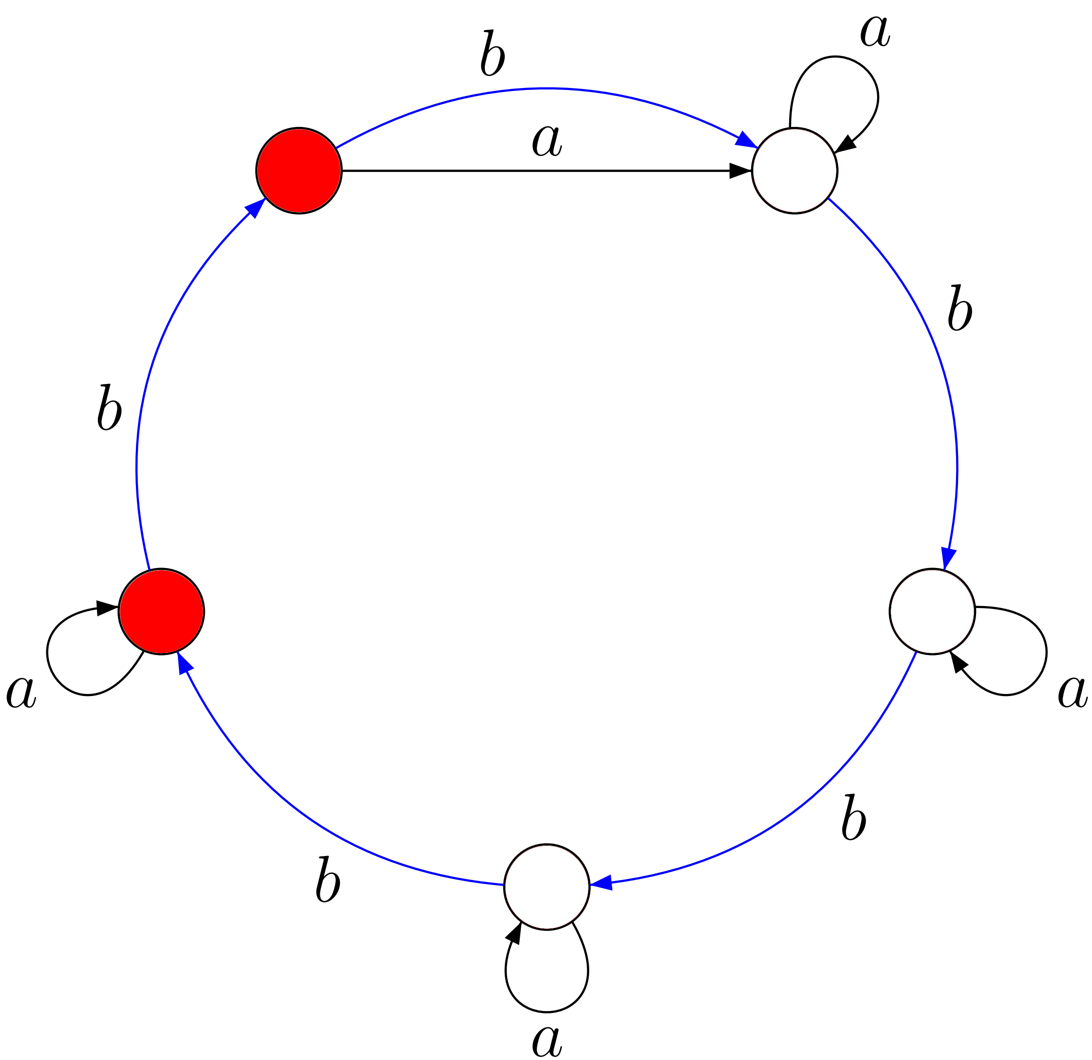

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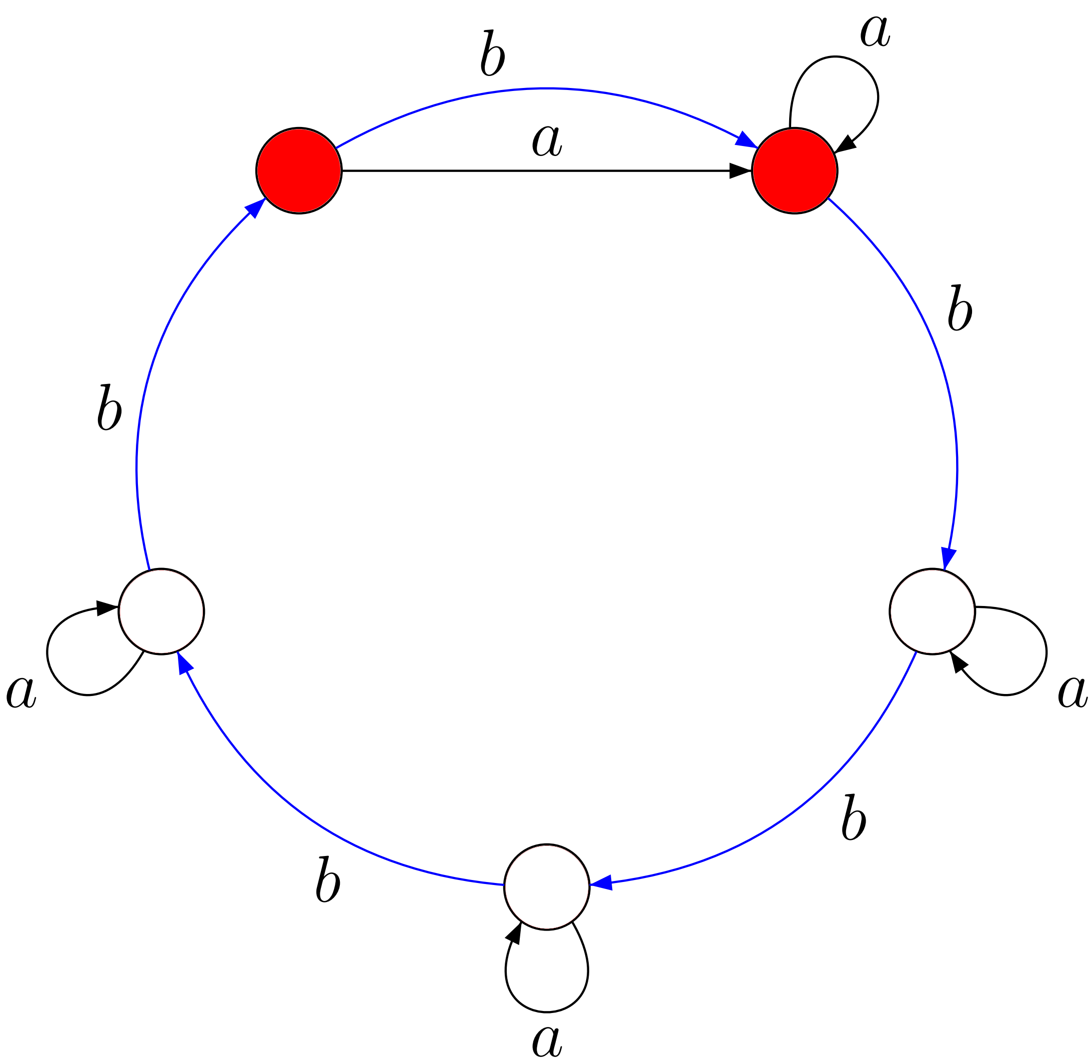

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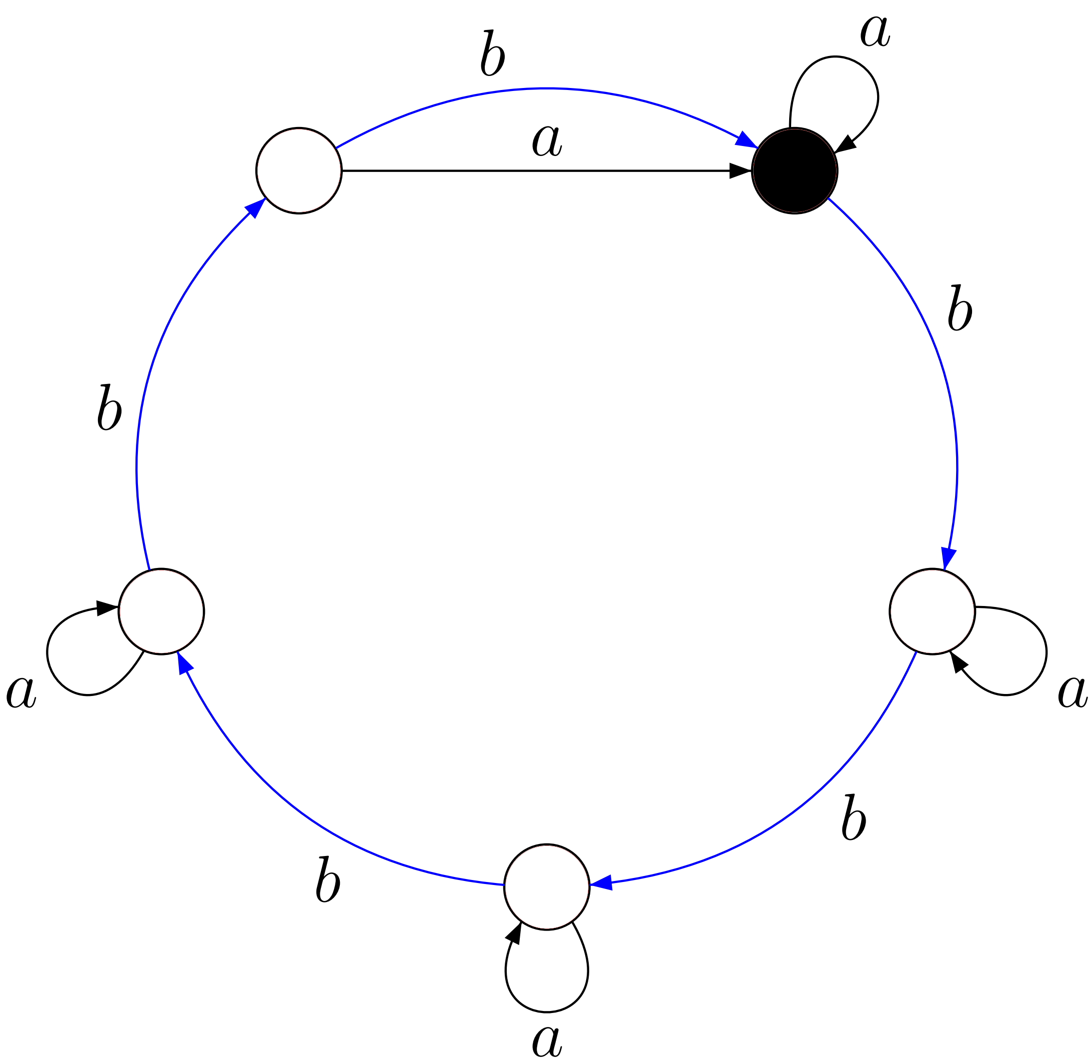

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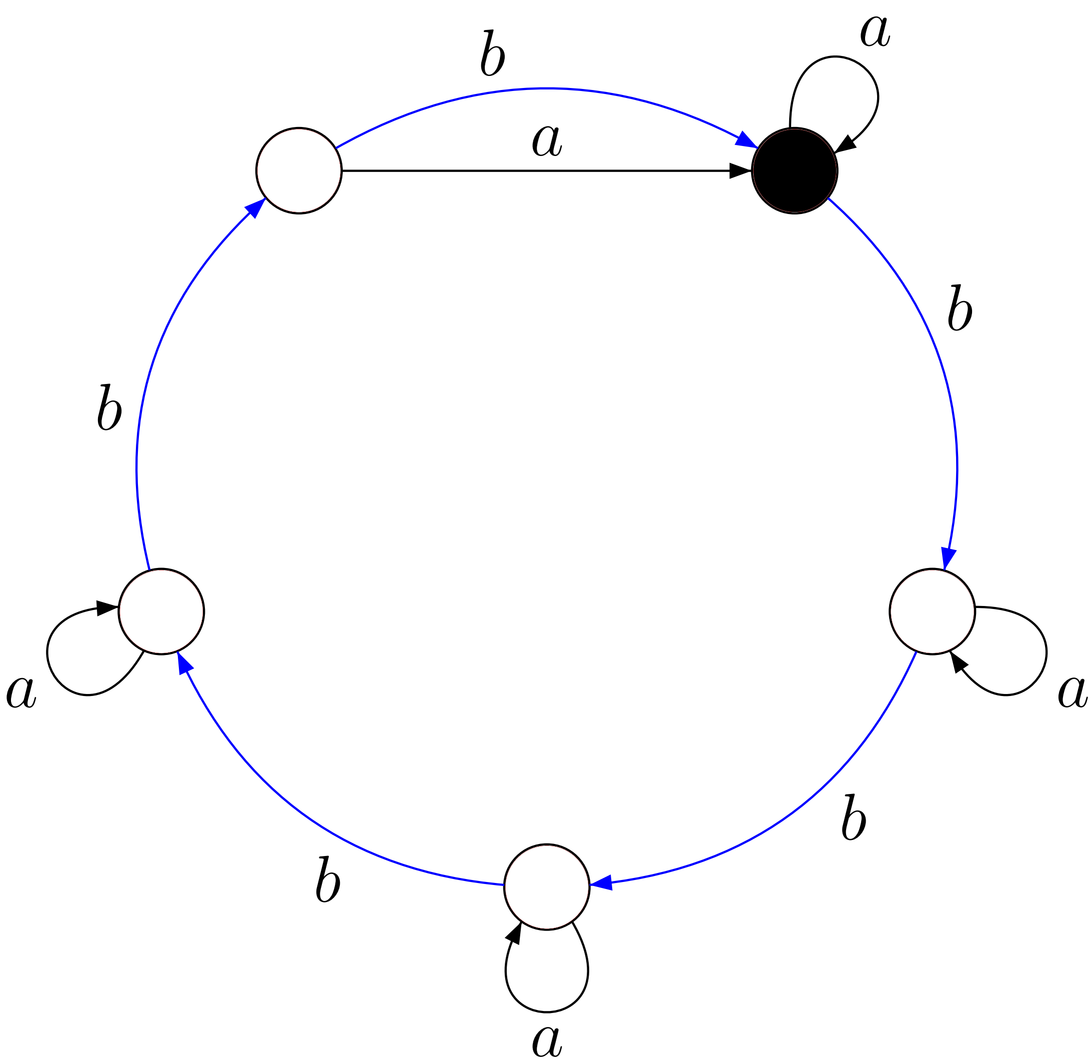

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approximation ratio $\frac{17}{16}$

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We apply it at most $\lceil \frac{n-1}{k-1} \rceil$ times. Therefore $|u| \leq |w| \cdot \lceil \frac{n-1}{k-1} \rceil$.

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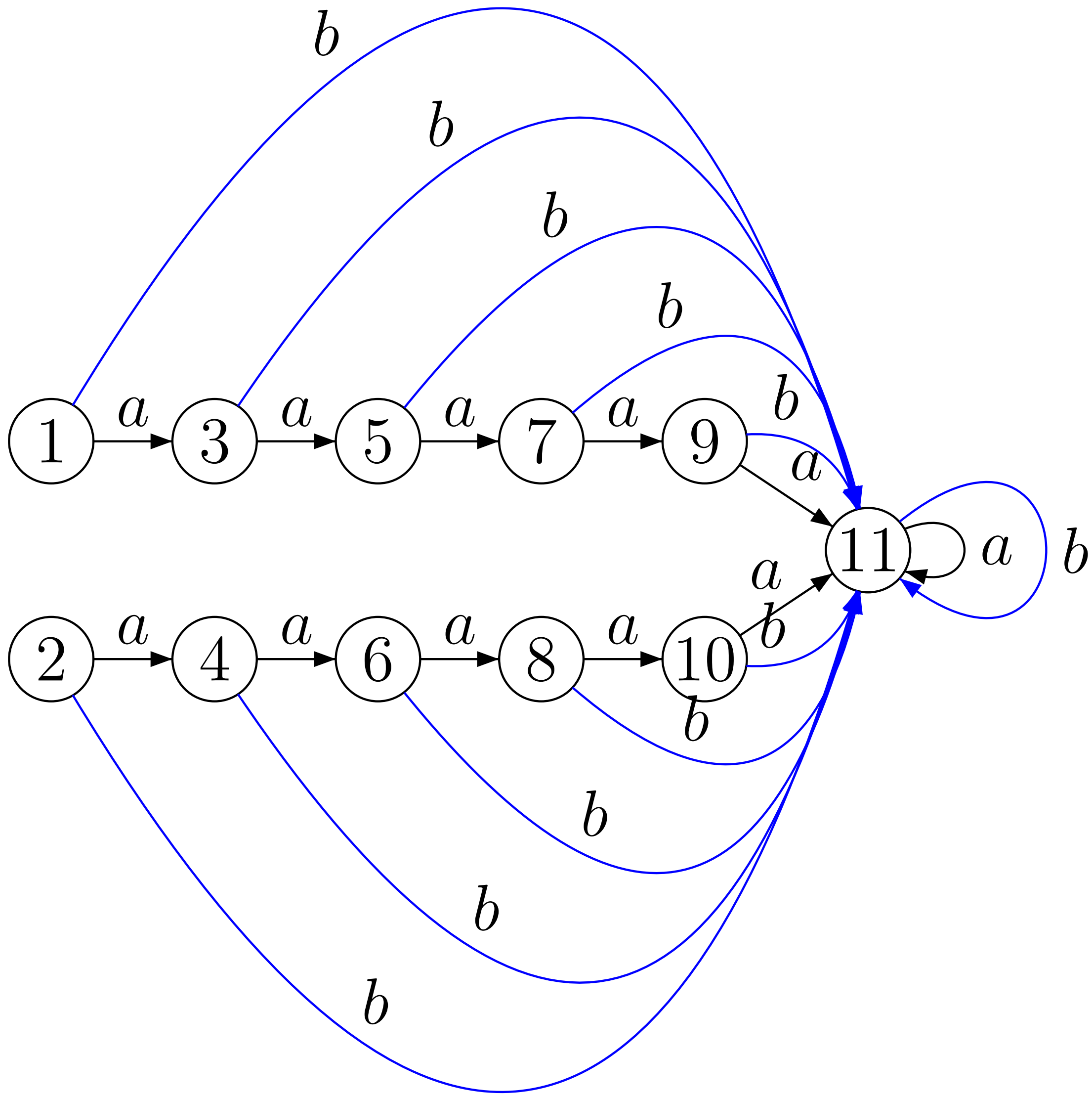
If for each set P the algorithm chooses the word a_P instead of a_Q to merge the states, then only k states are merged at a time. In this case, the word u that the algorithm gives has length $\lceil \frac{n-1}{k-1} \rceil$.

Example: 2 letters and greedy choices (k=3)



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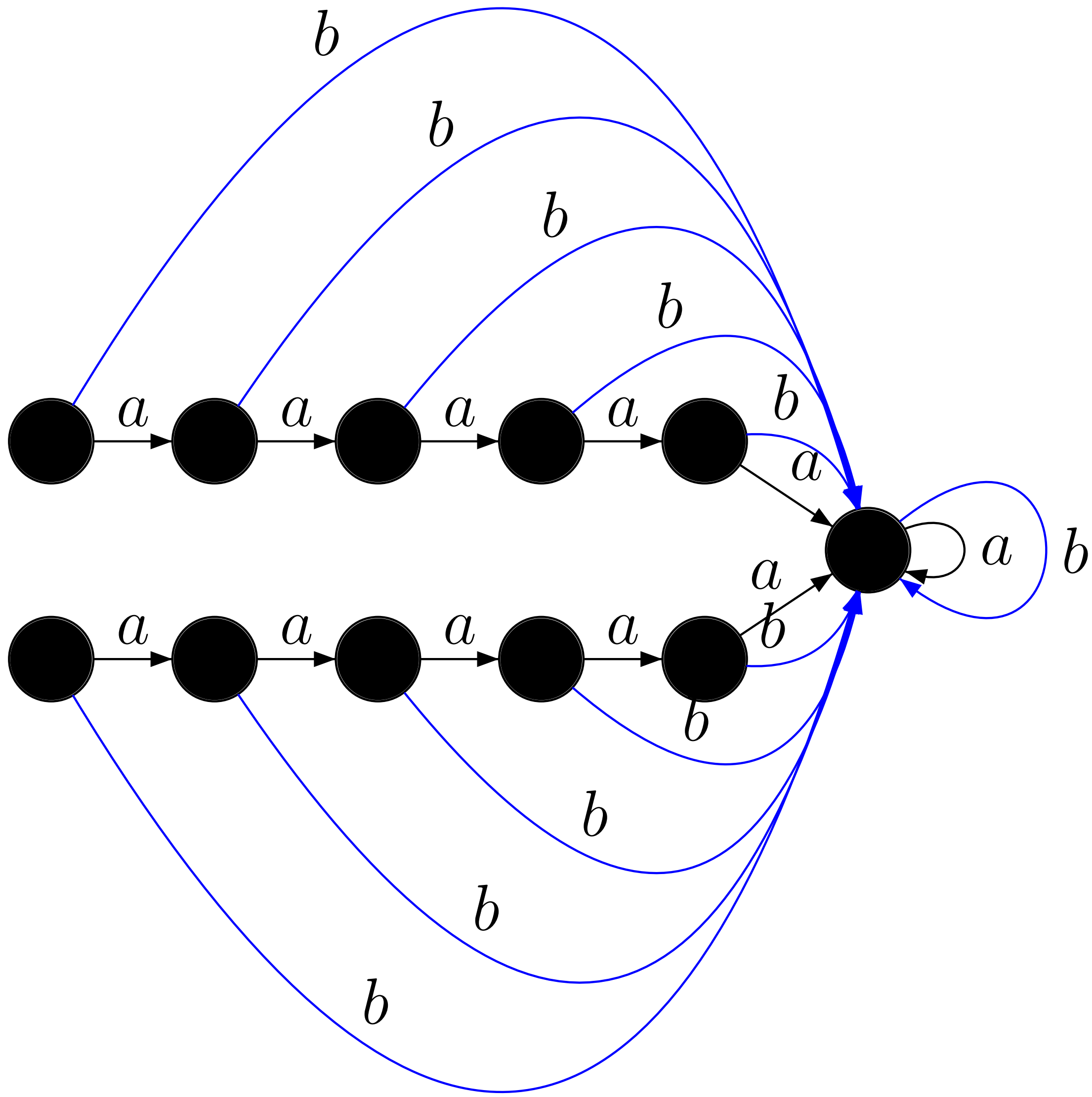

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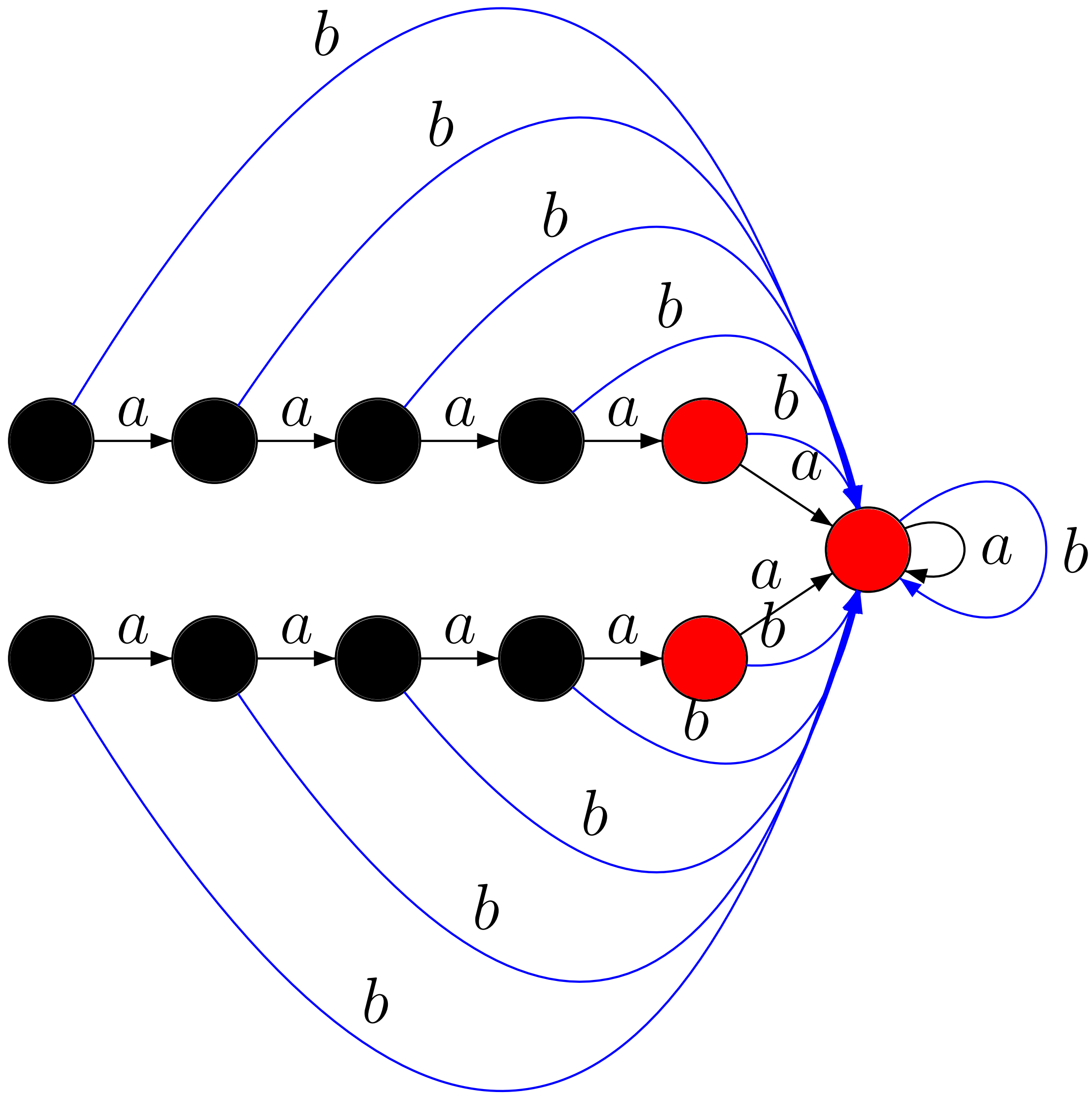

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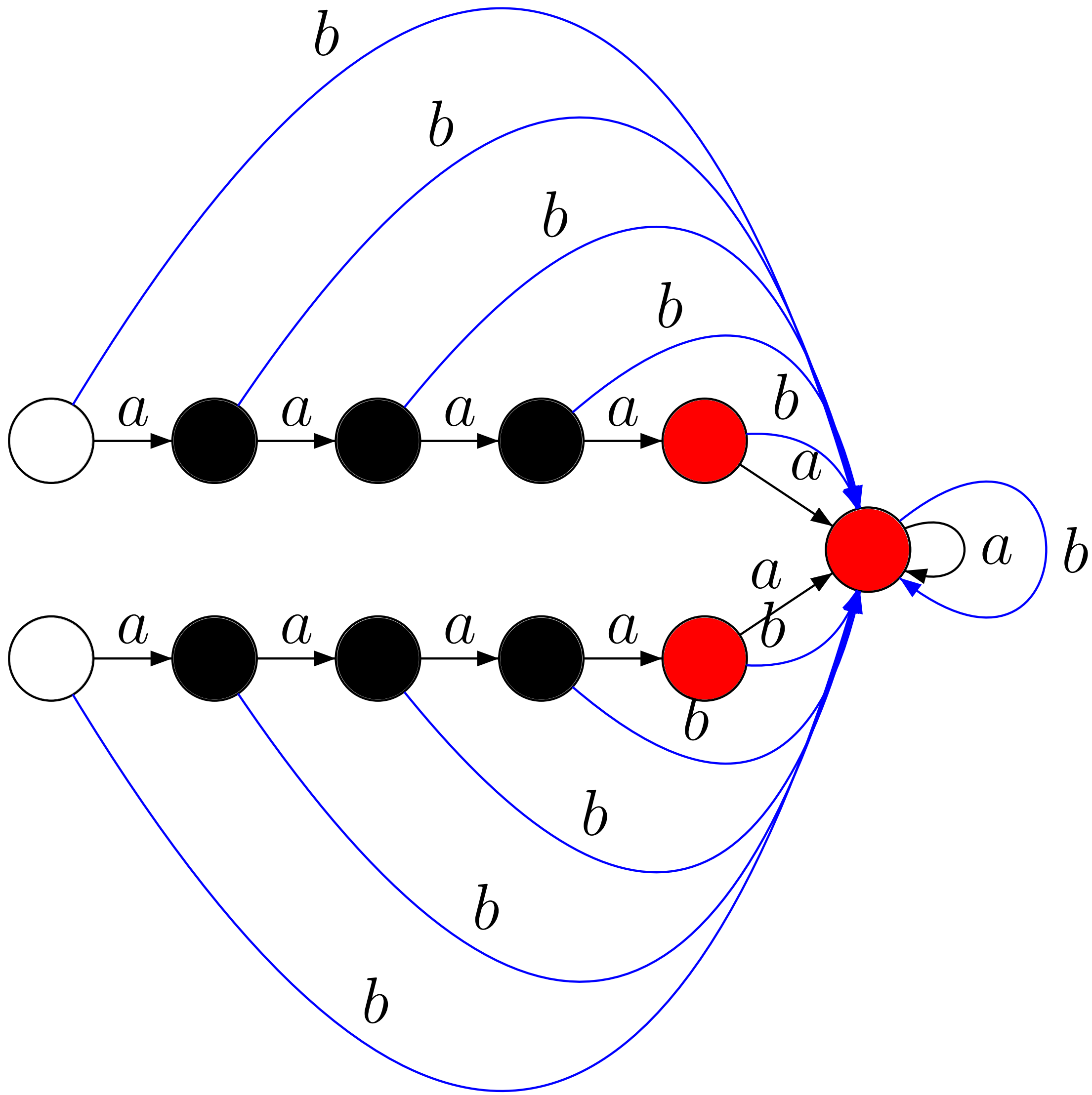

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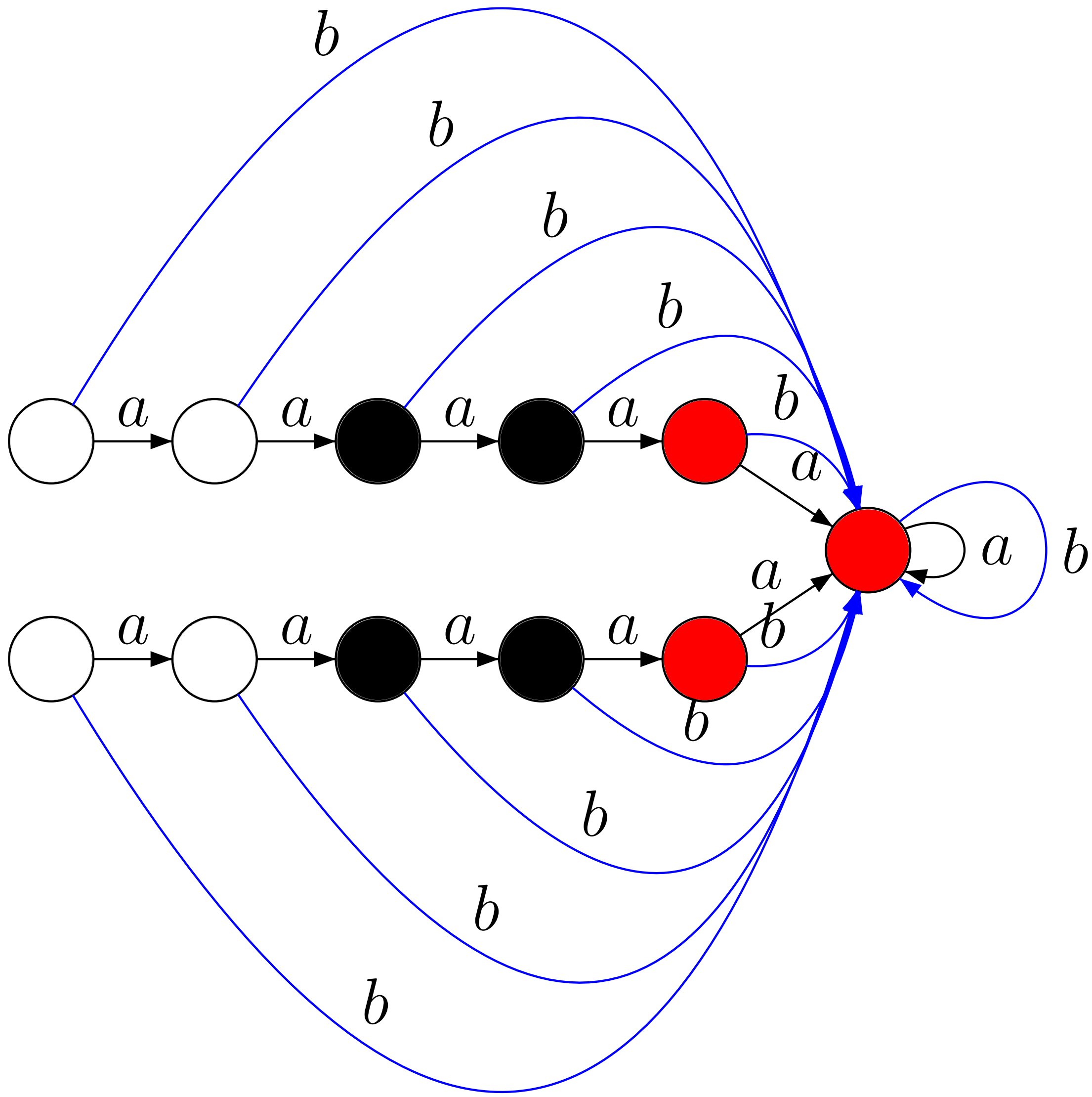

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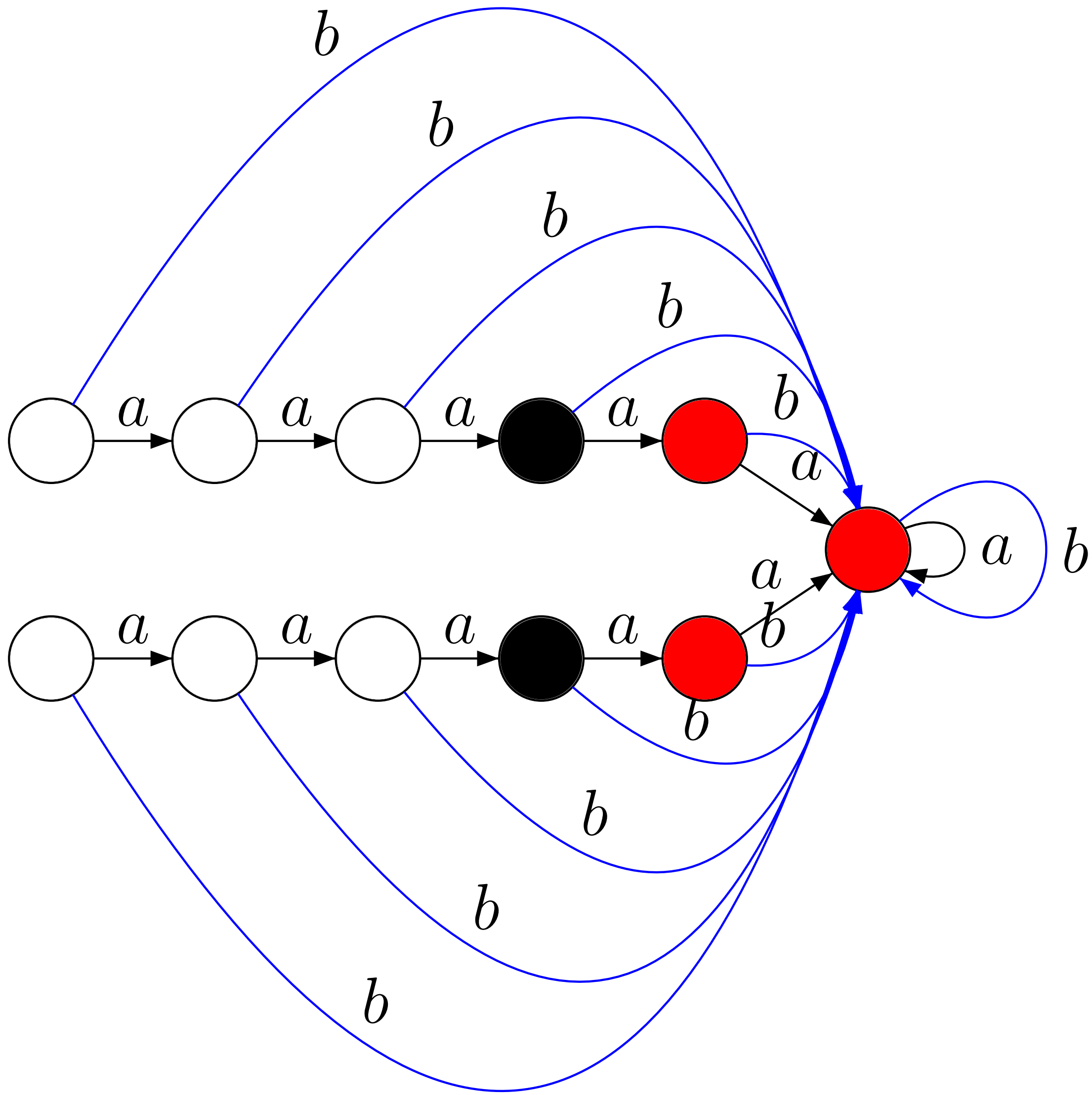

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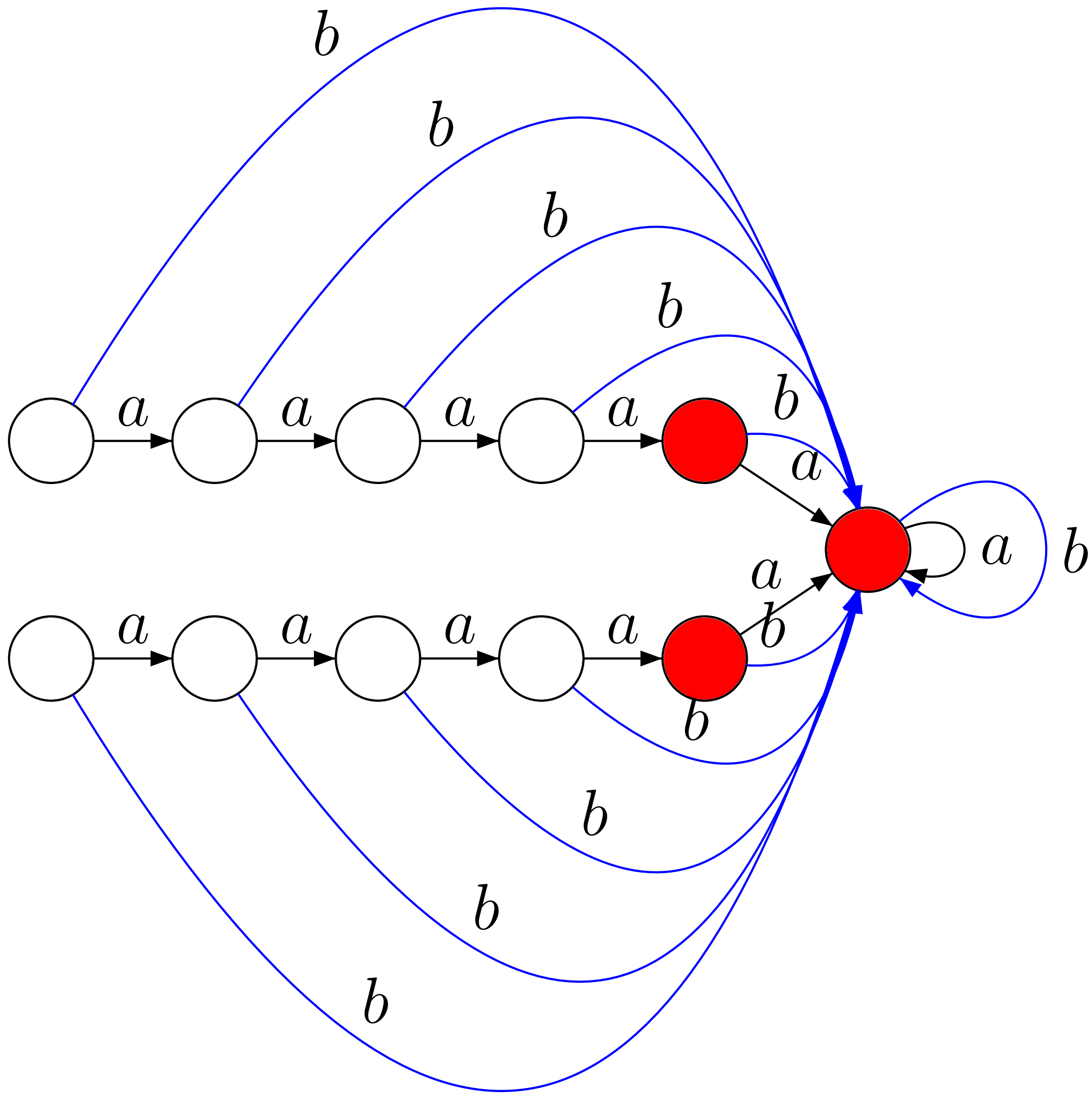

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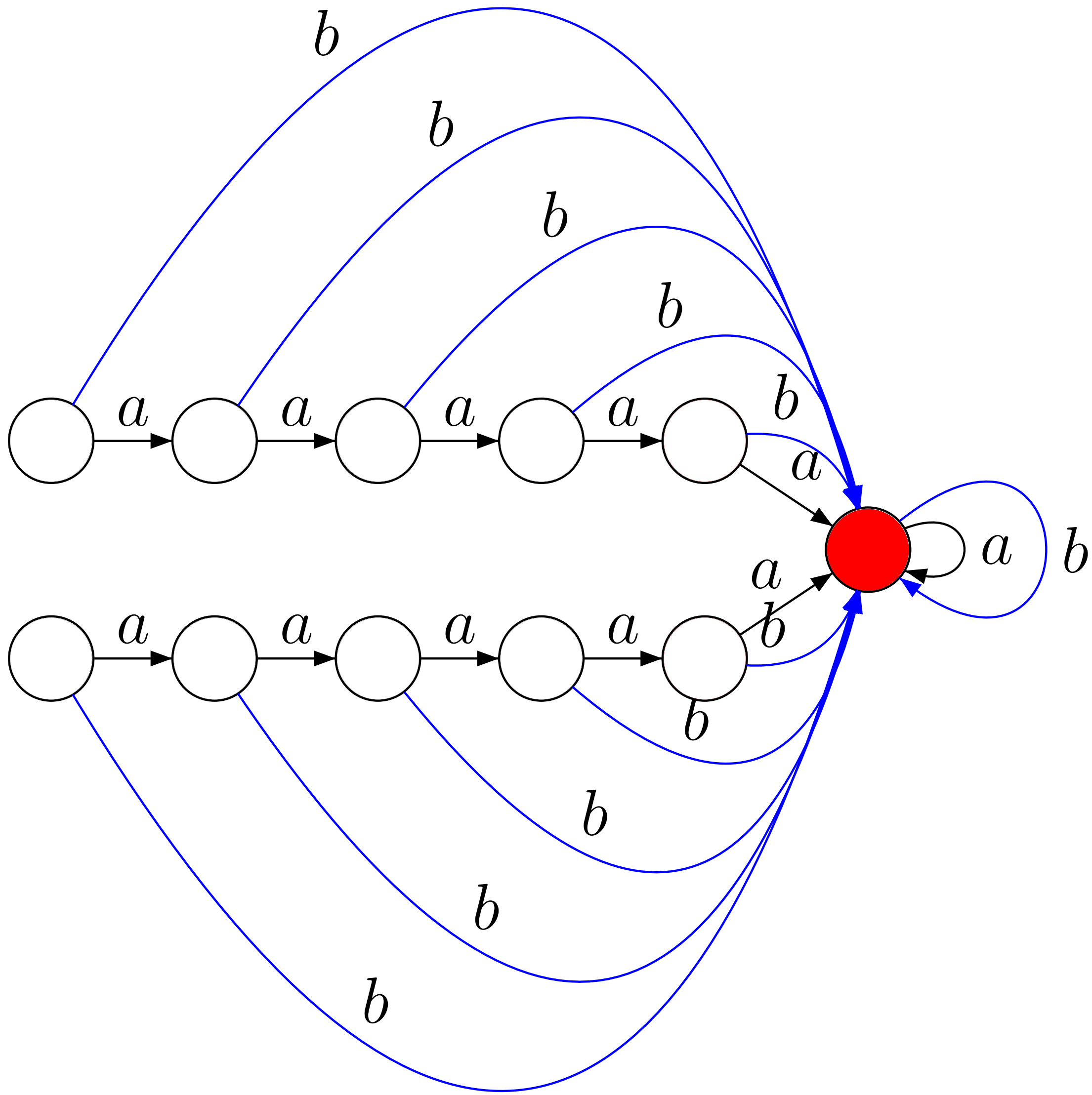

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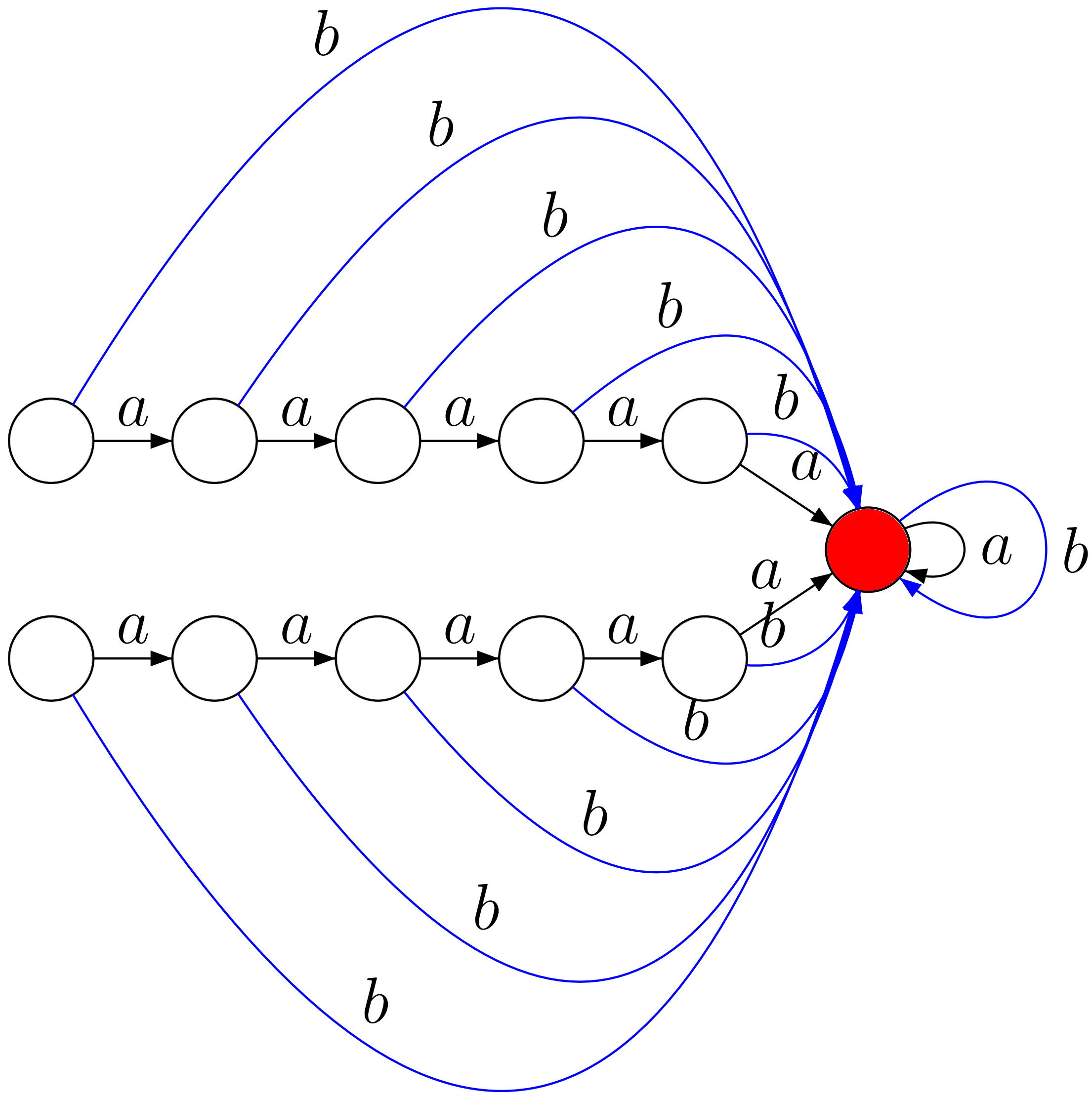

approximation ratio 5 or $\lceil \frac{n-1}{k-1} \rceil$

Perfect greedy algorithm

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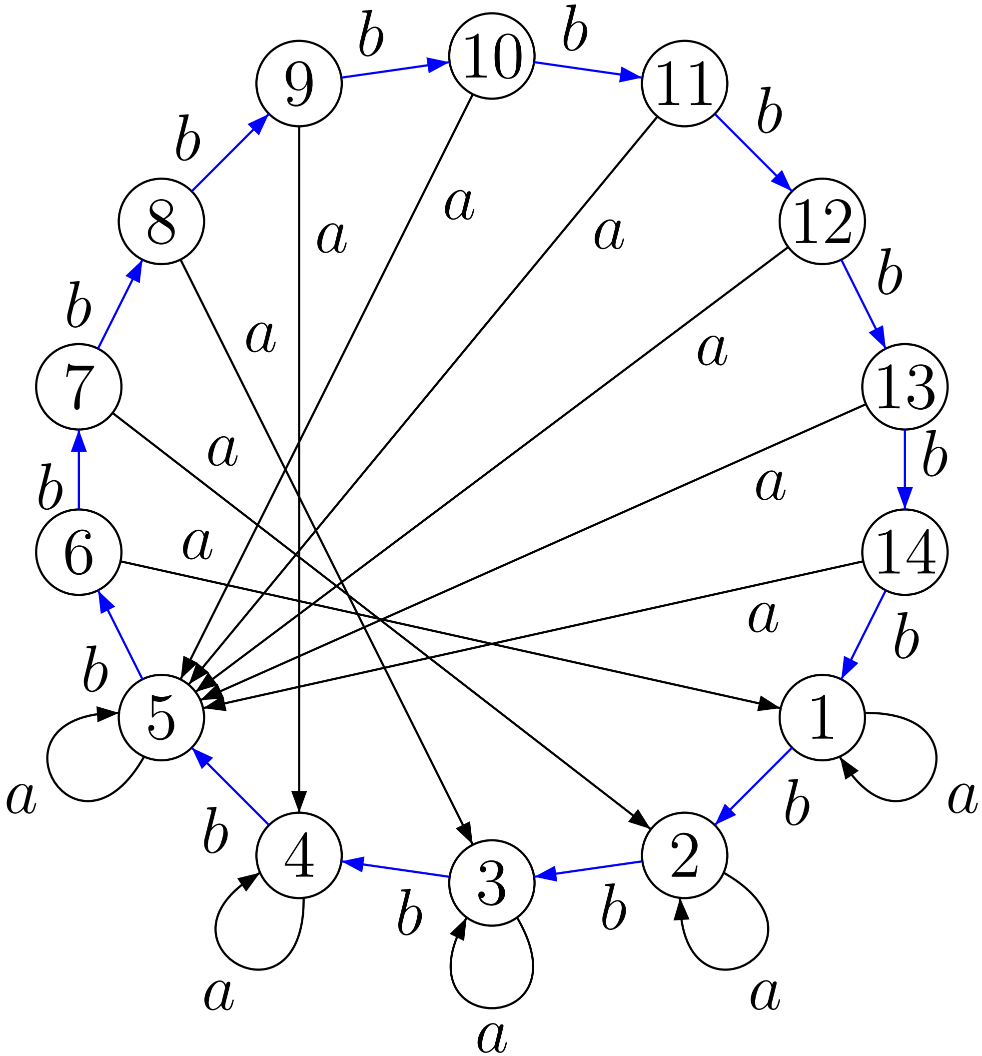
The example for perfect greedy algorithm gives the lower bound of approximation ratio for arbitrary greedy algorithm

“Honest” example: shortest reset word



$abbbbbbbba$

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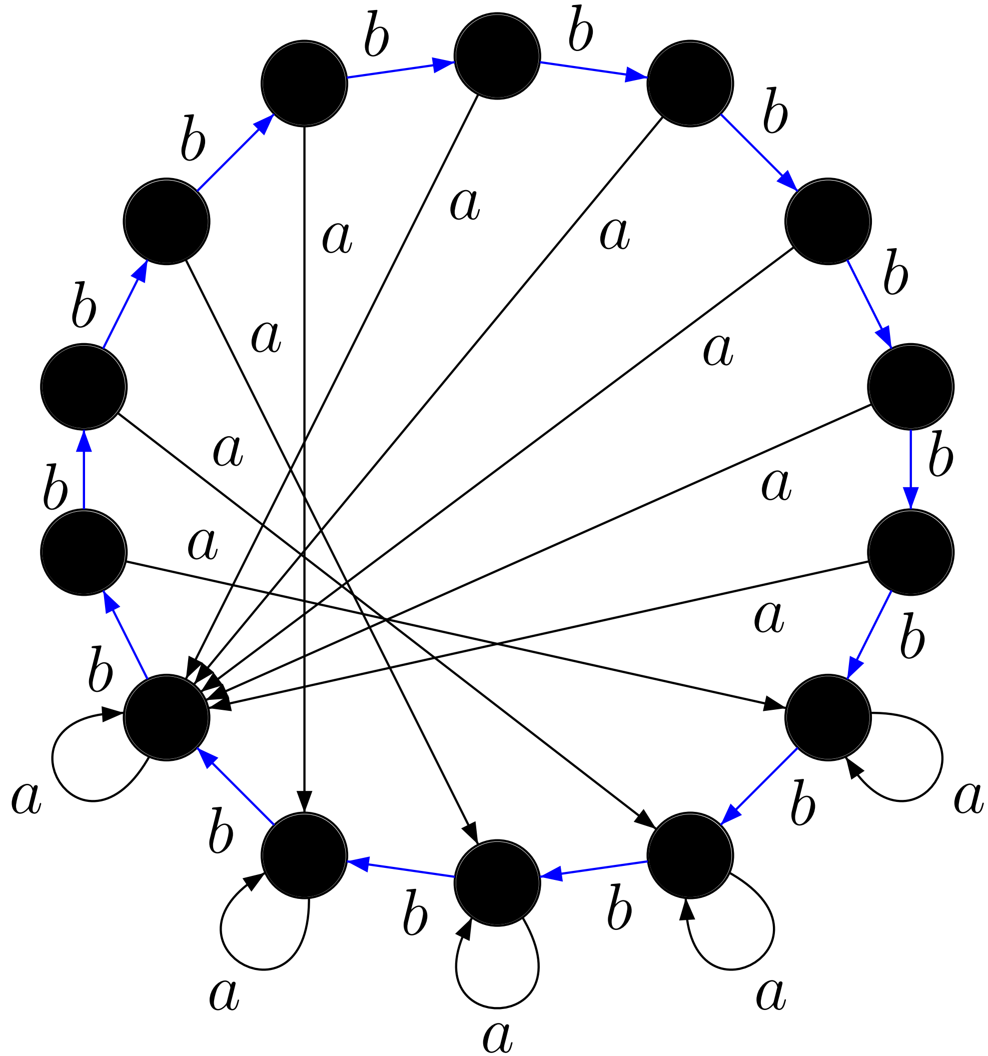

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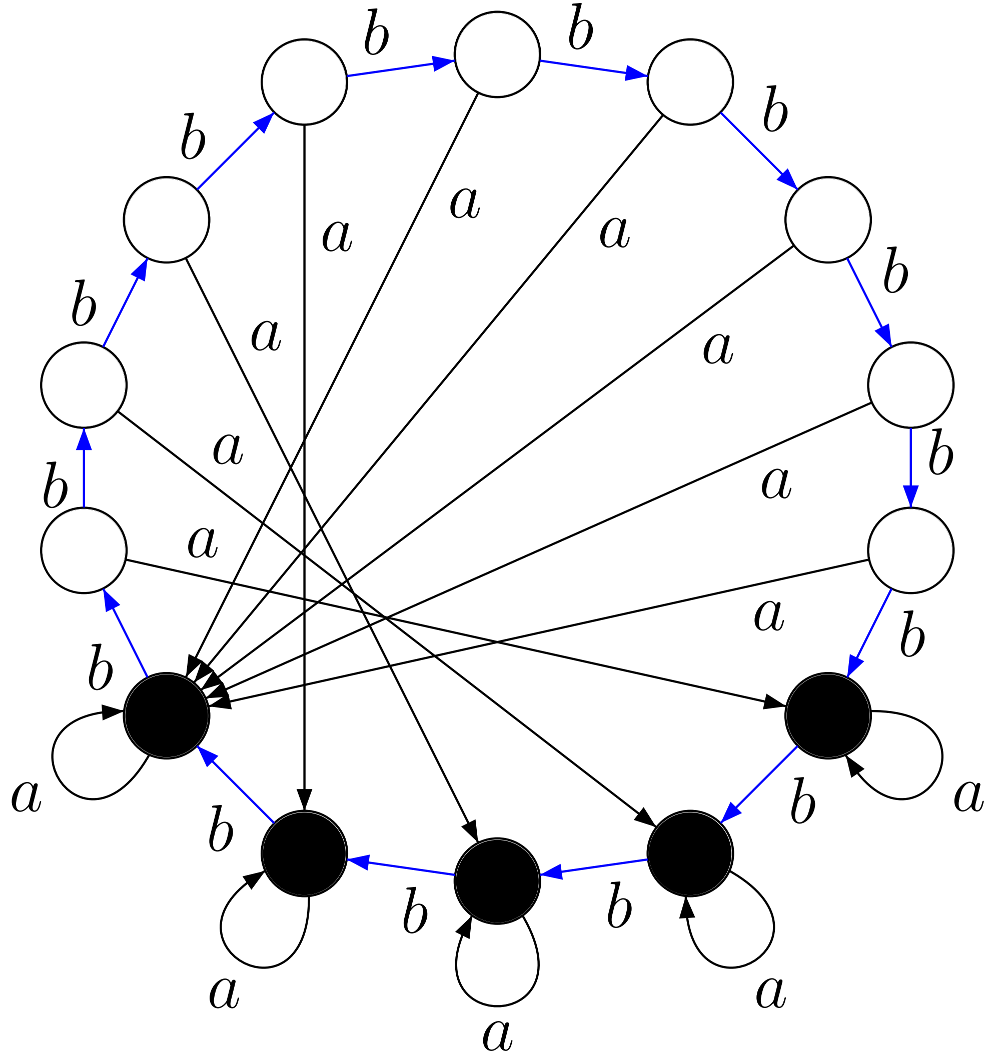

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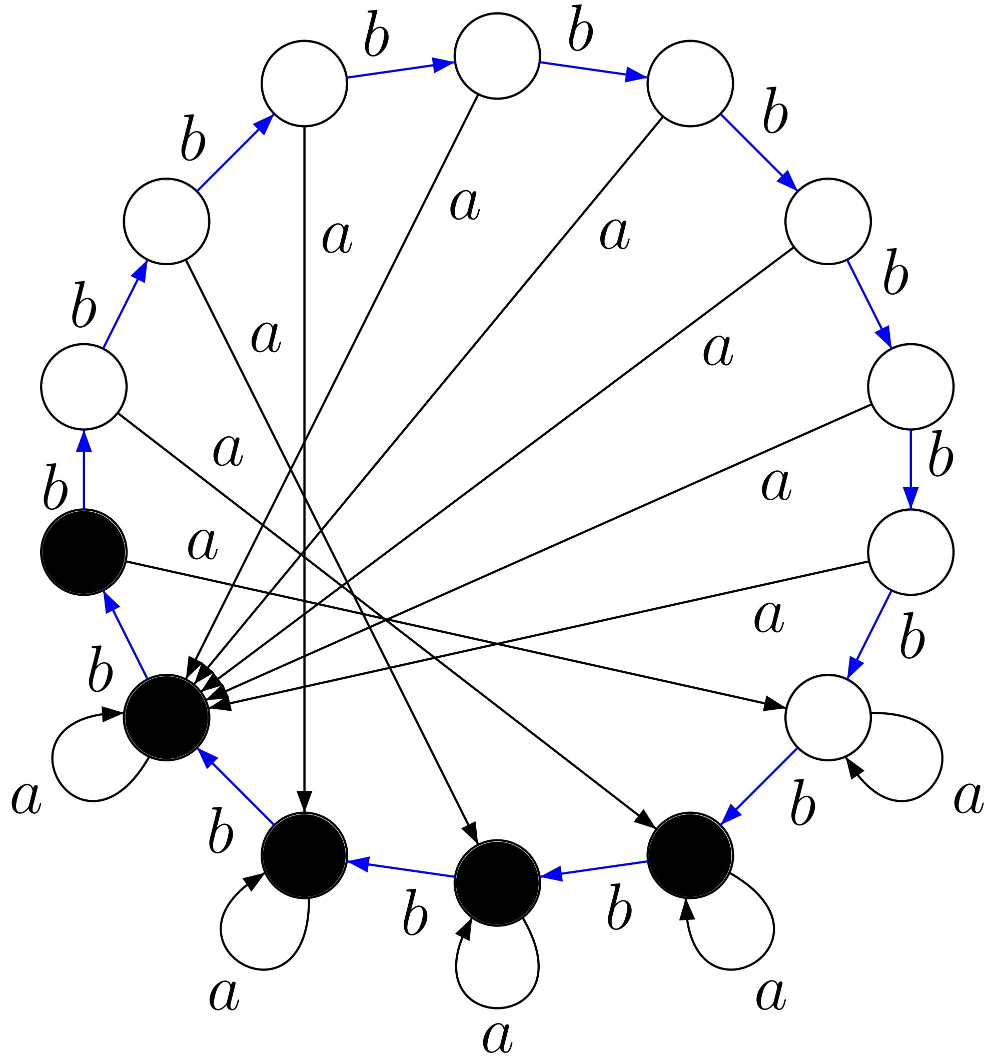

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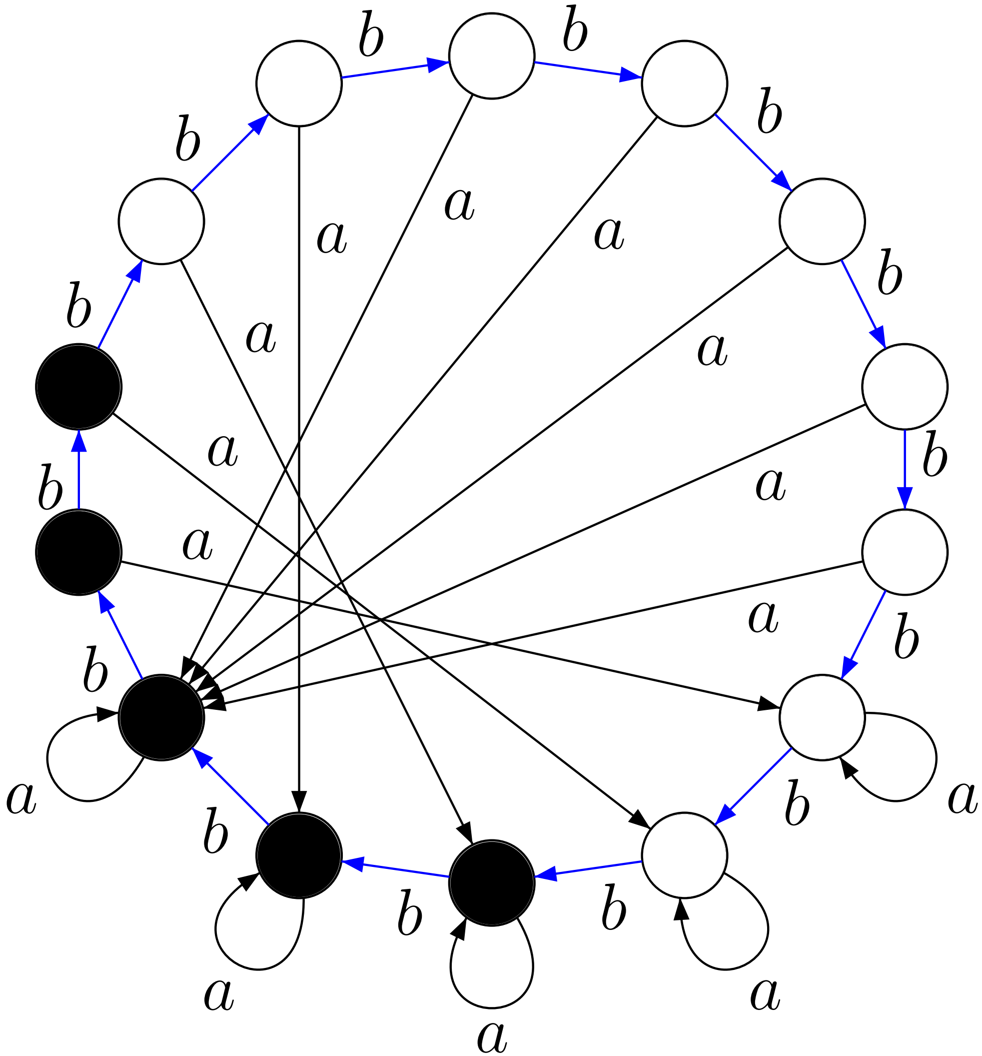

$ab\textcolor{red}{b}bbbbbbba$

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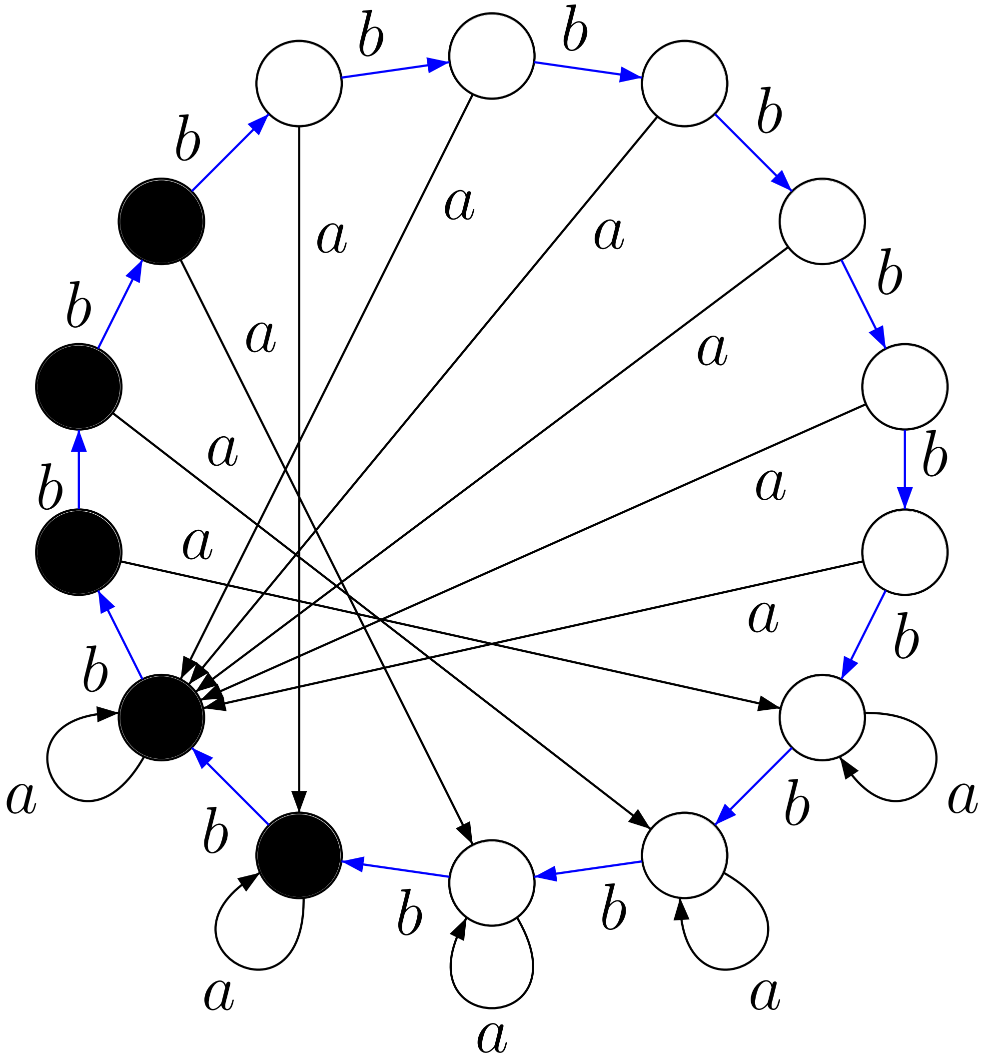

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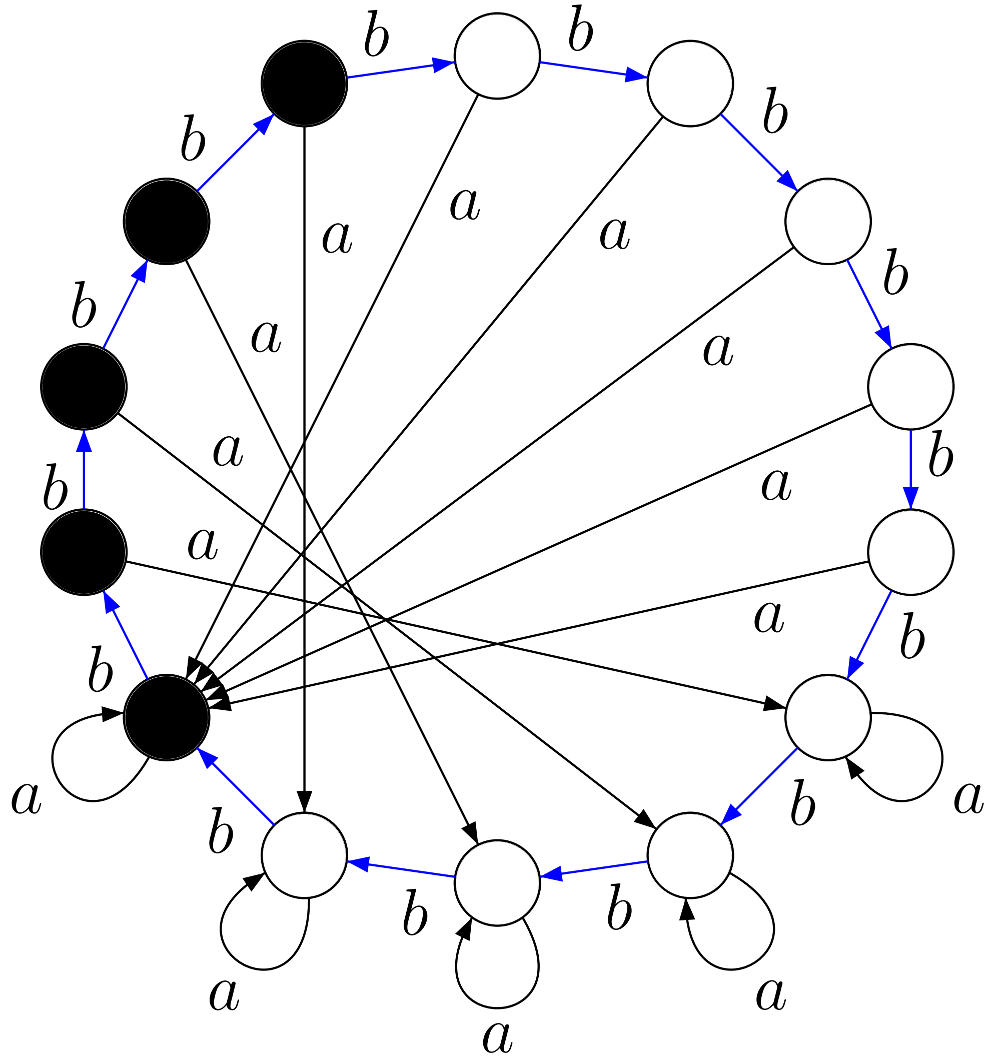

$abbbbbbbba$

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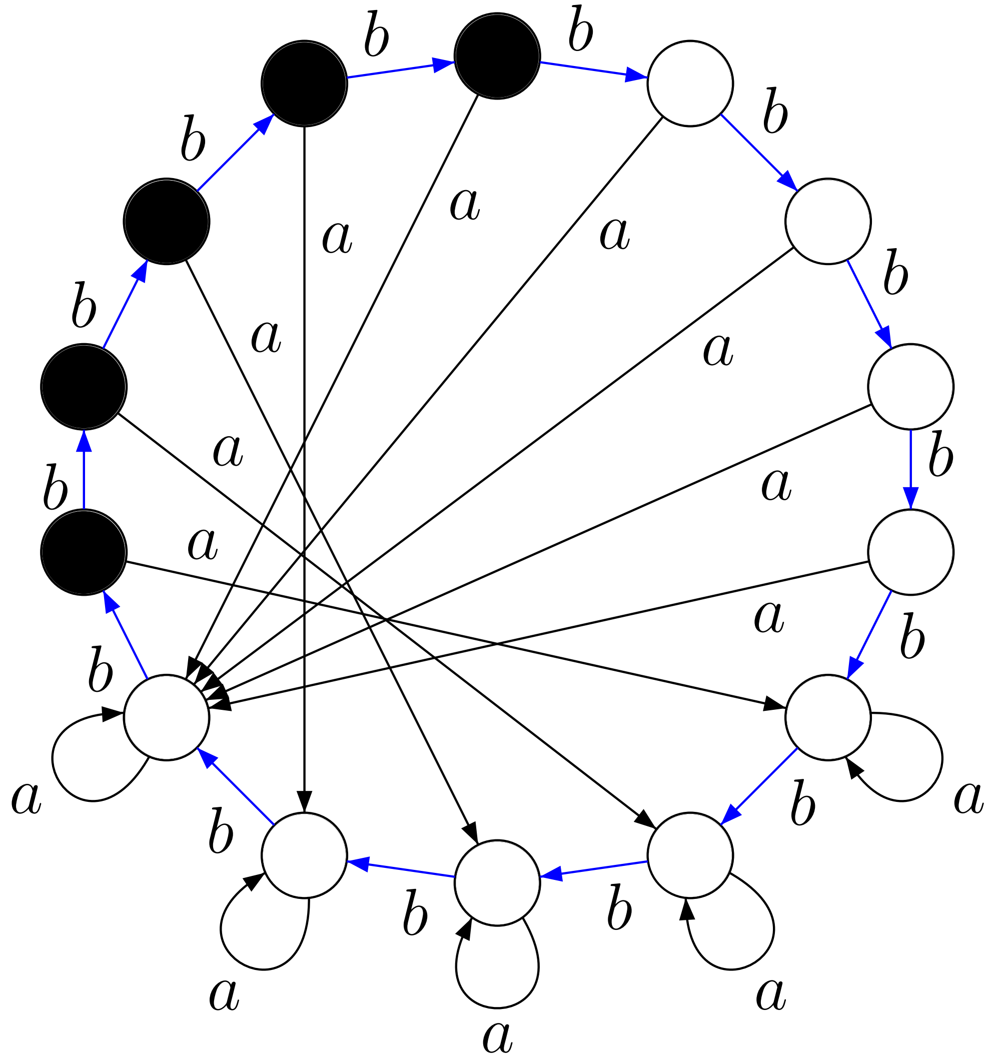

$abbbb\color{red}{b}\color{black}{bbbba}$

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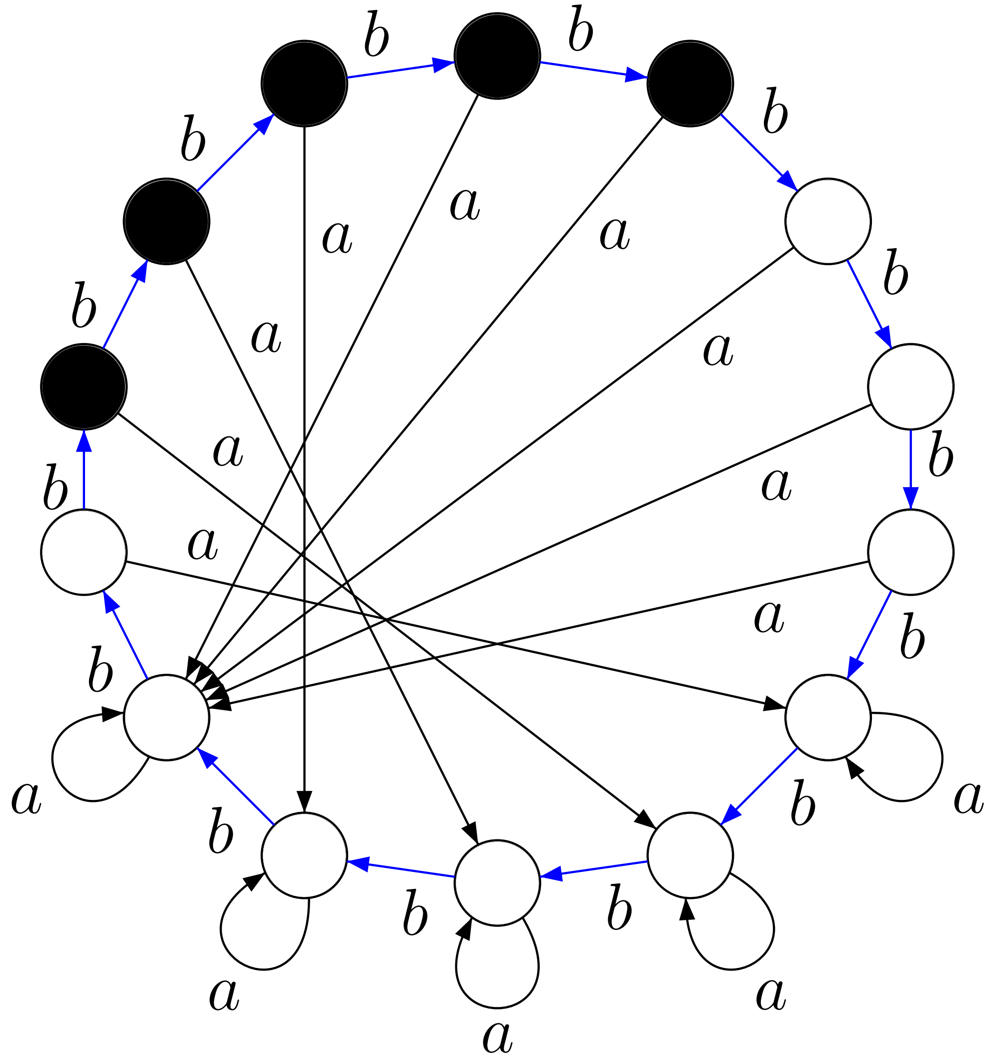

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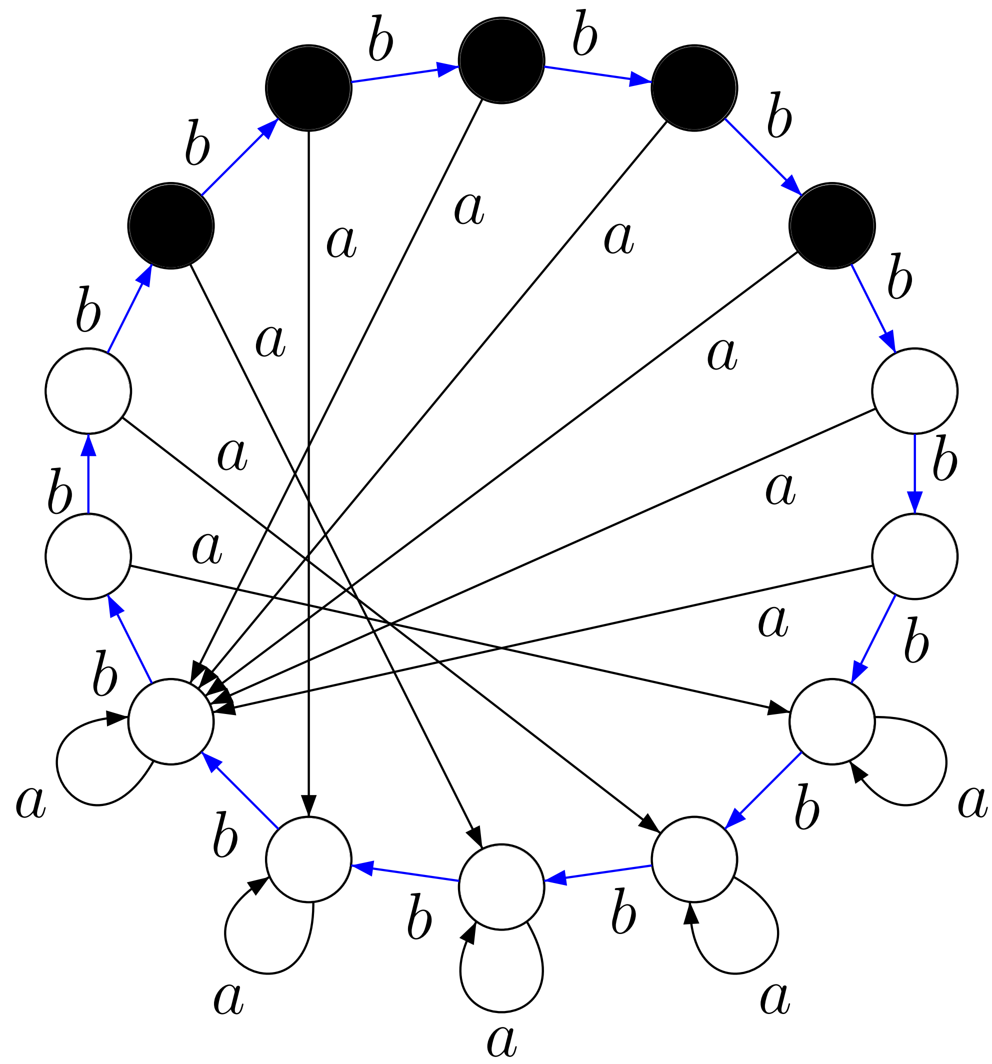

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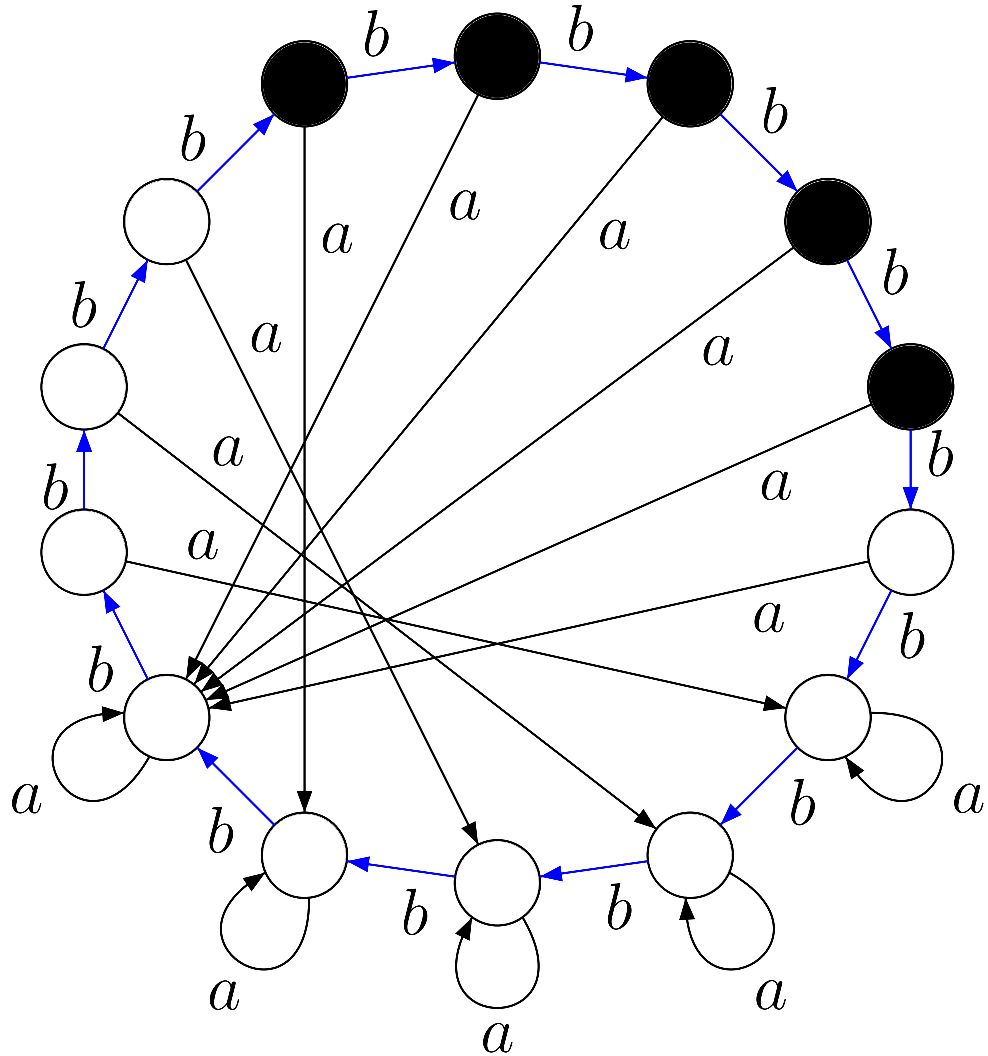

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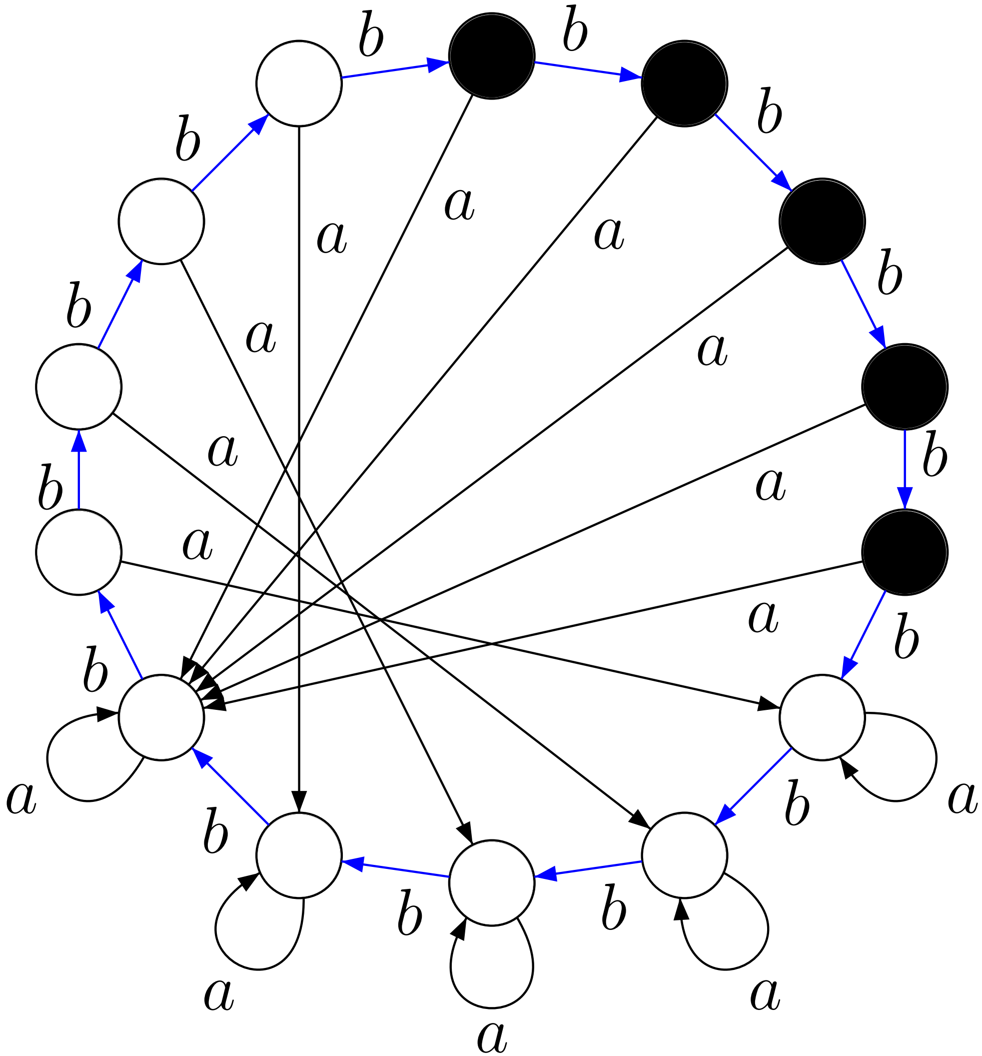

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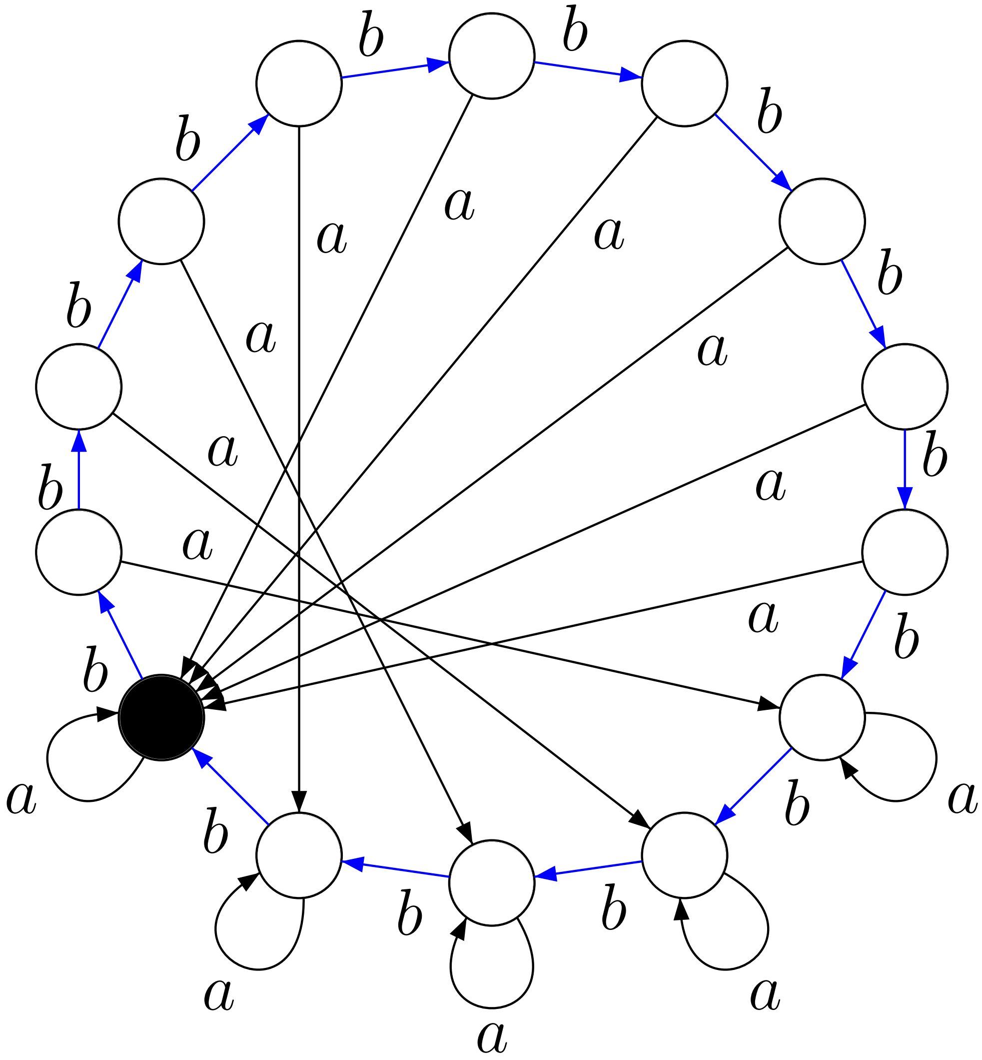

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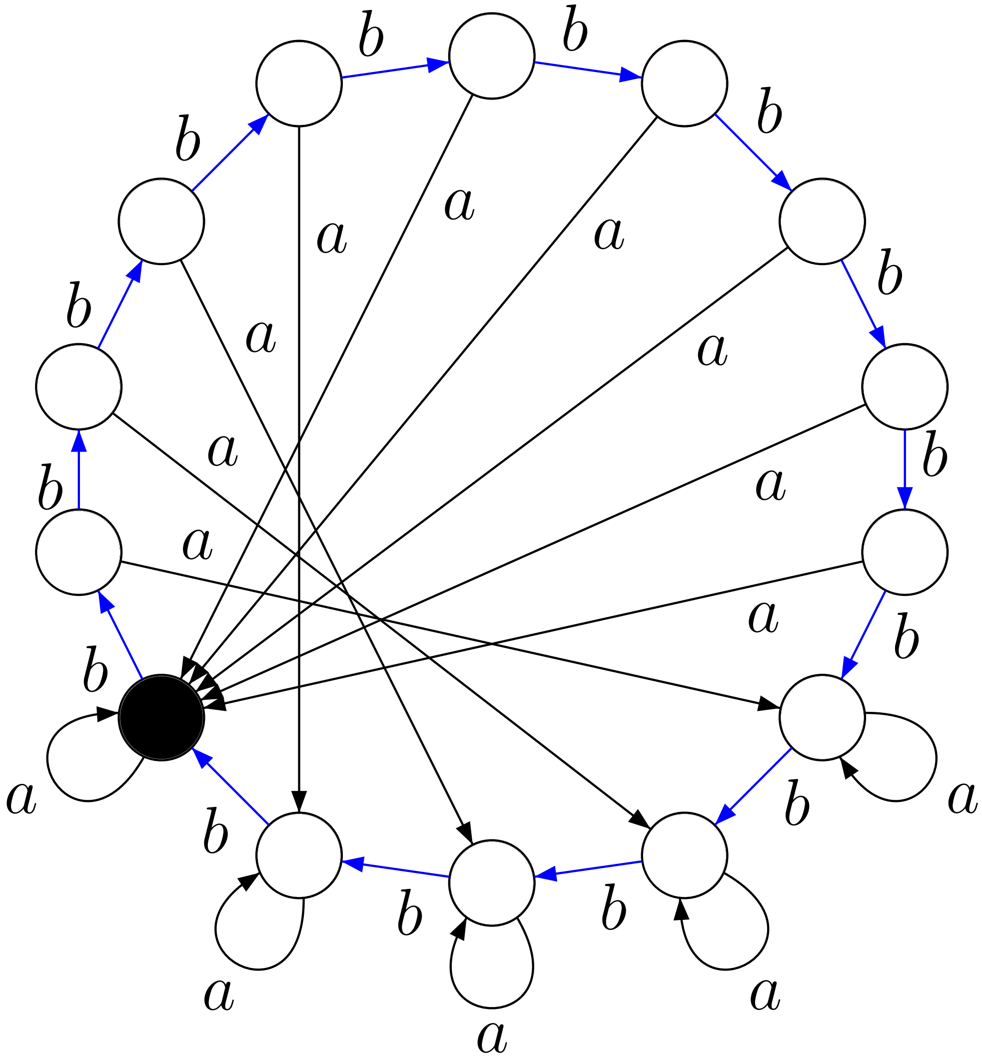
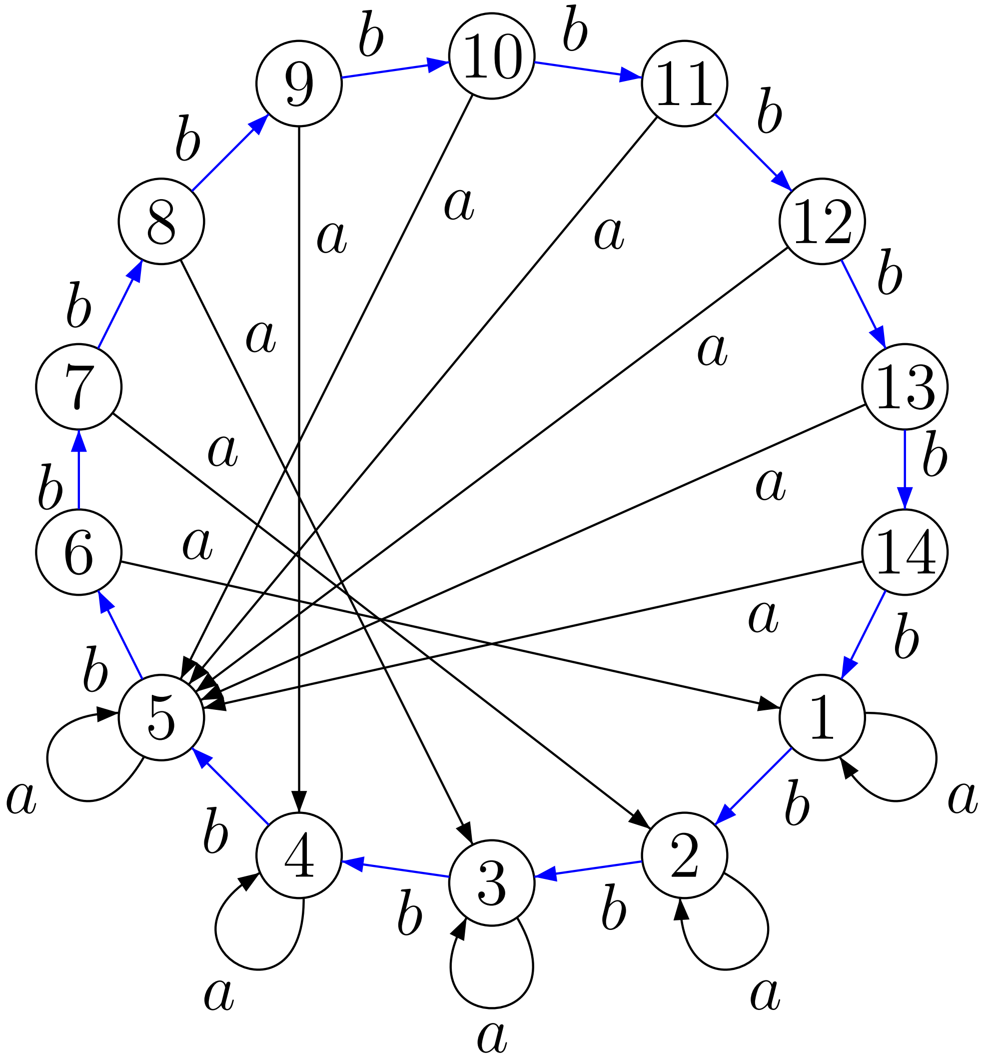


$ab b b b b b b b b \textcolor{red}{a}$

reset threshold 11 or $\frac{2(n+1)}{3} + 1$

“Honest” example: “greedy” reset word (k=2)



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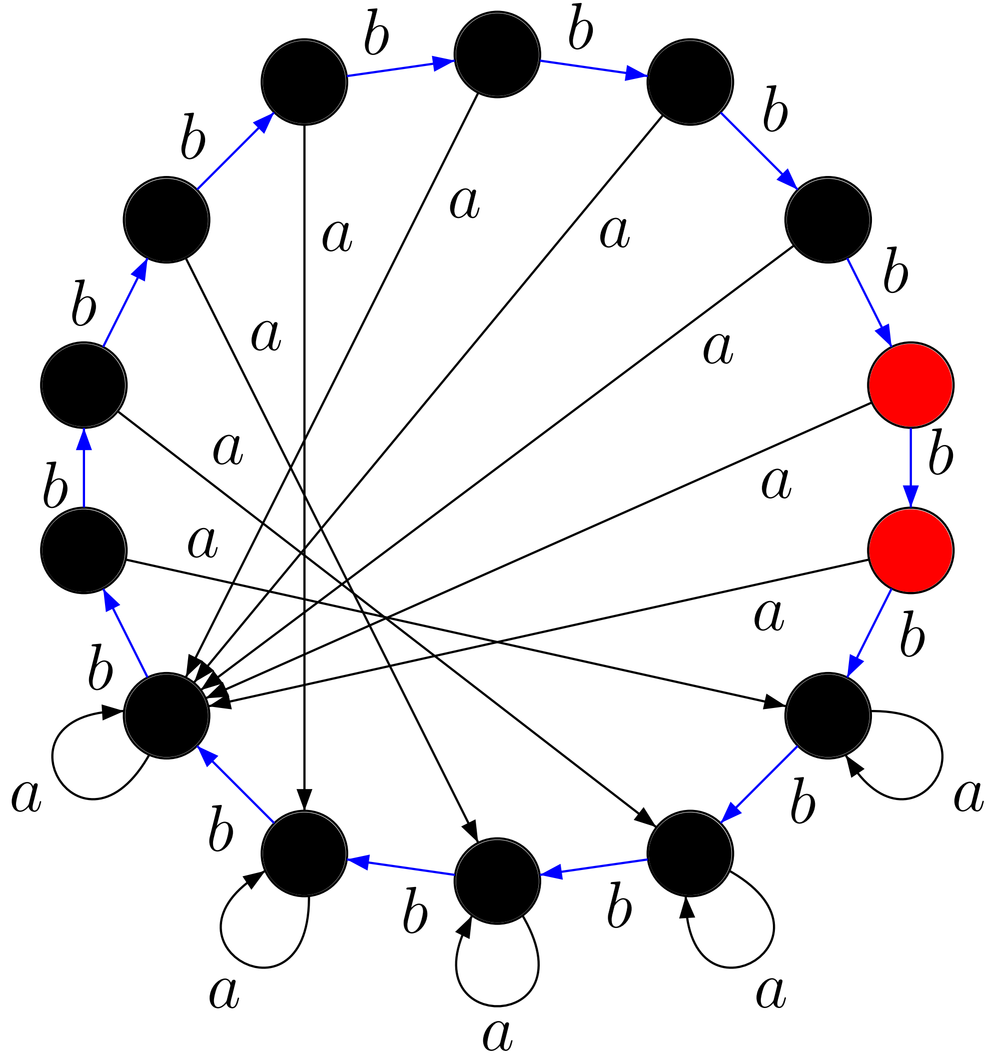

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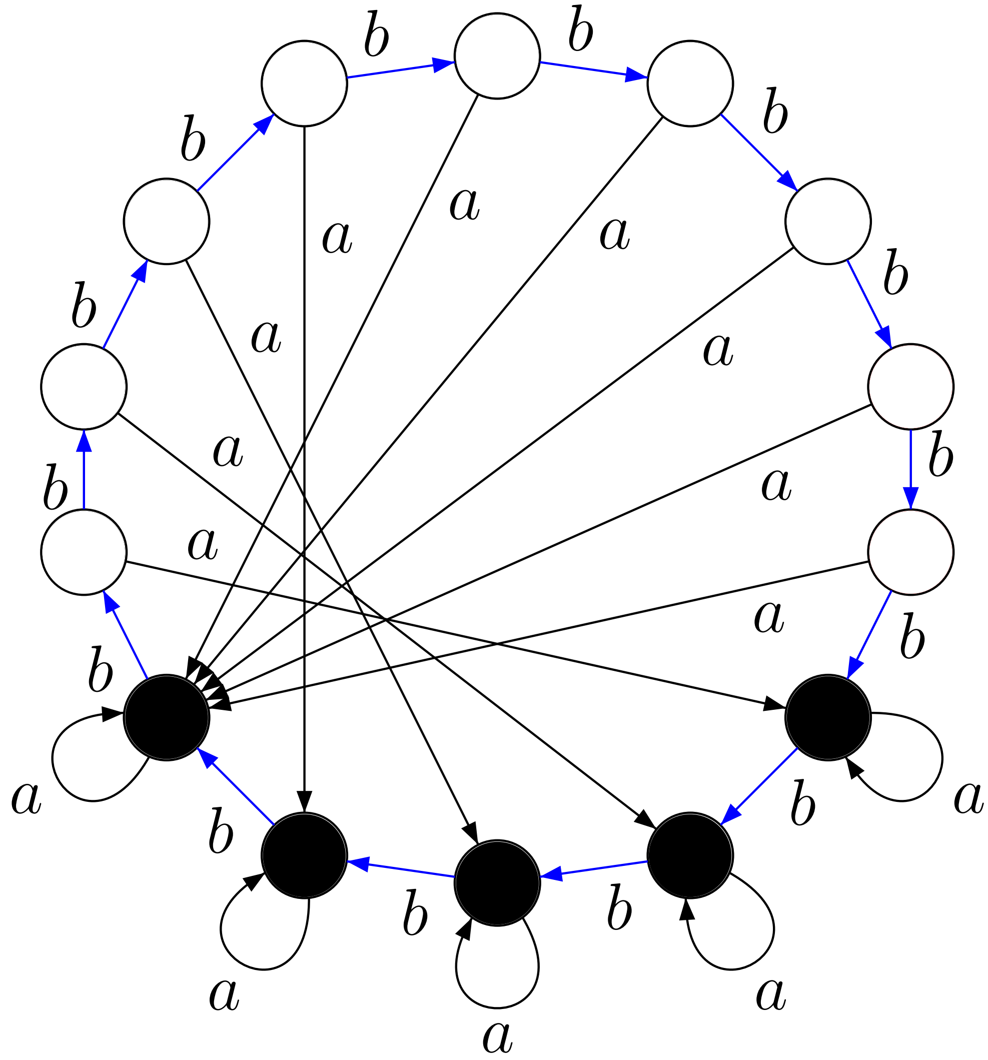

a

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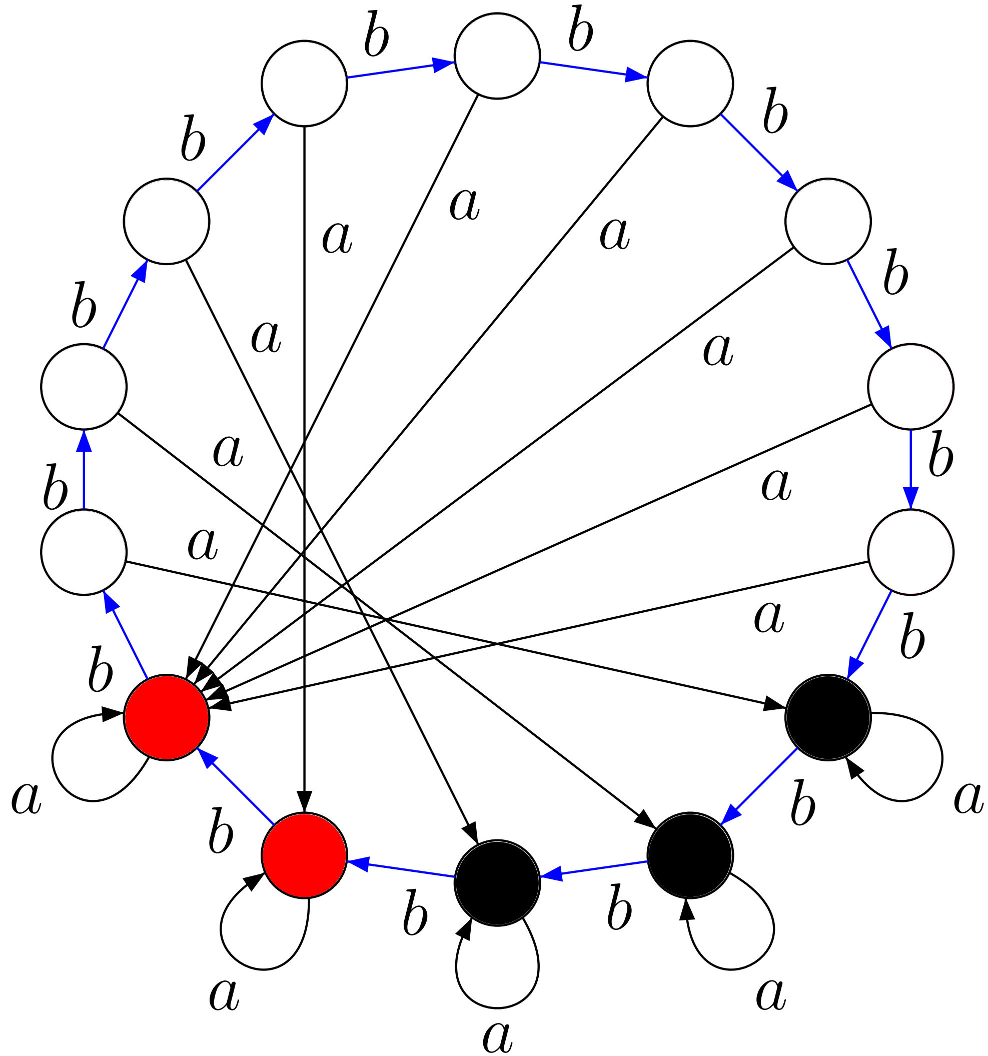

$abbbbbba$

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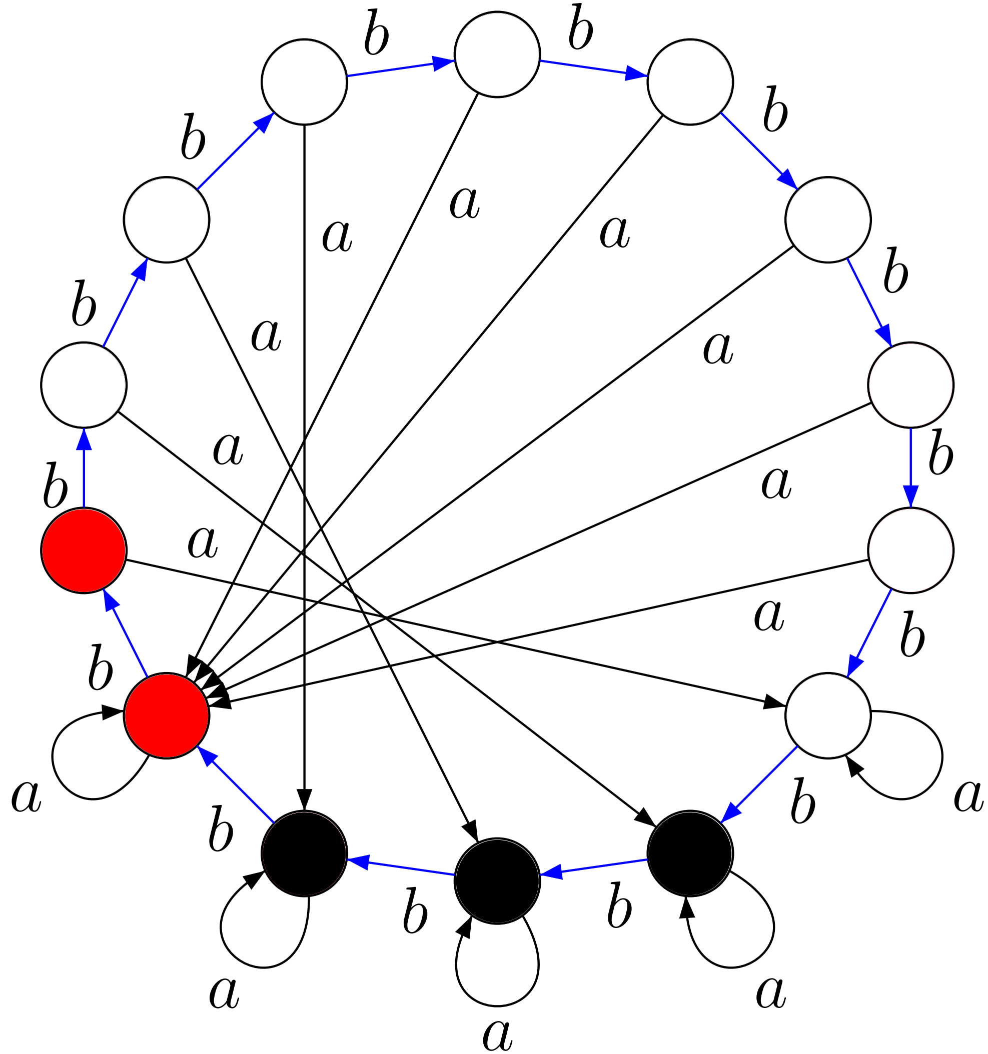

$a\textcolor{red}{b}bbbbba$

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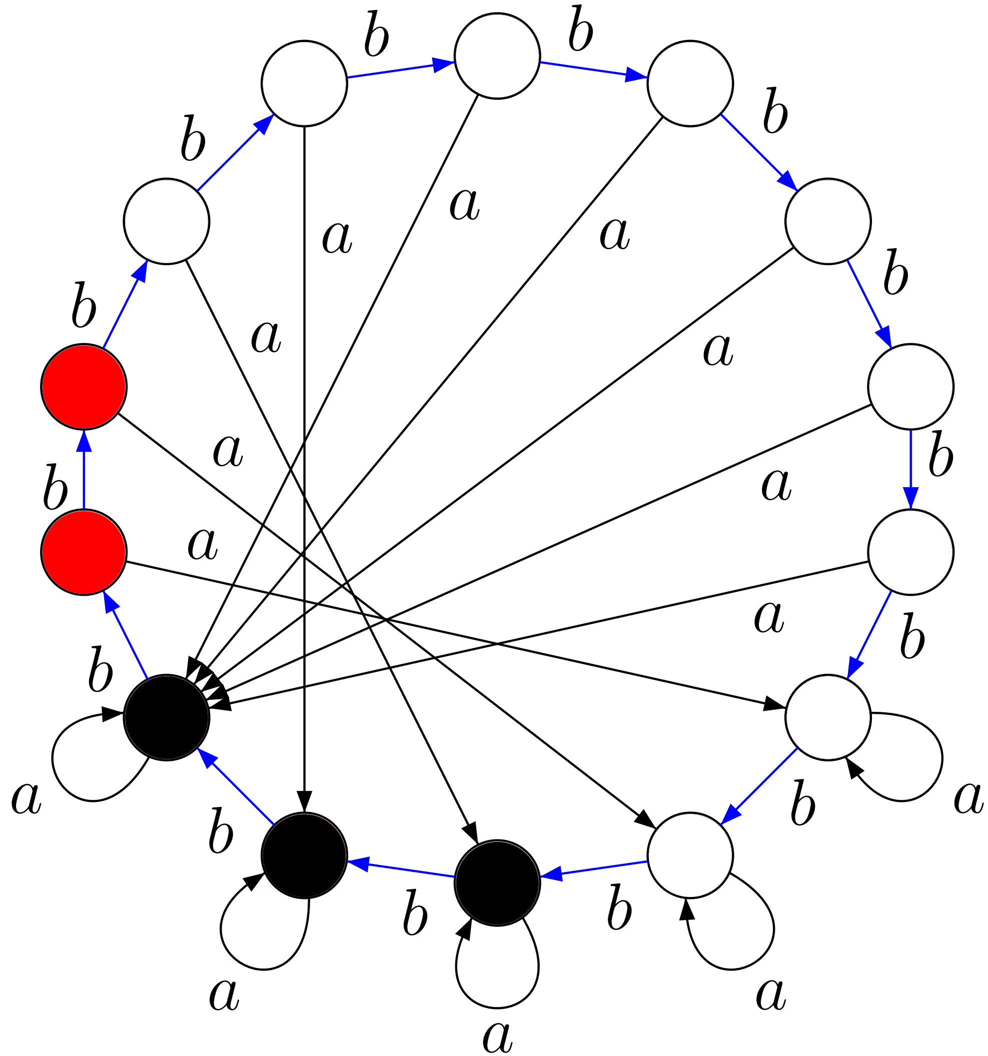

$abbbbbba$

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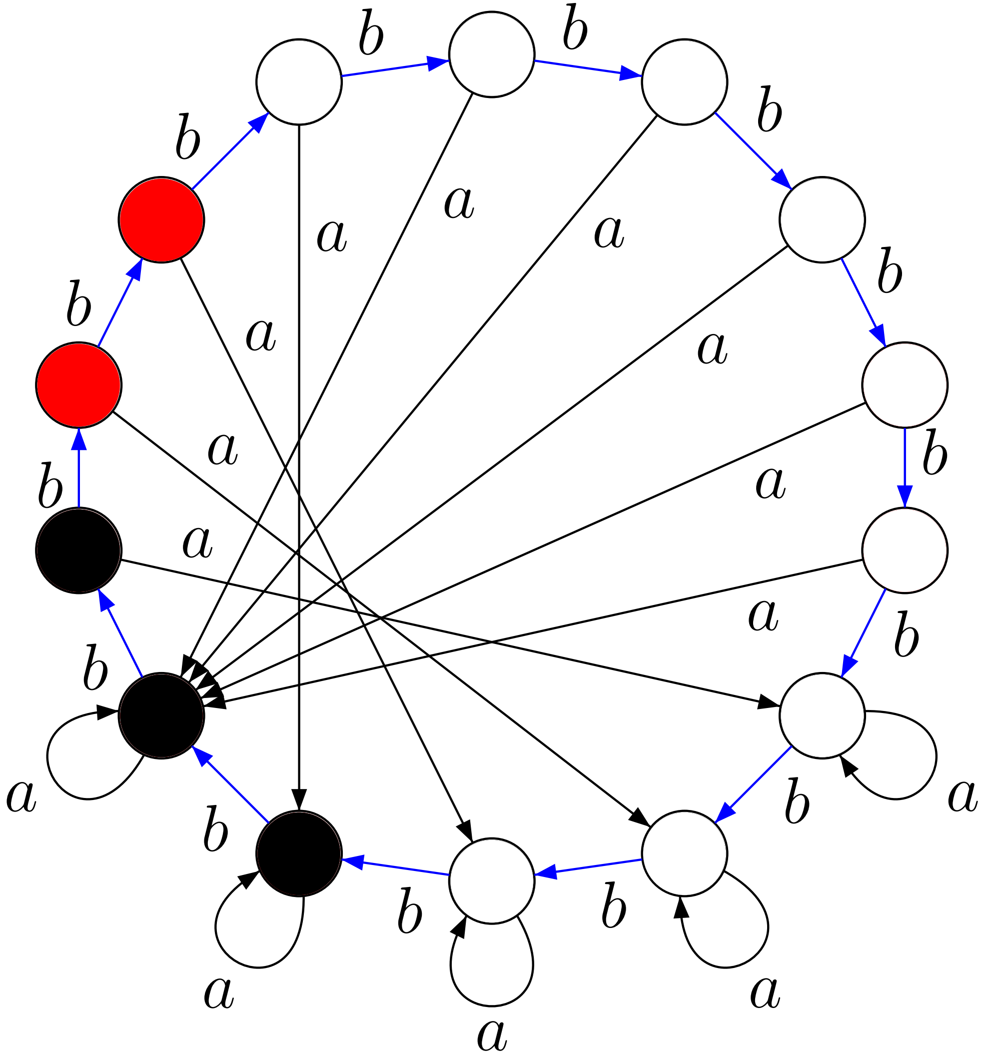

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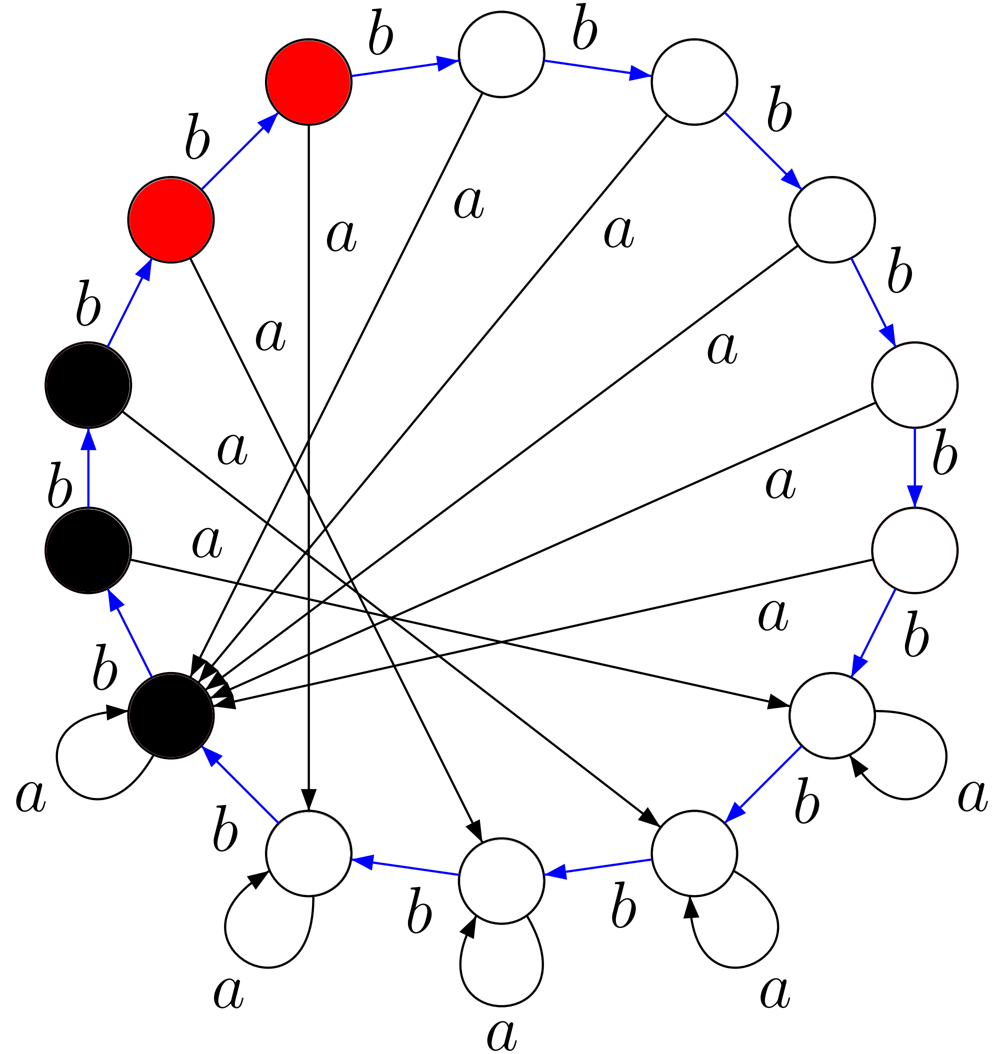

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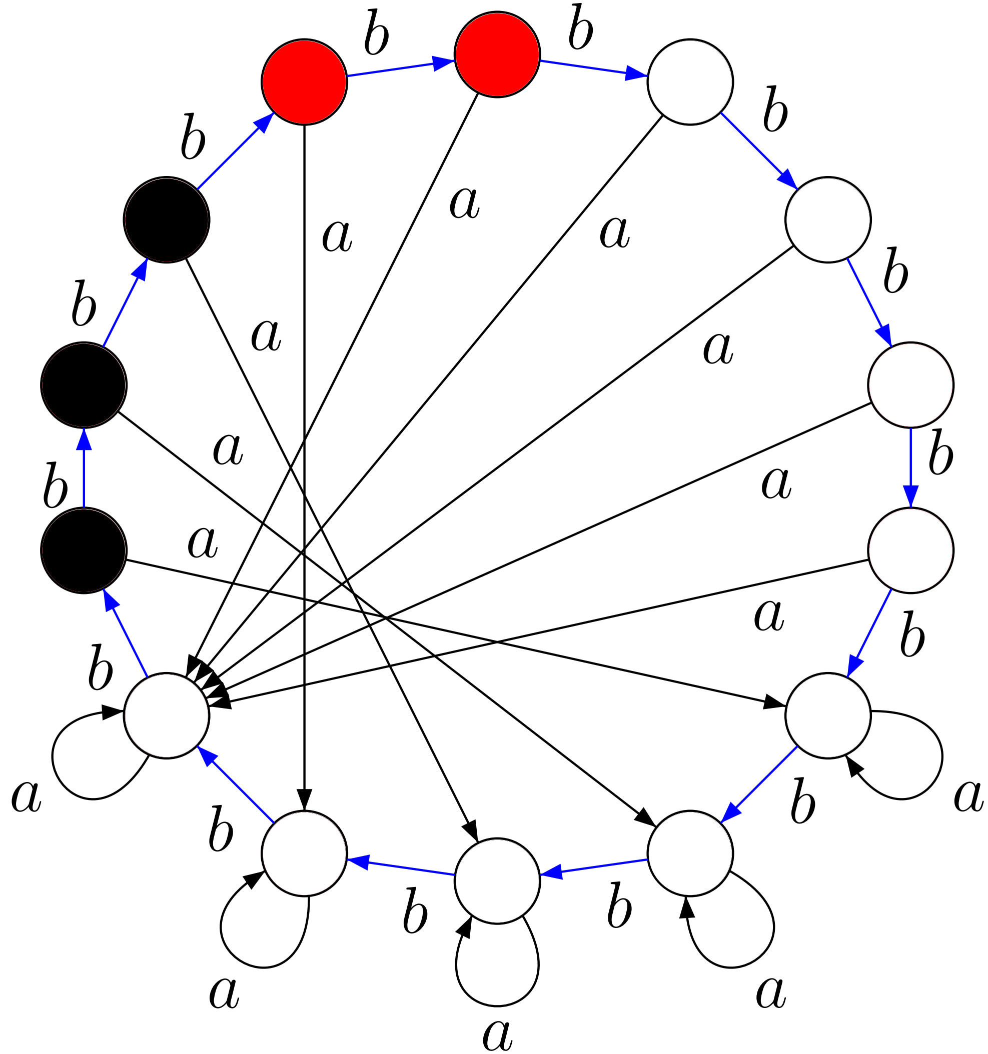

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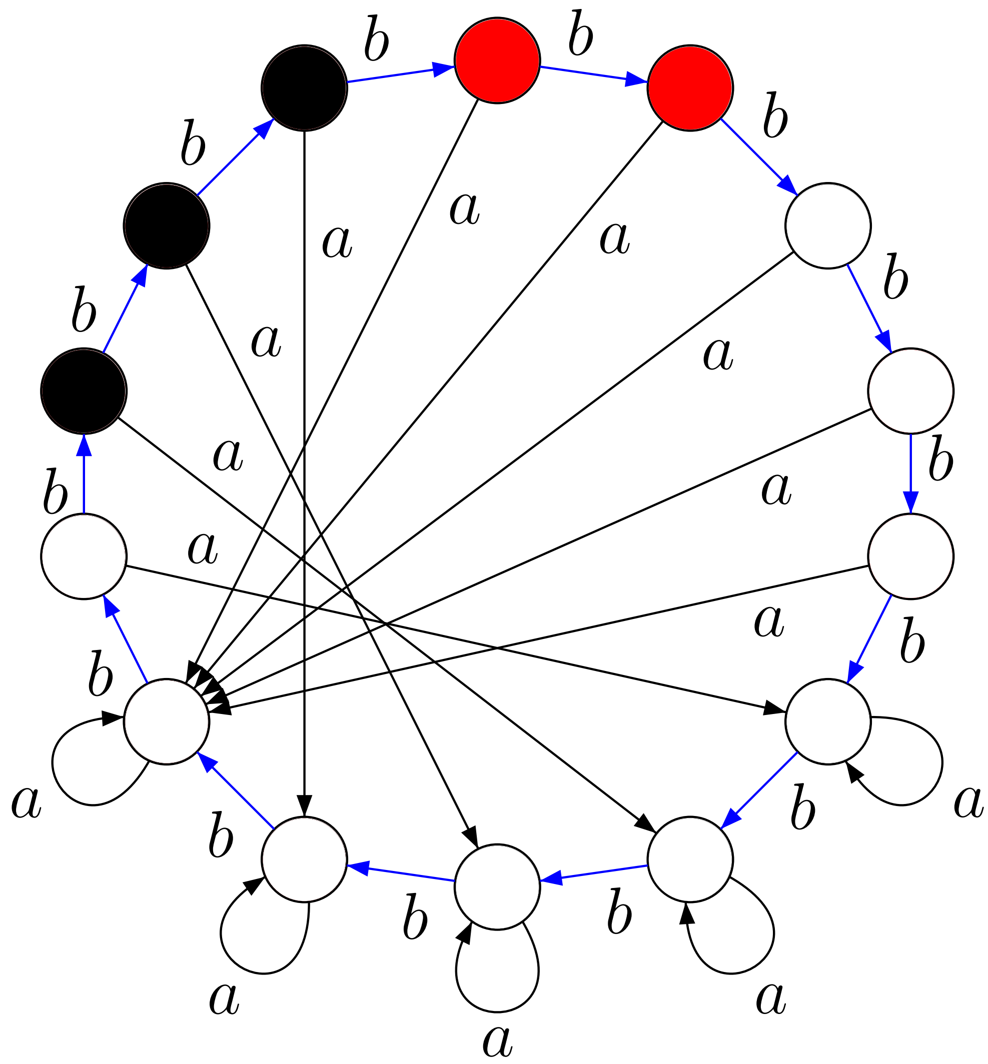

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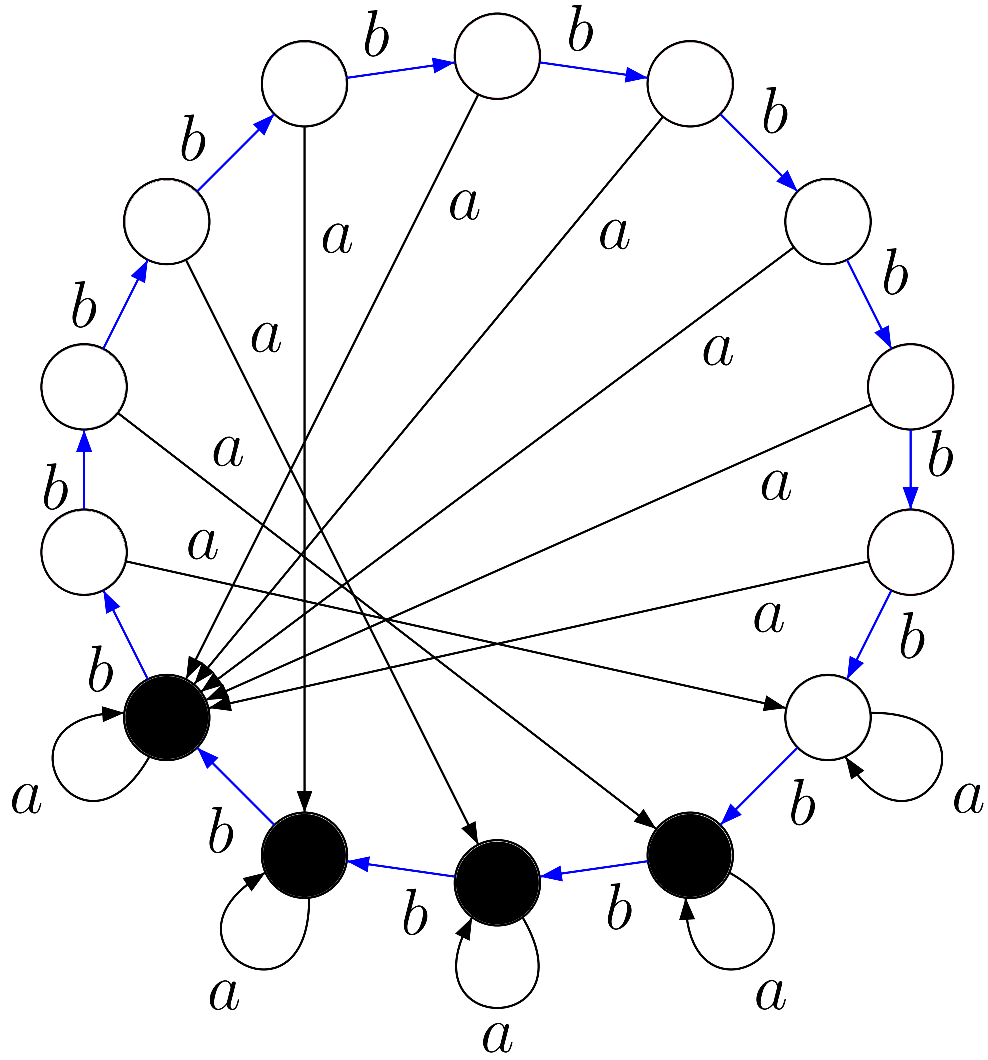

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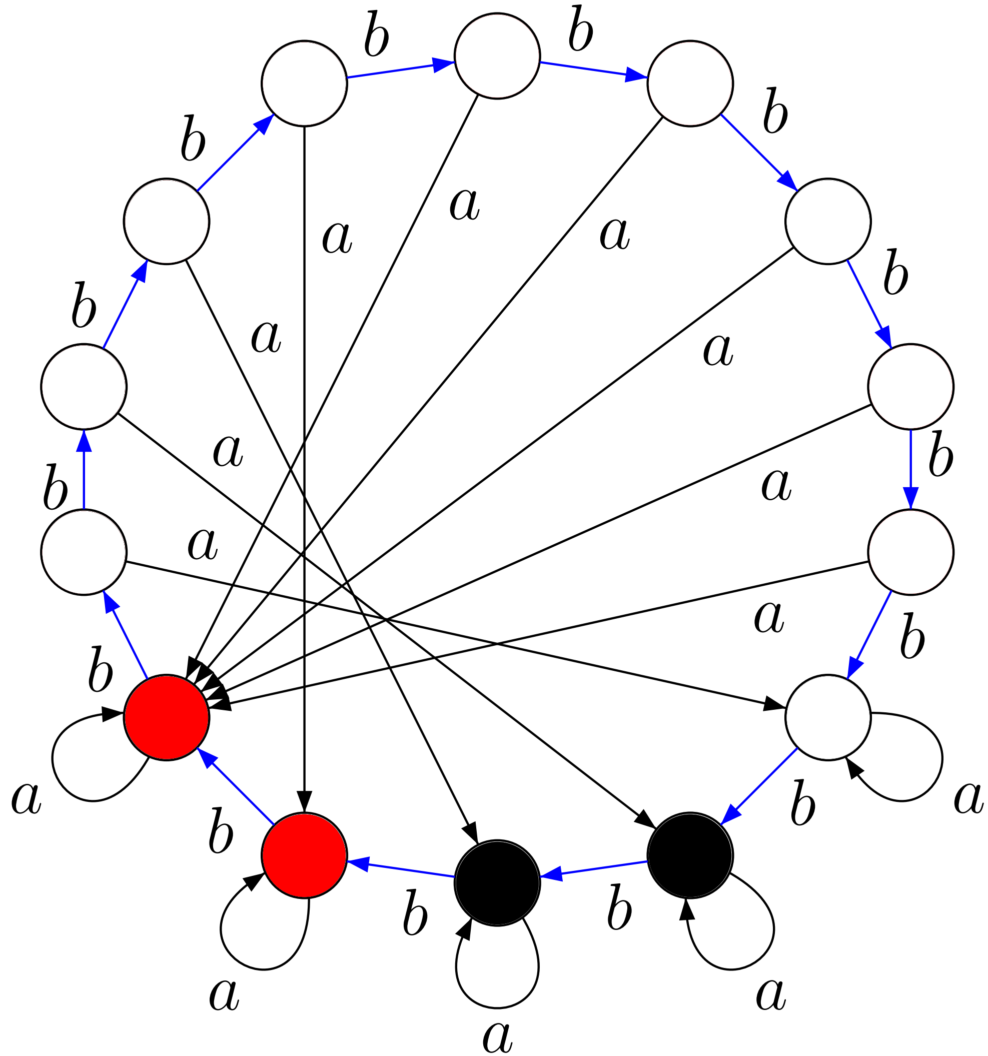

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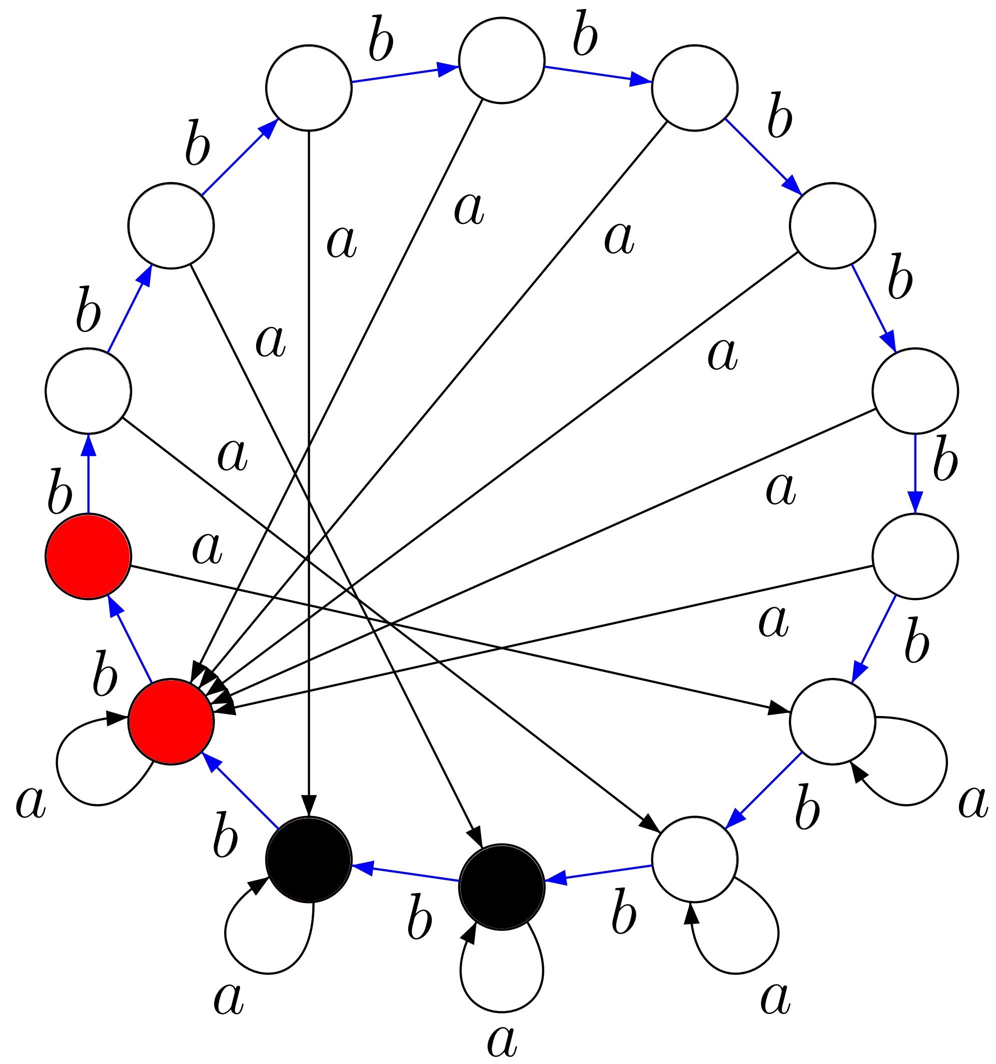

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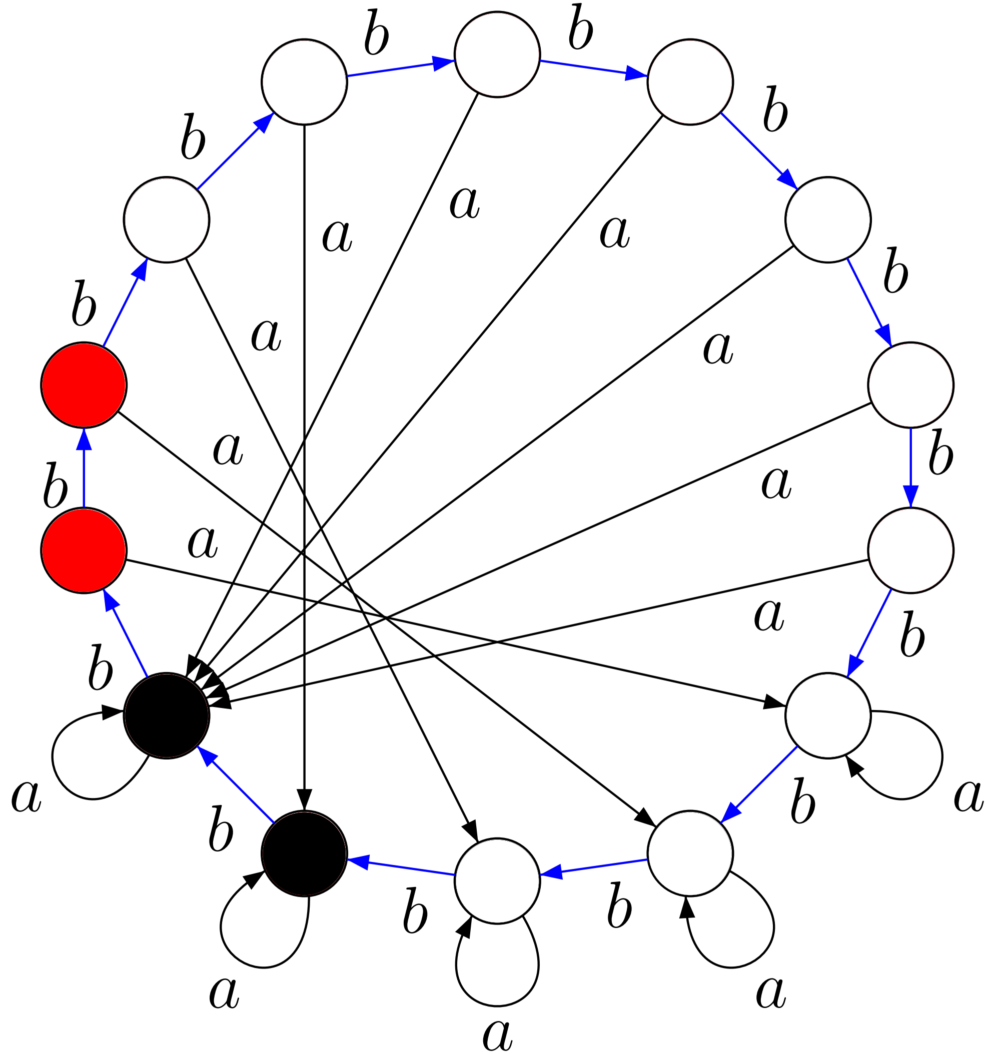

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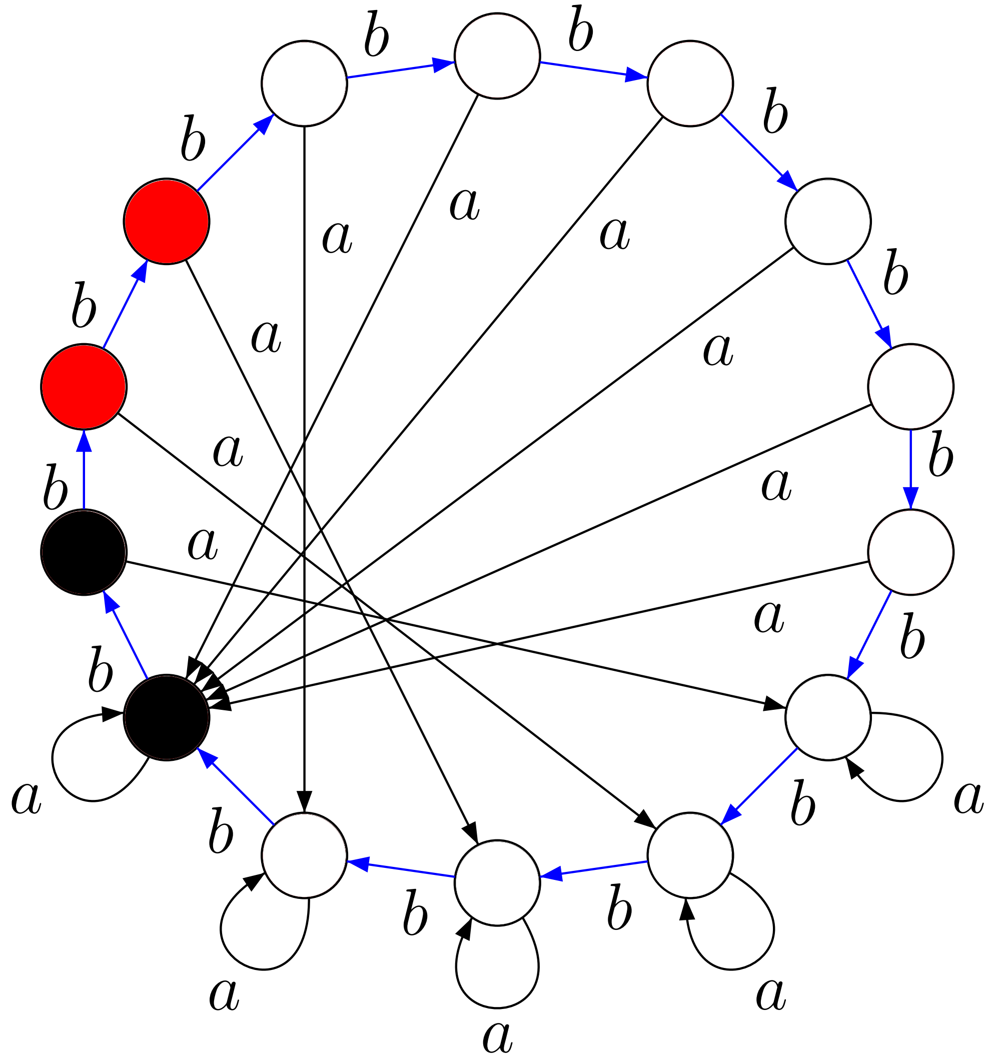

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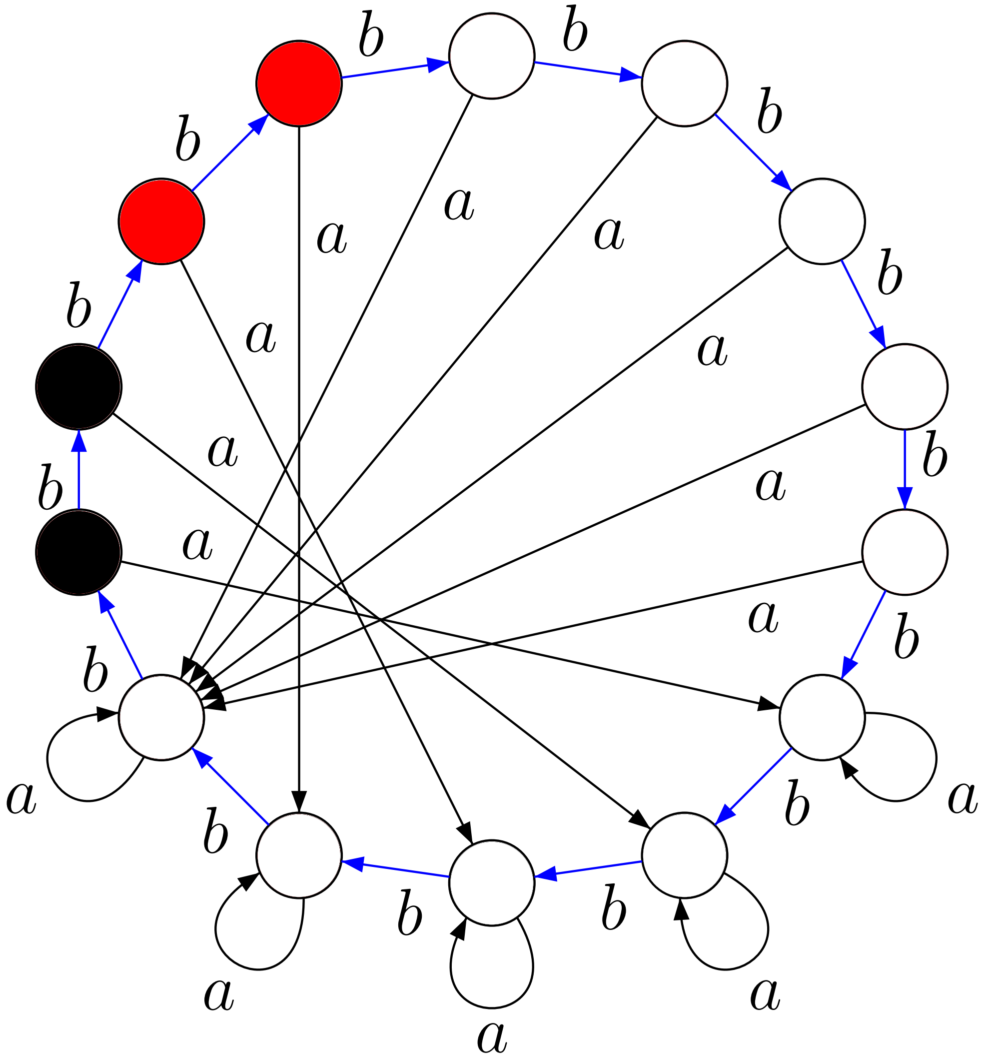

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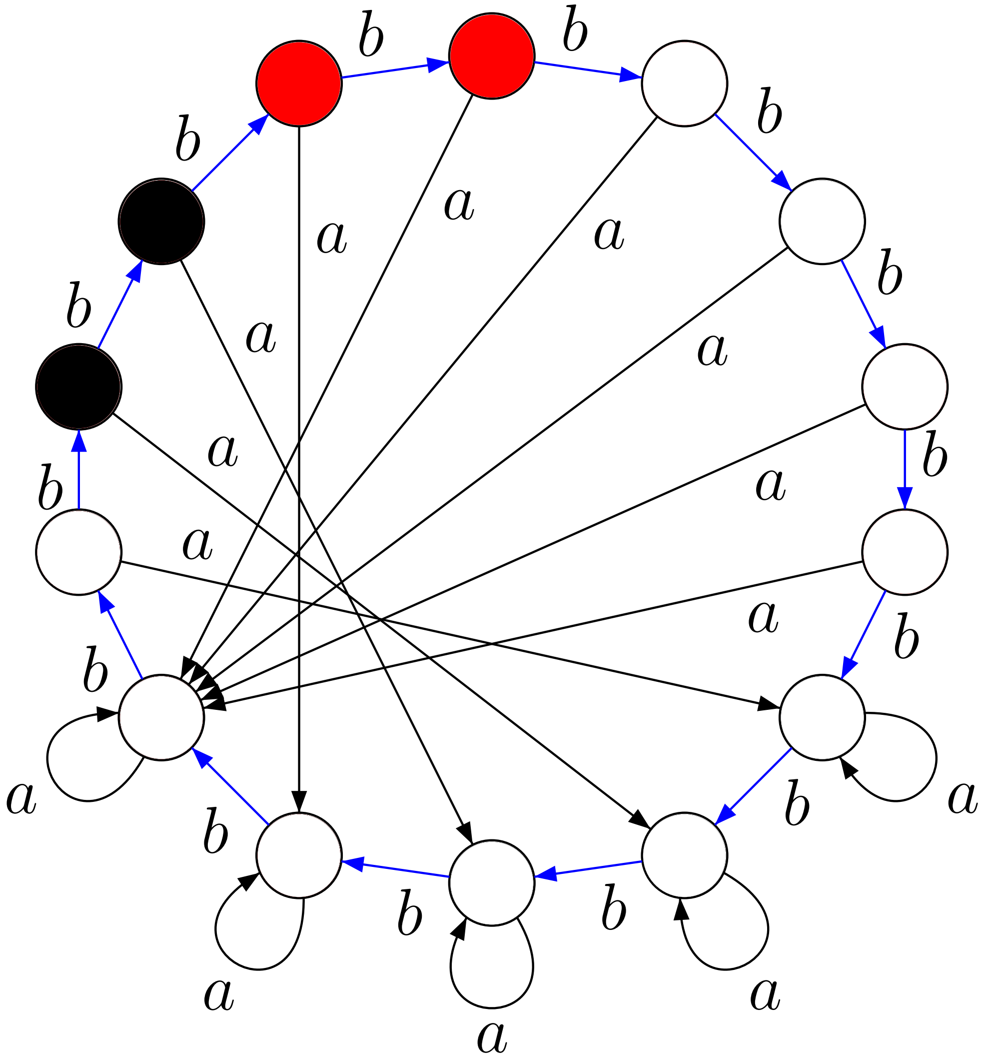

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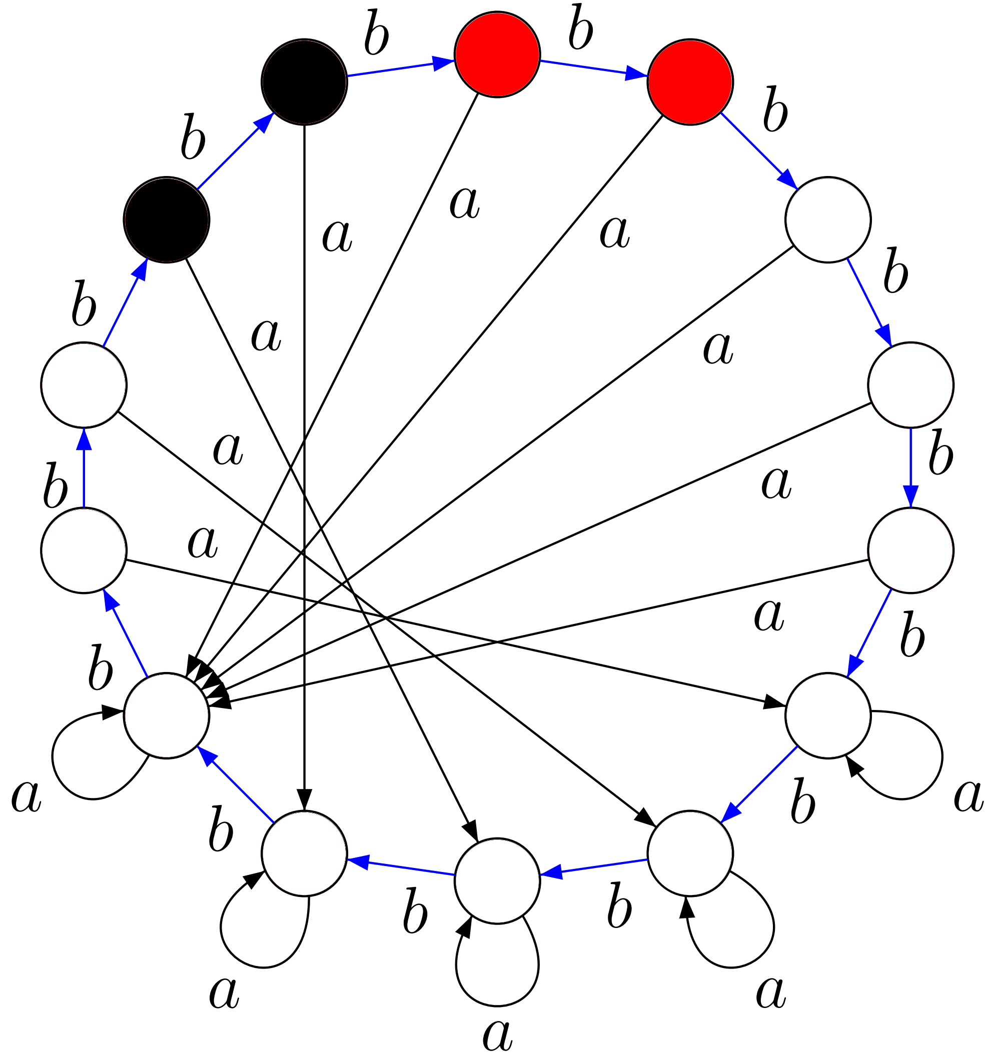

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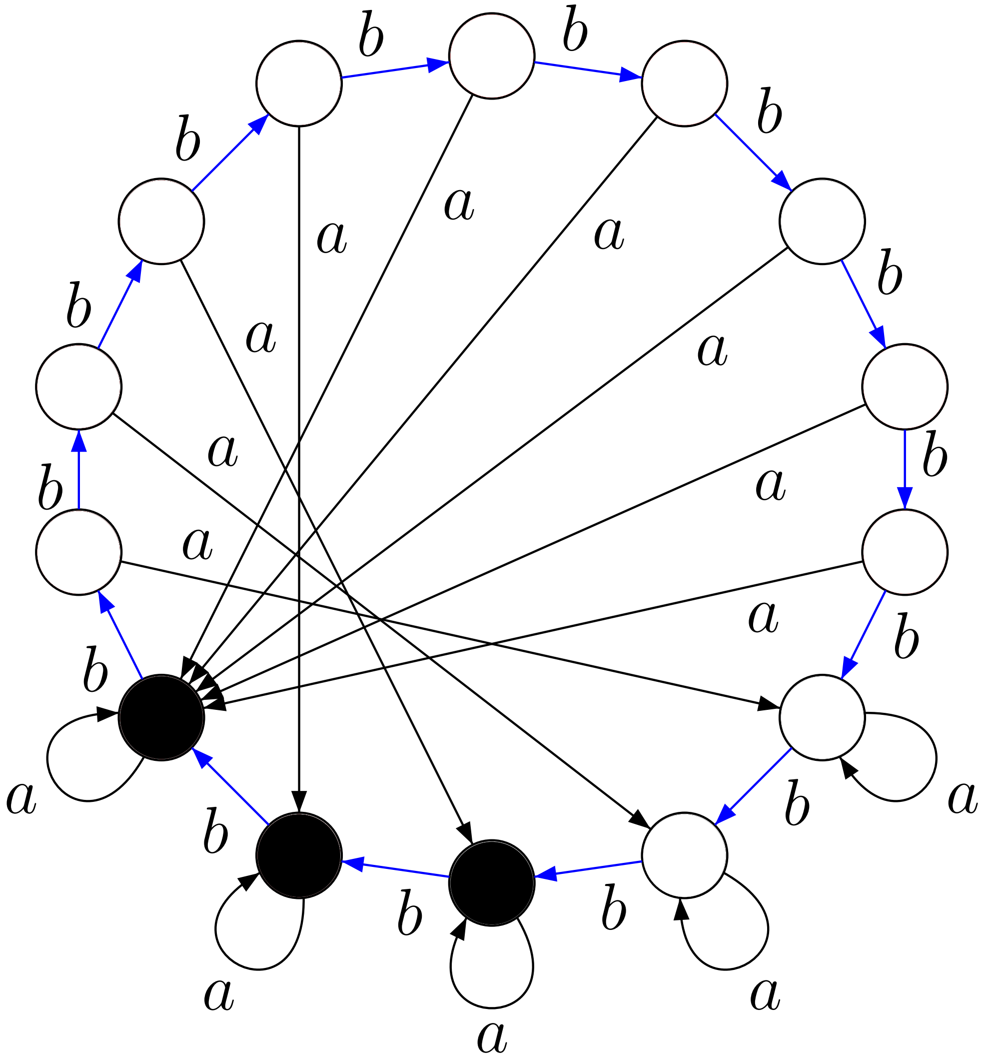

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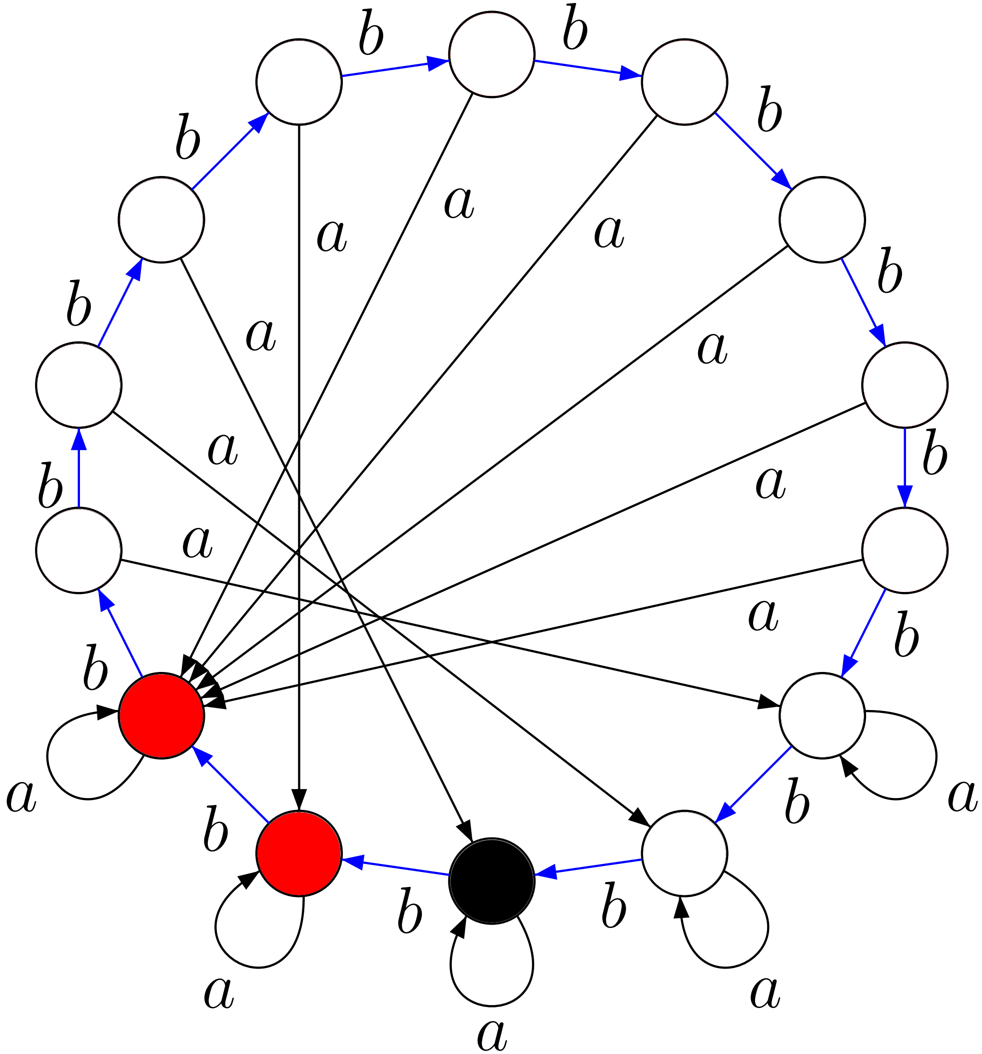

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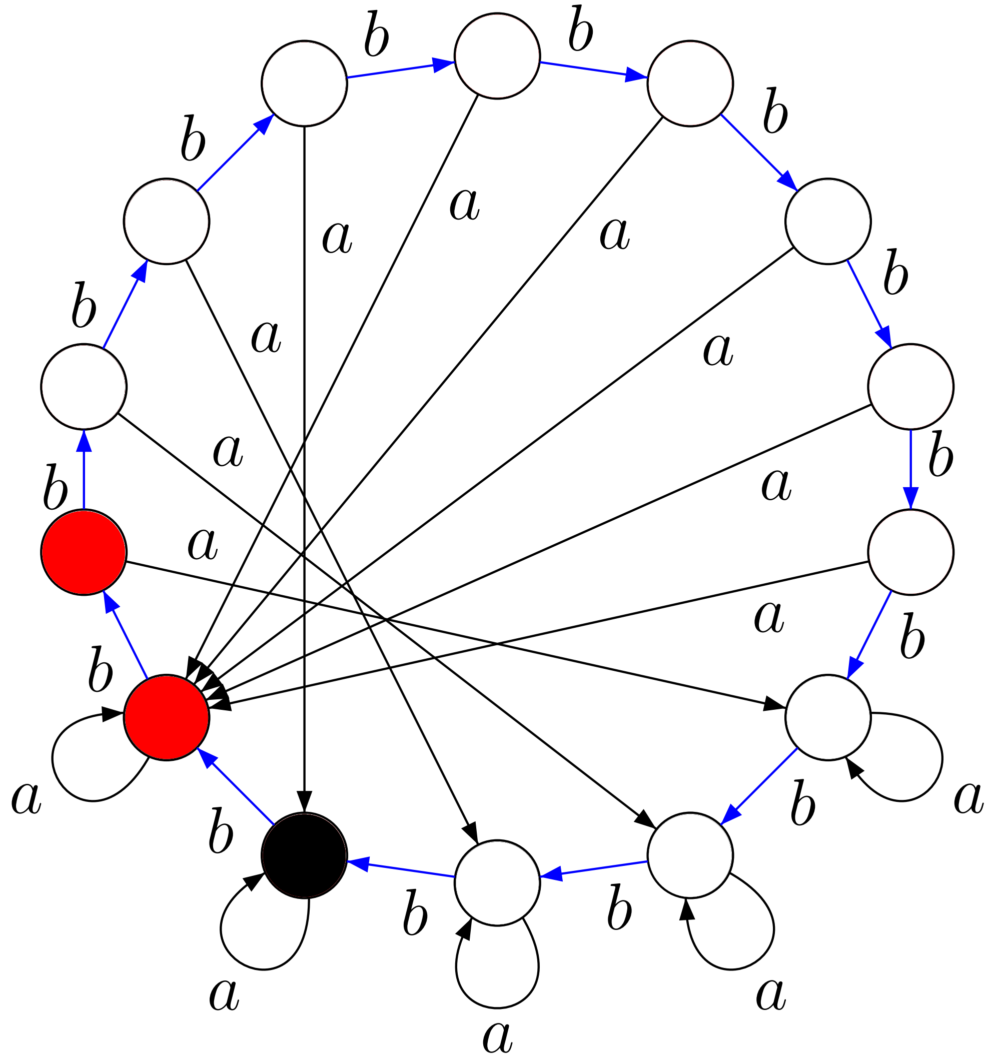

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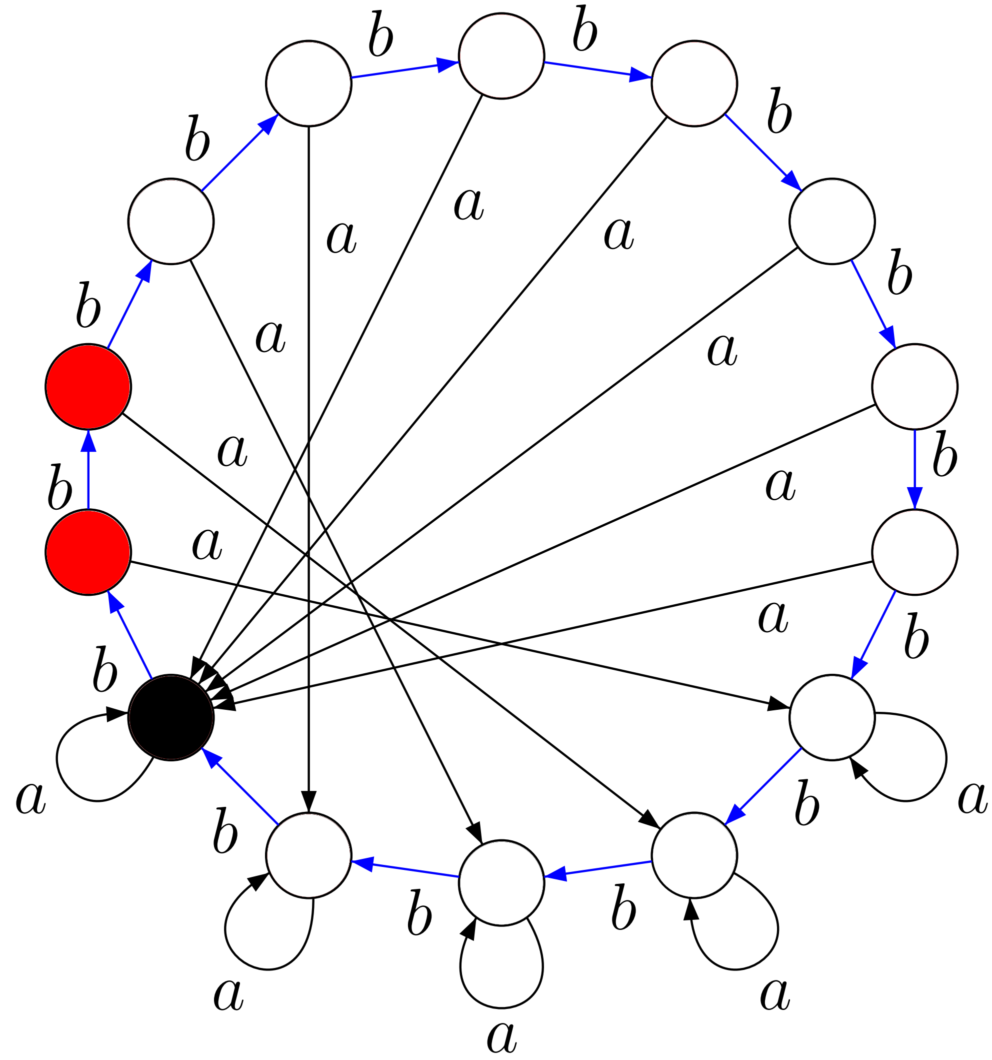

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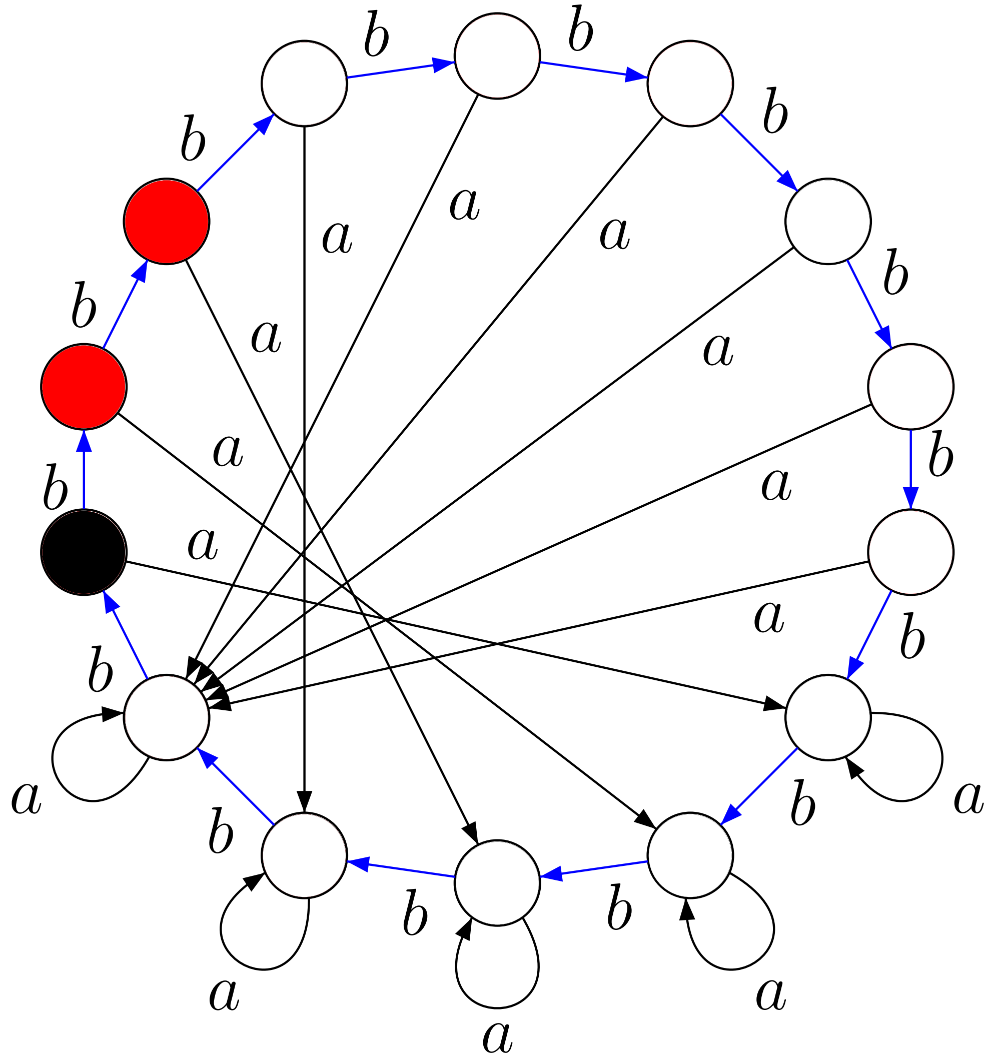

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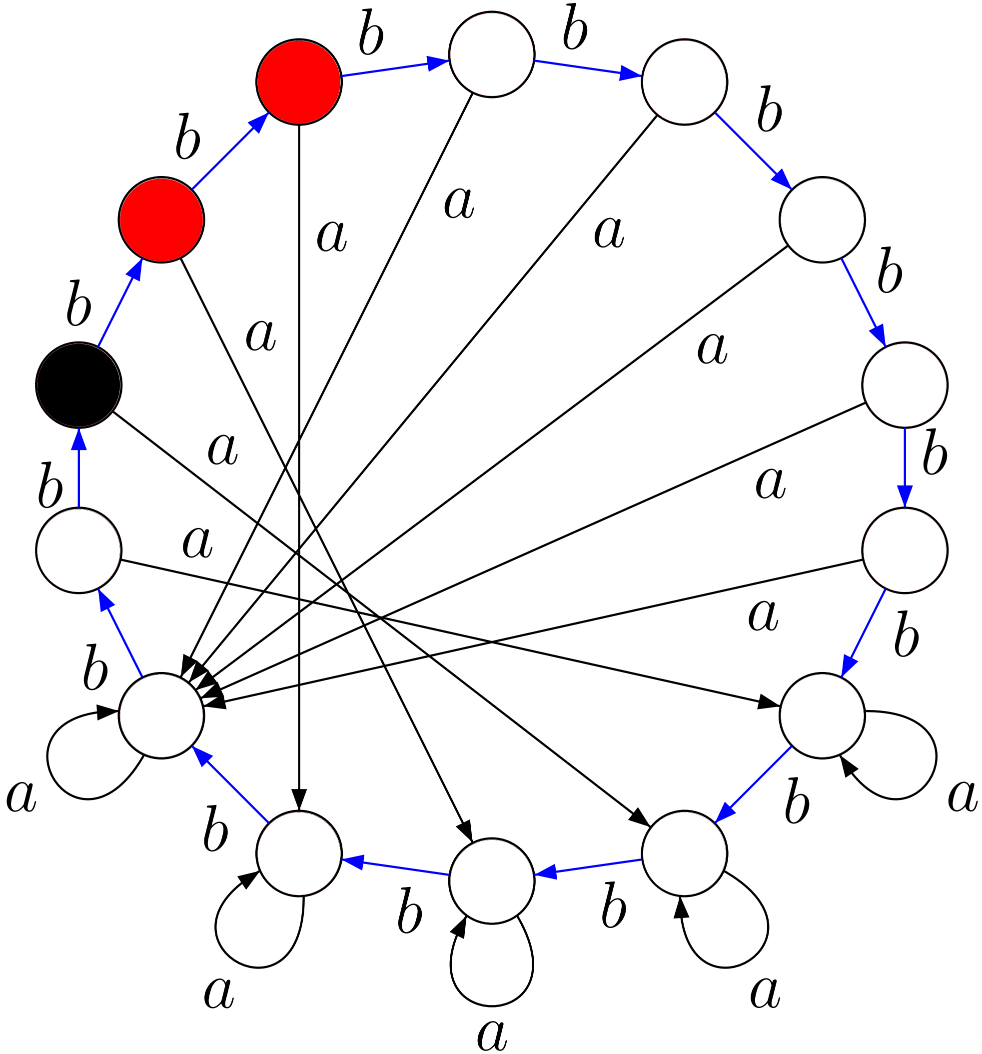

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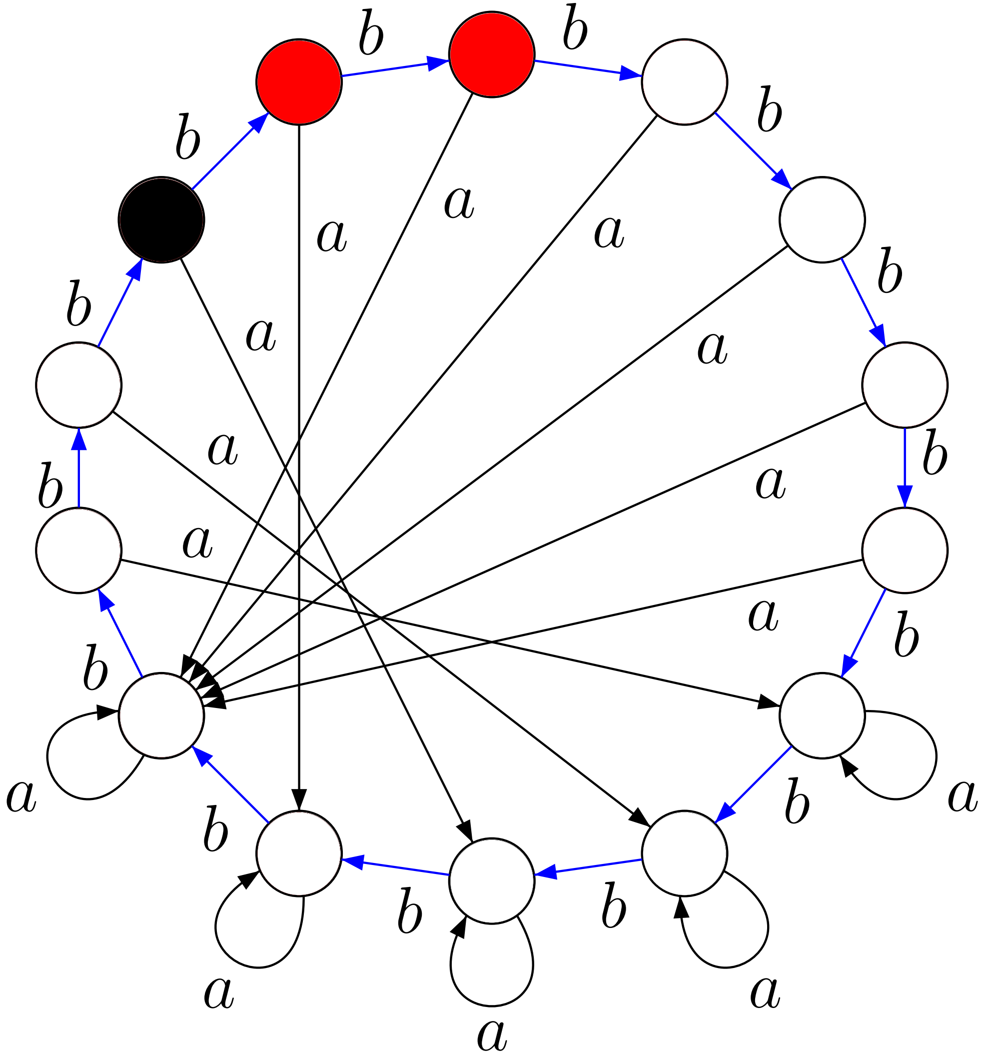

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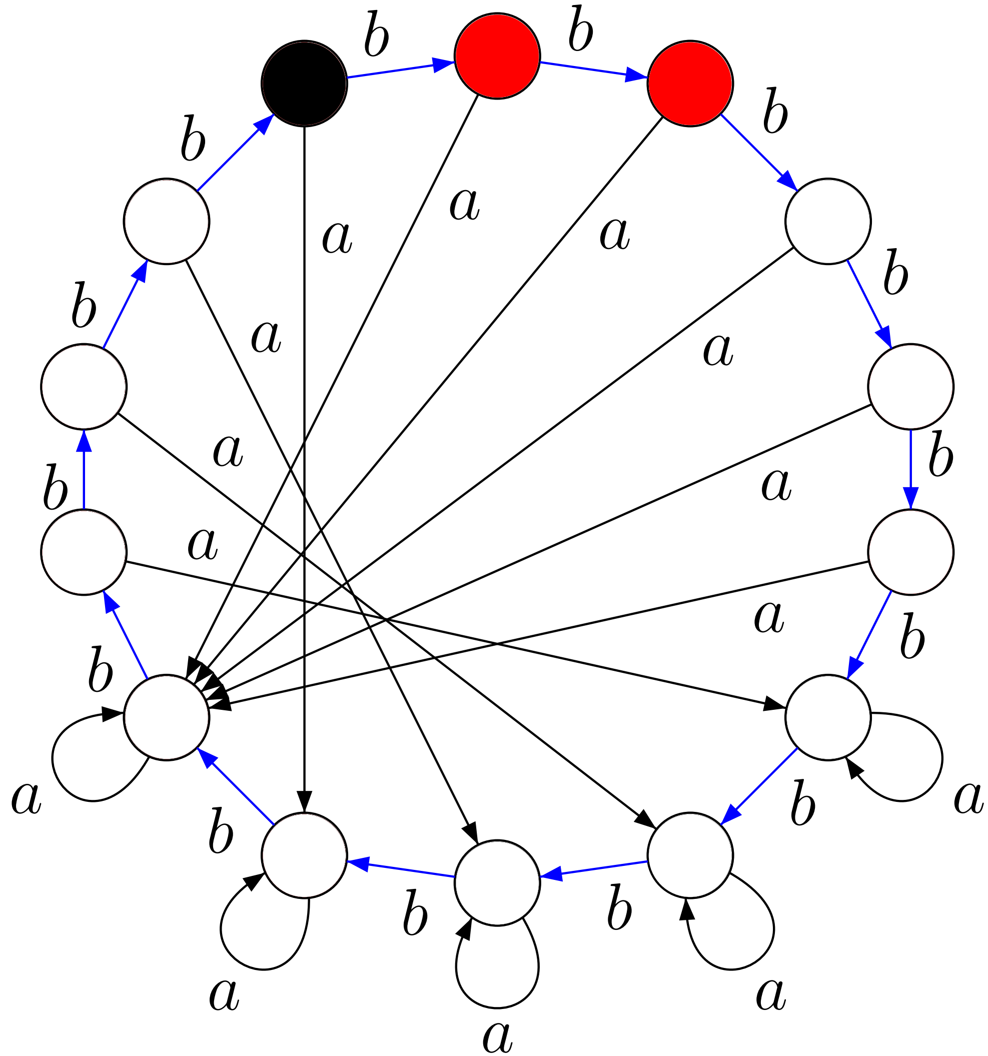

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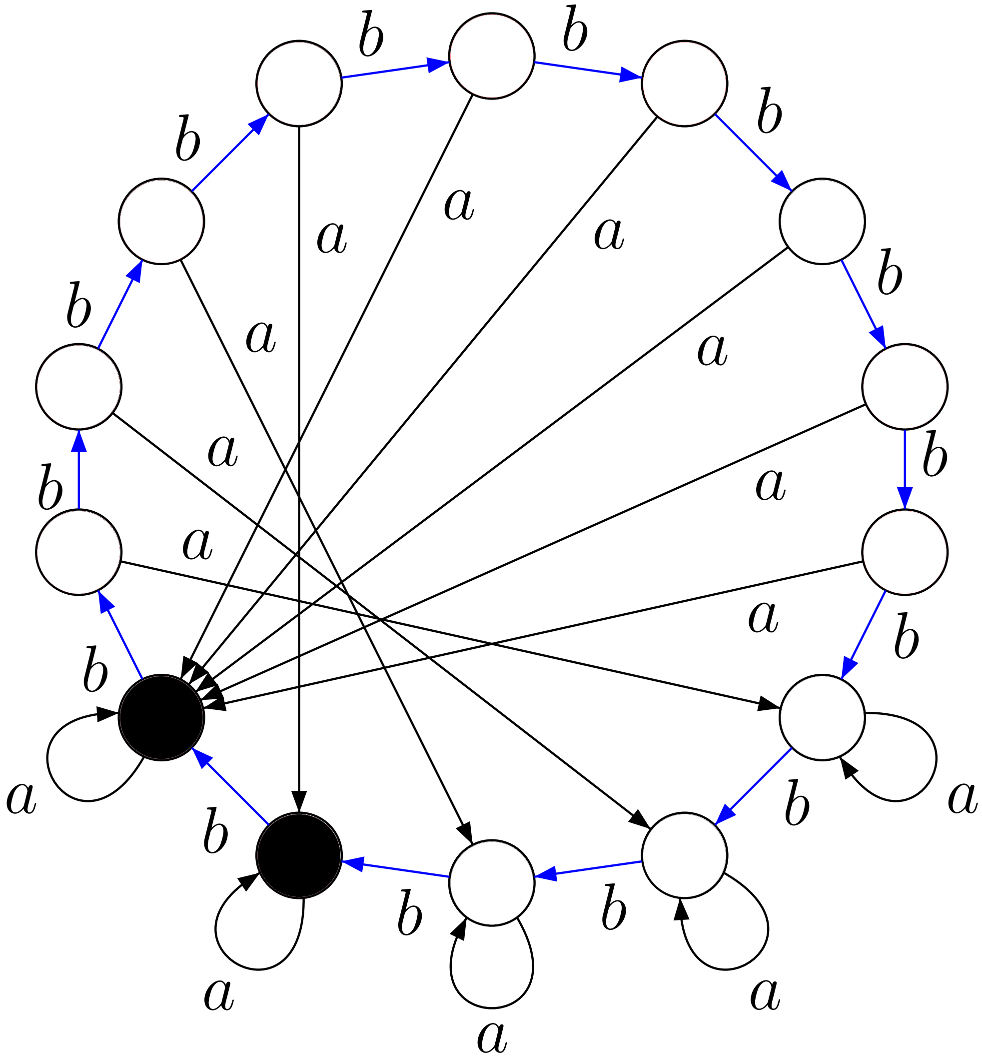

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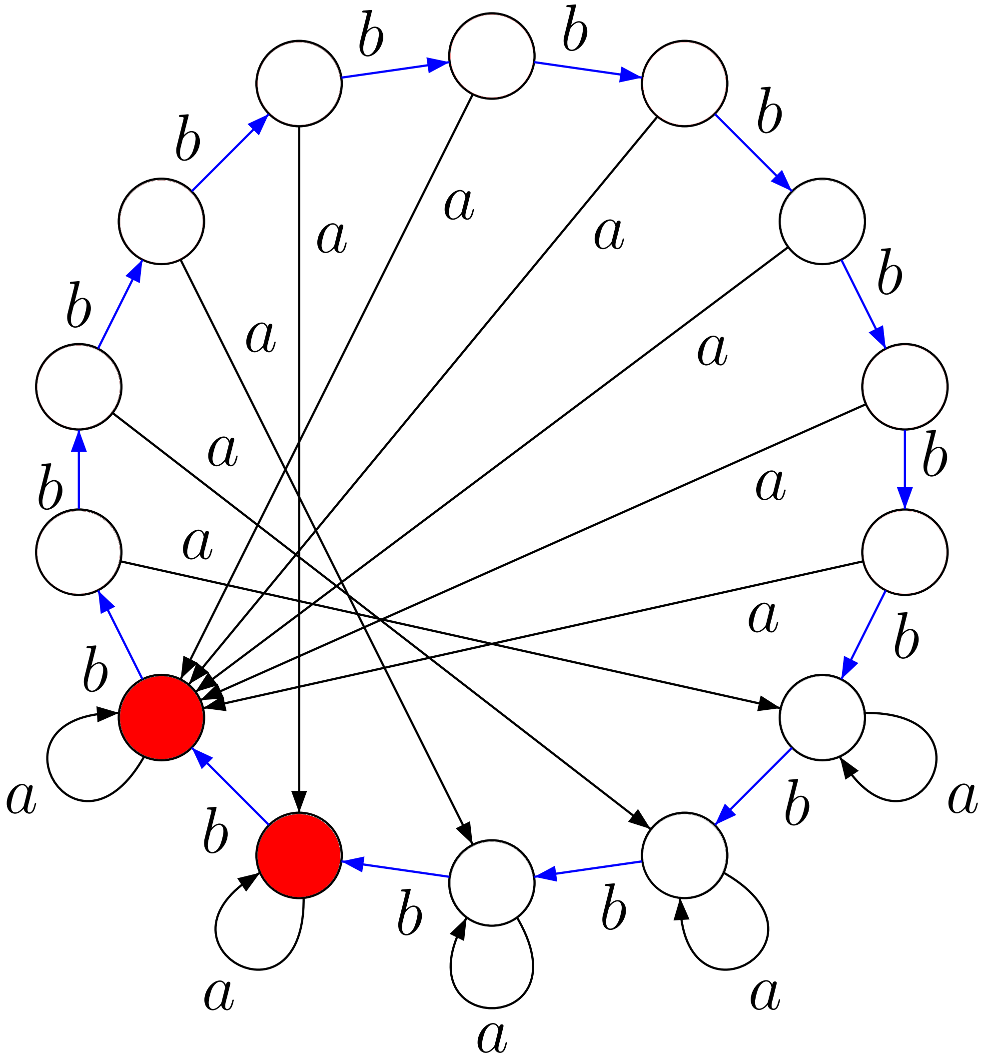

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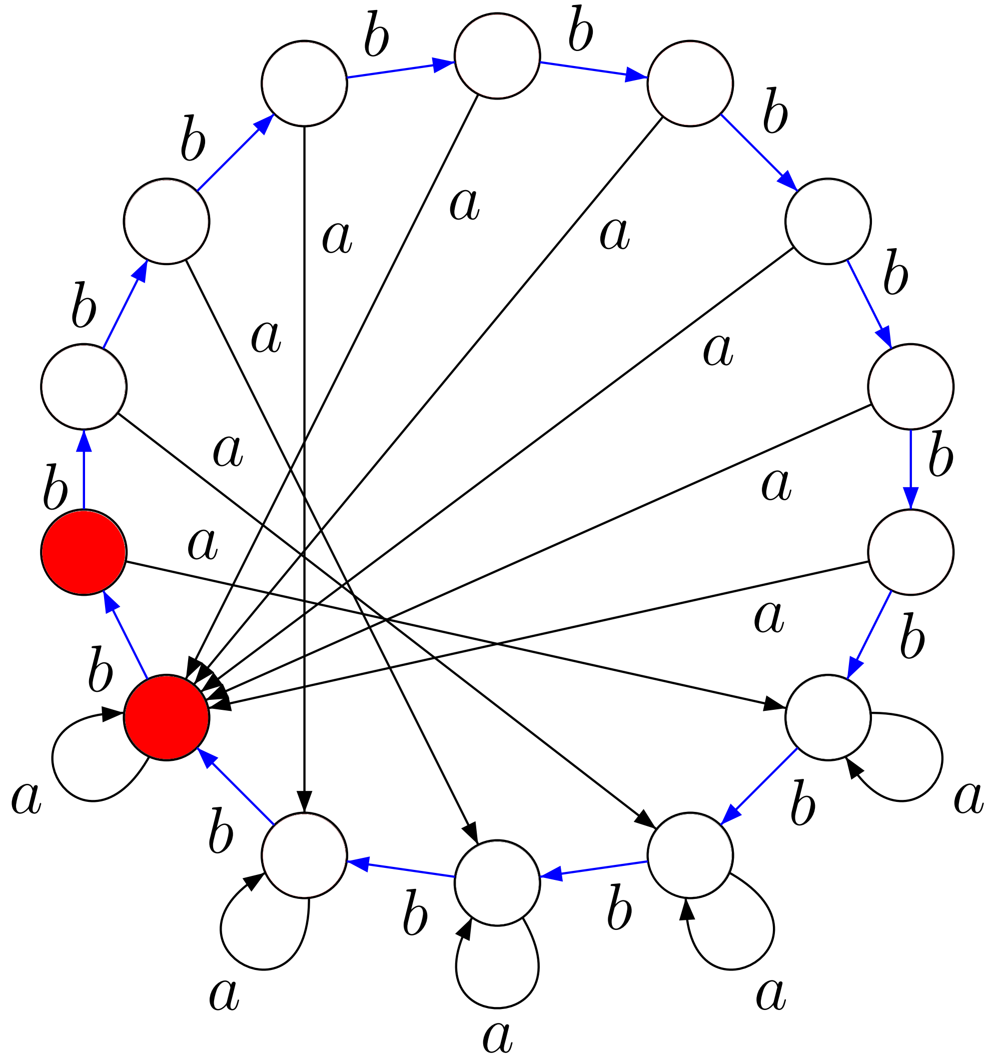

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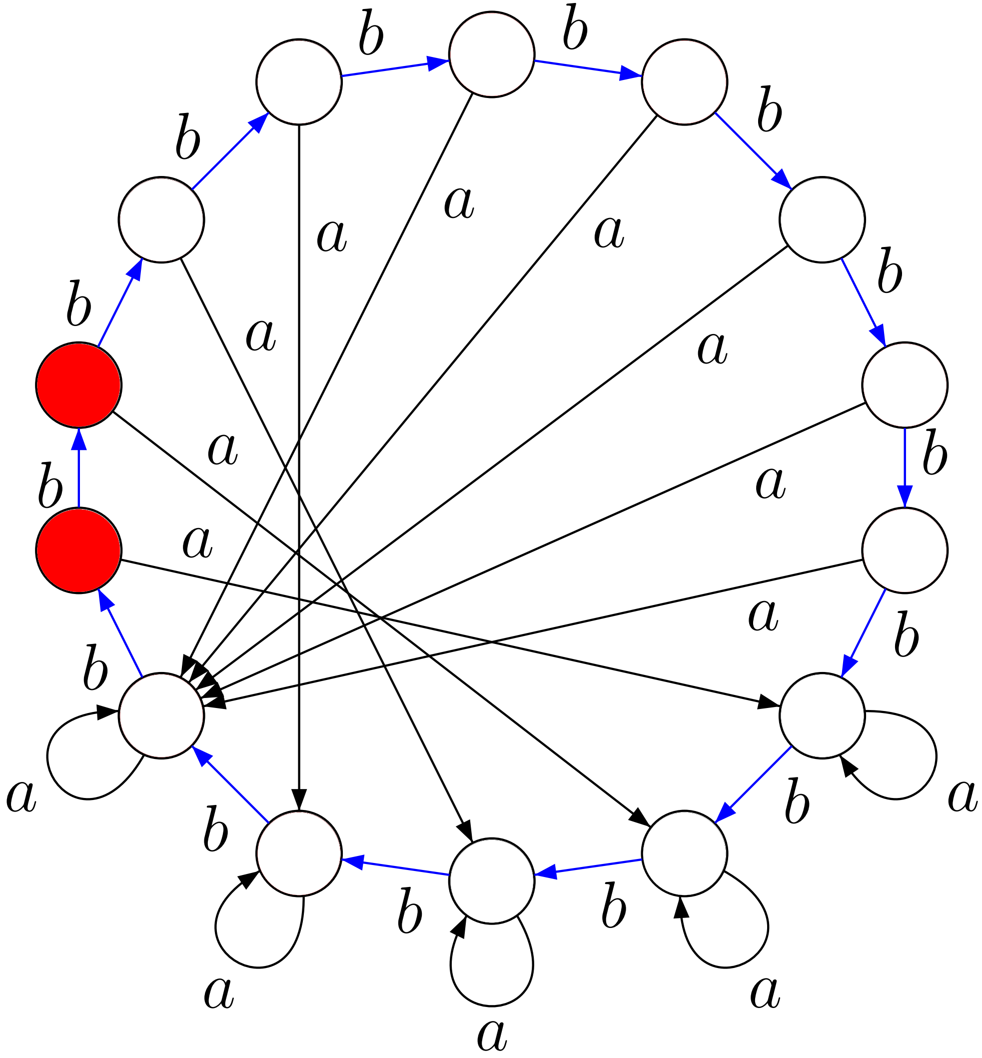

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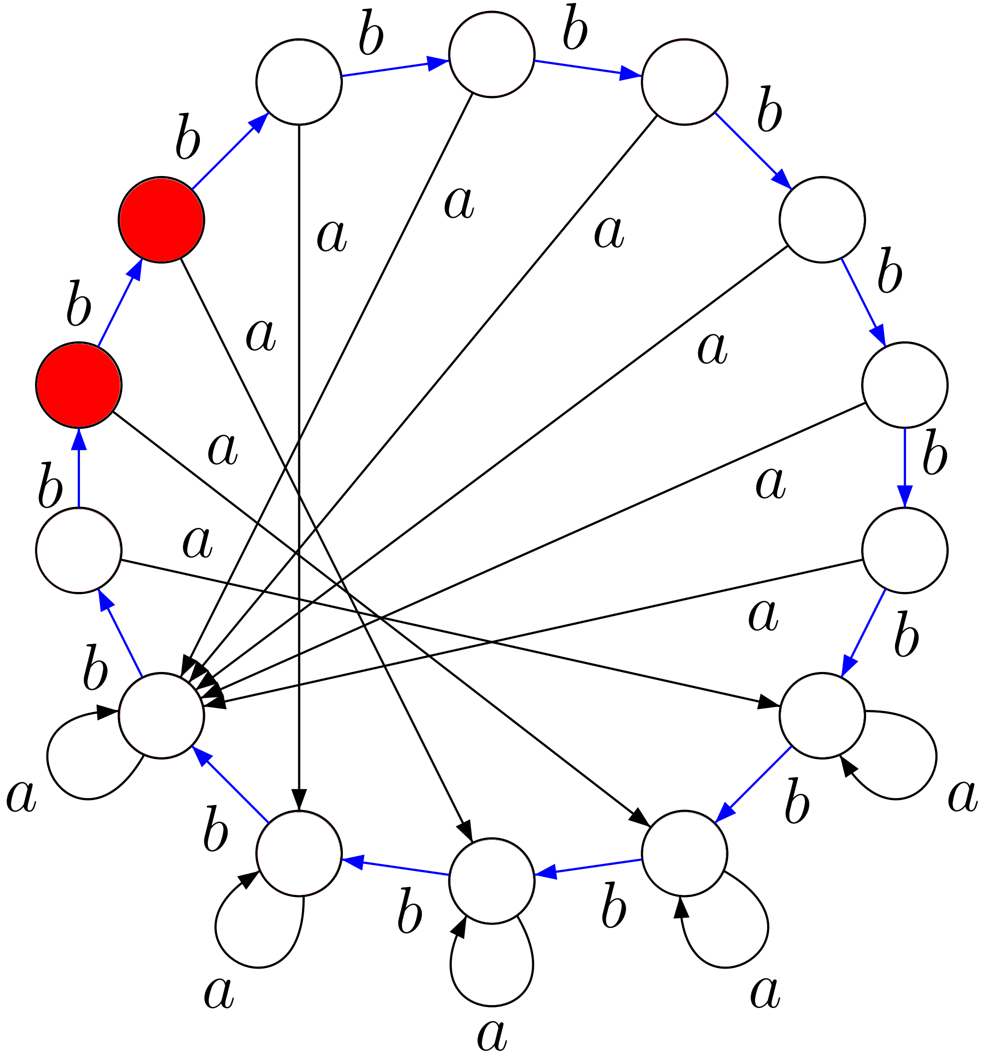

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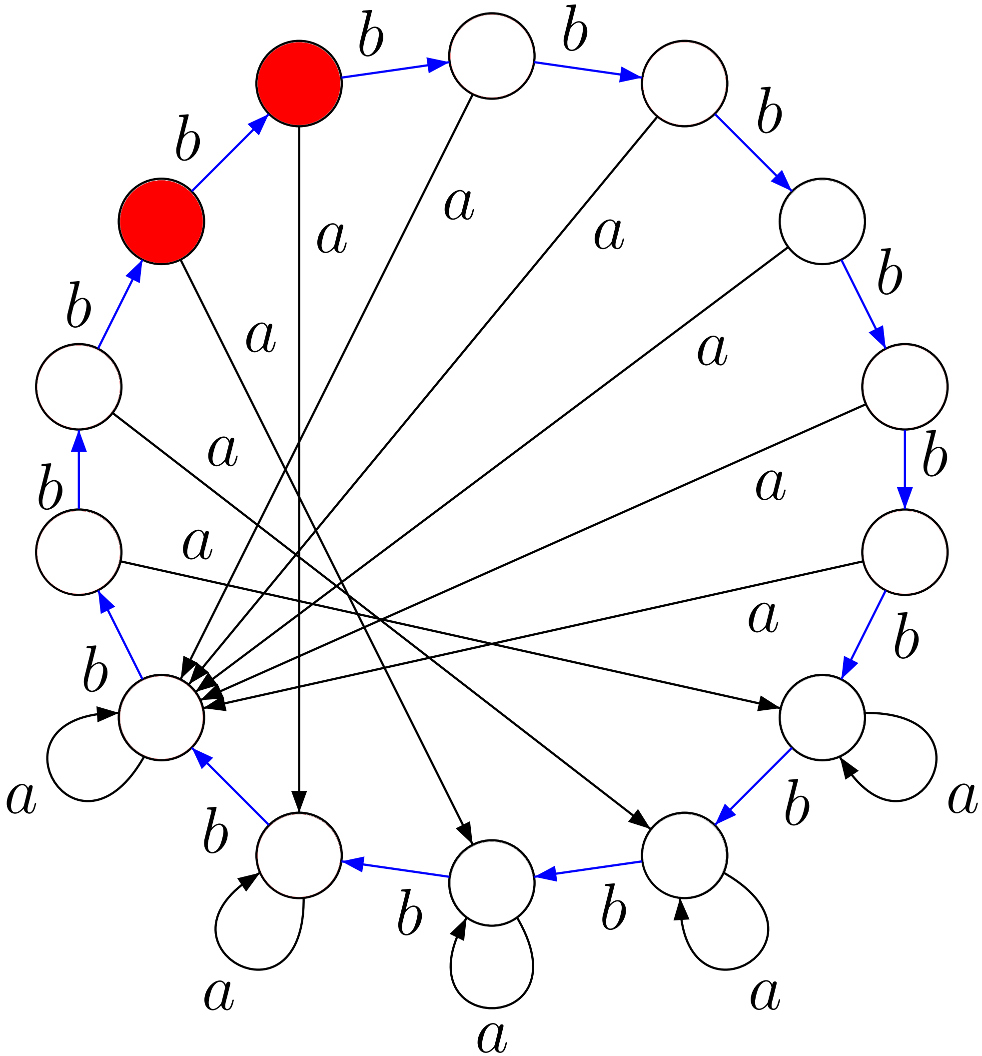

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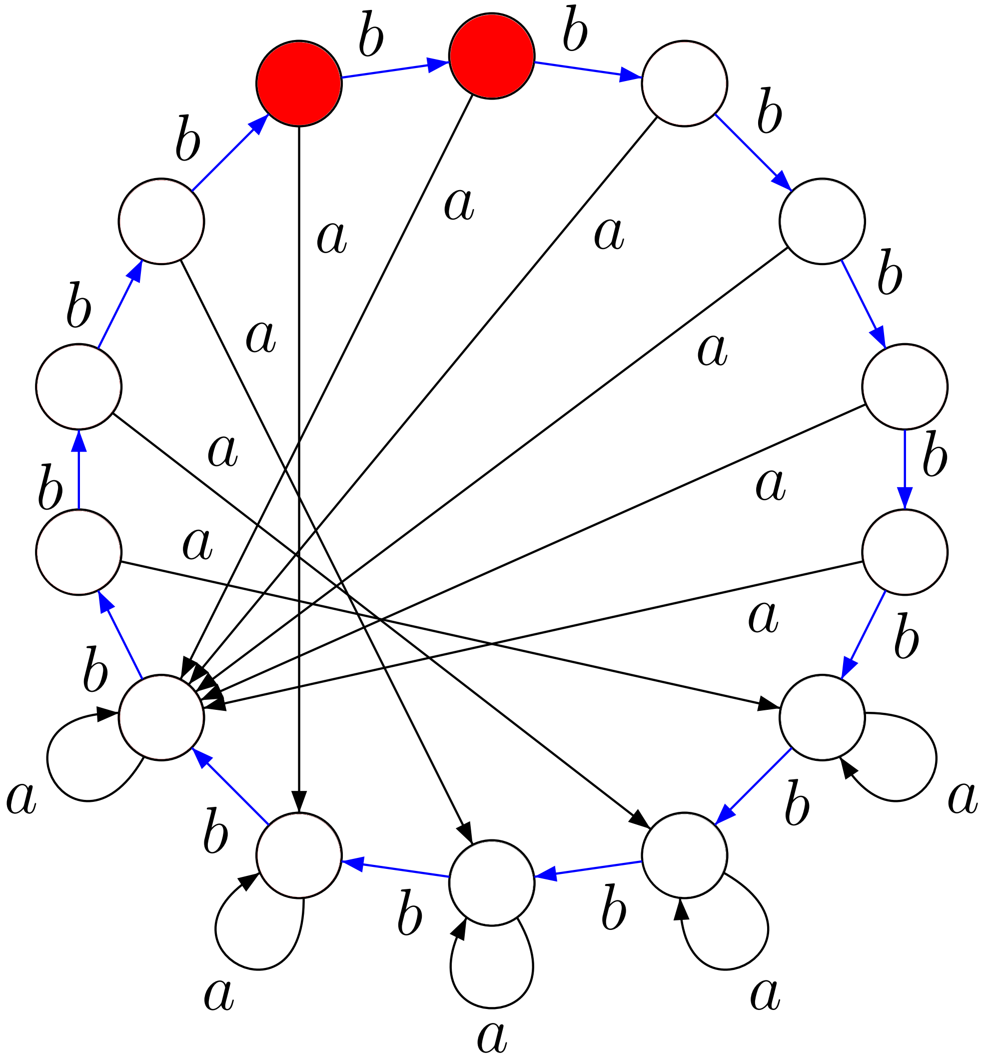

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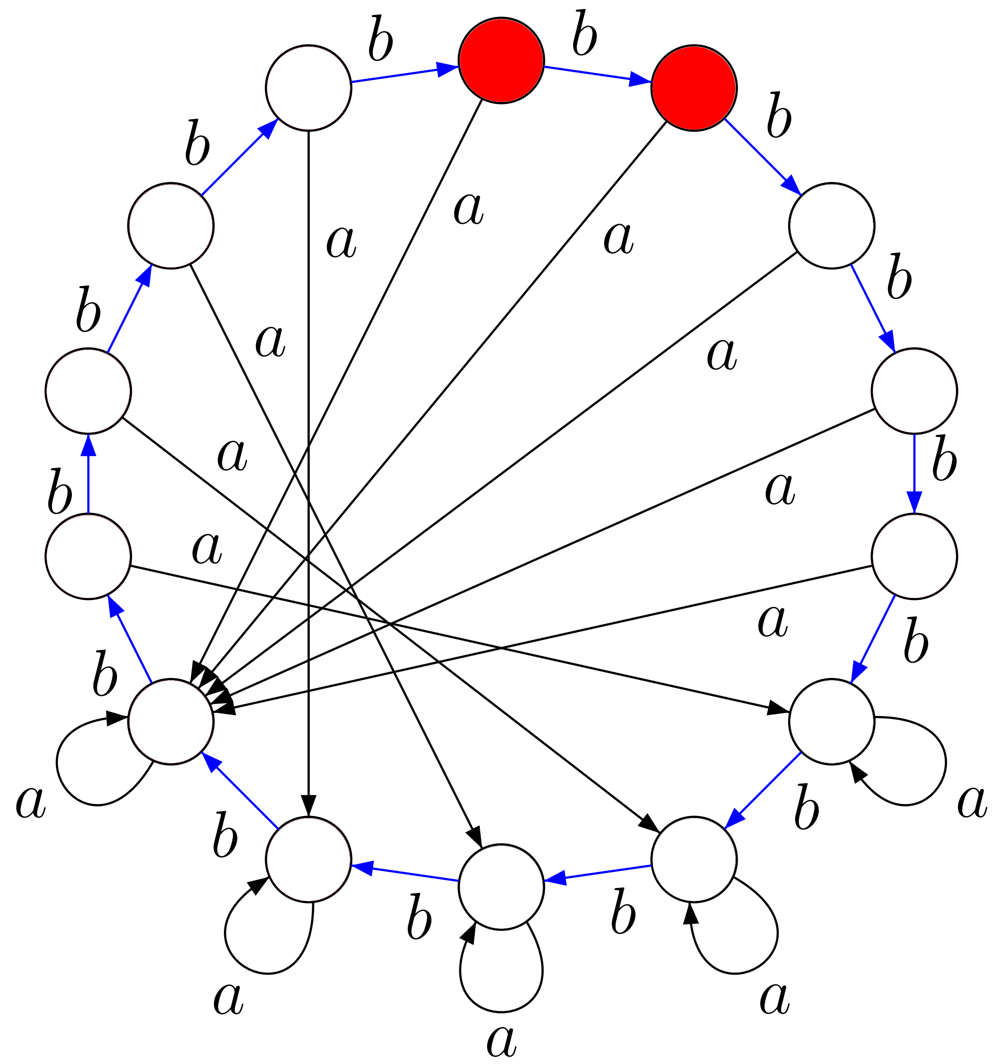

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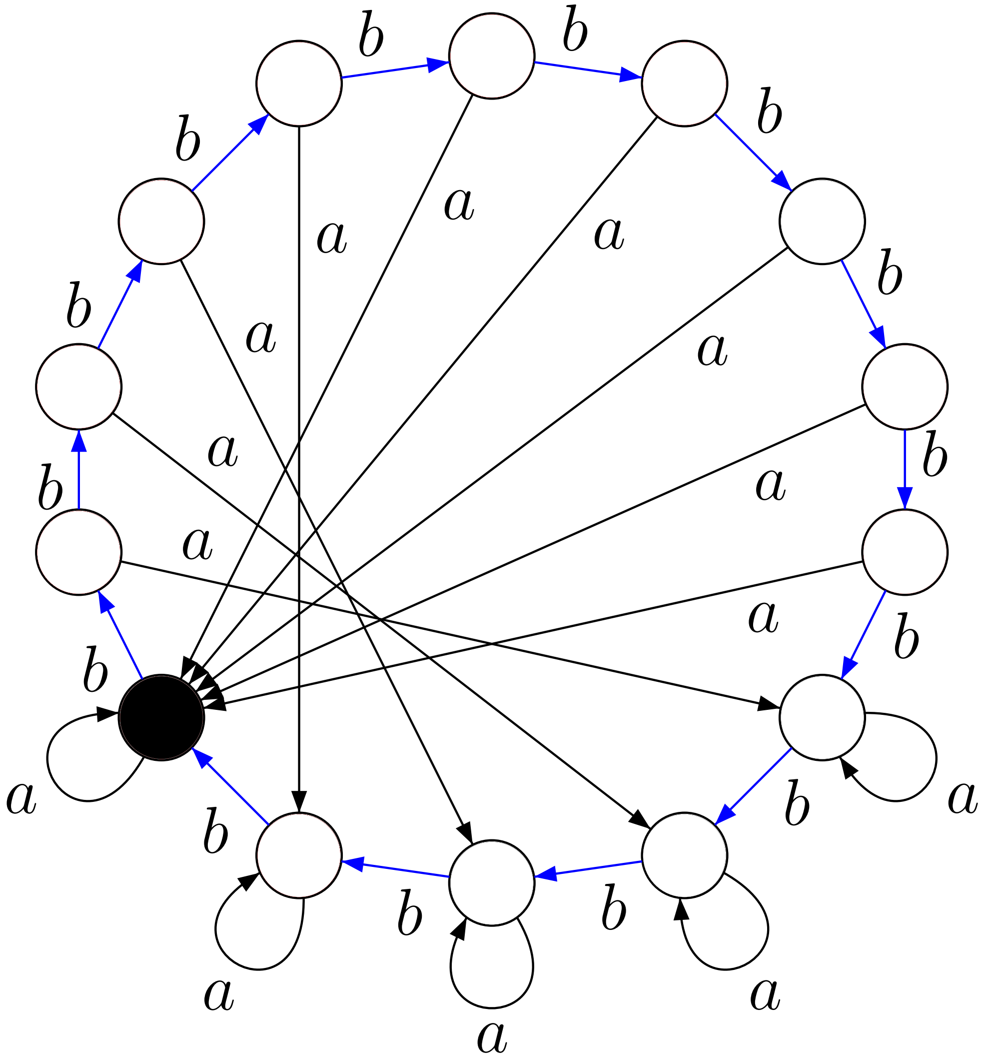

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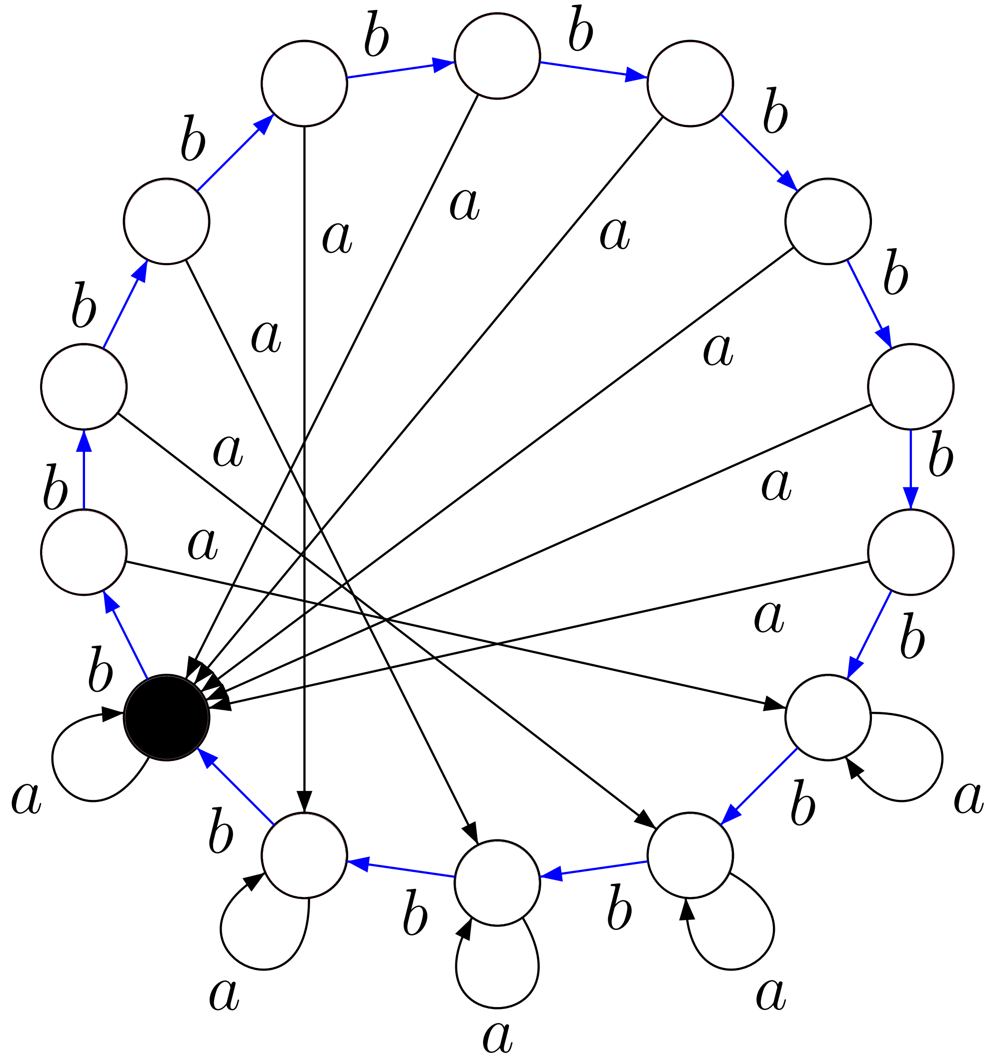

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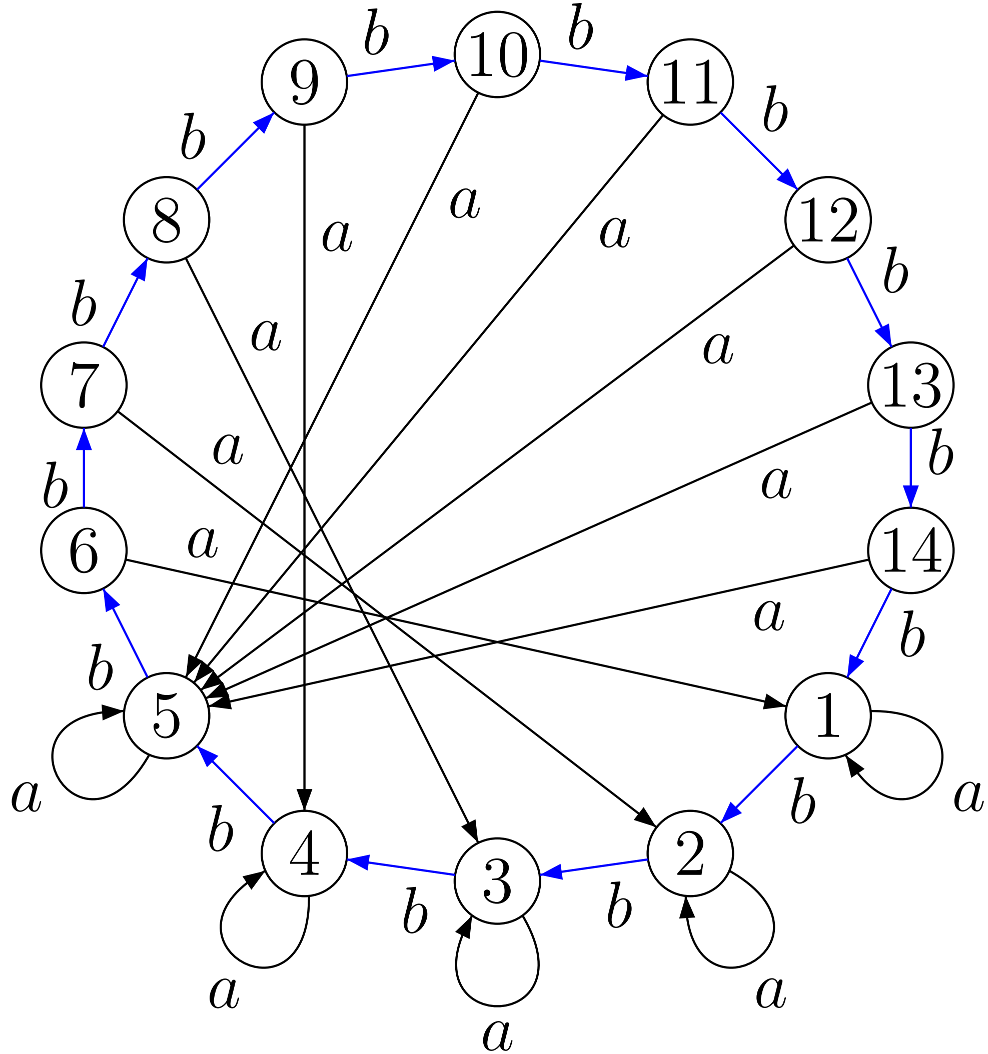
approximation ratio $\frac{29}{11}$ or $\geq \frac{n-1}{6(k-1)}$

“Honest” example: “greedy” reset word (k=3)



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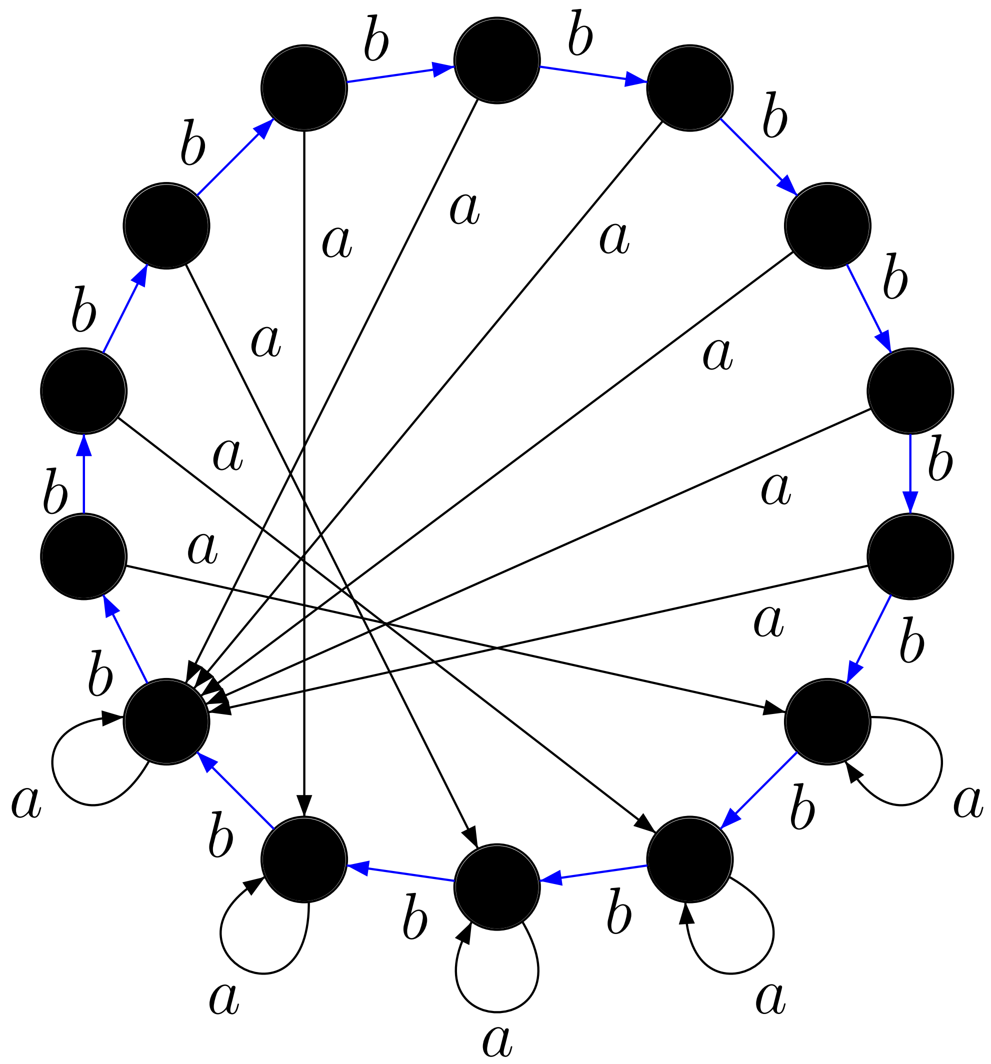

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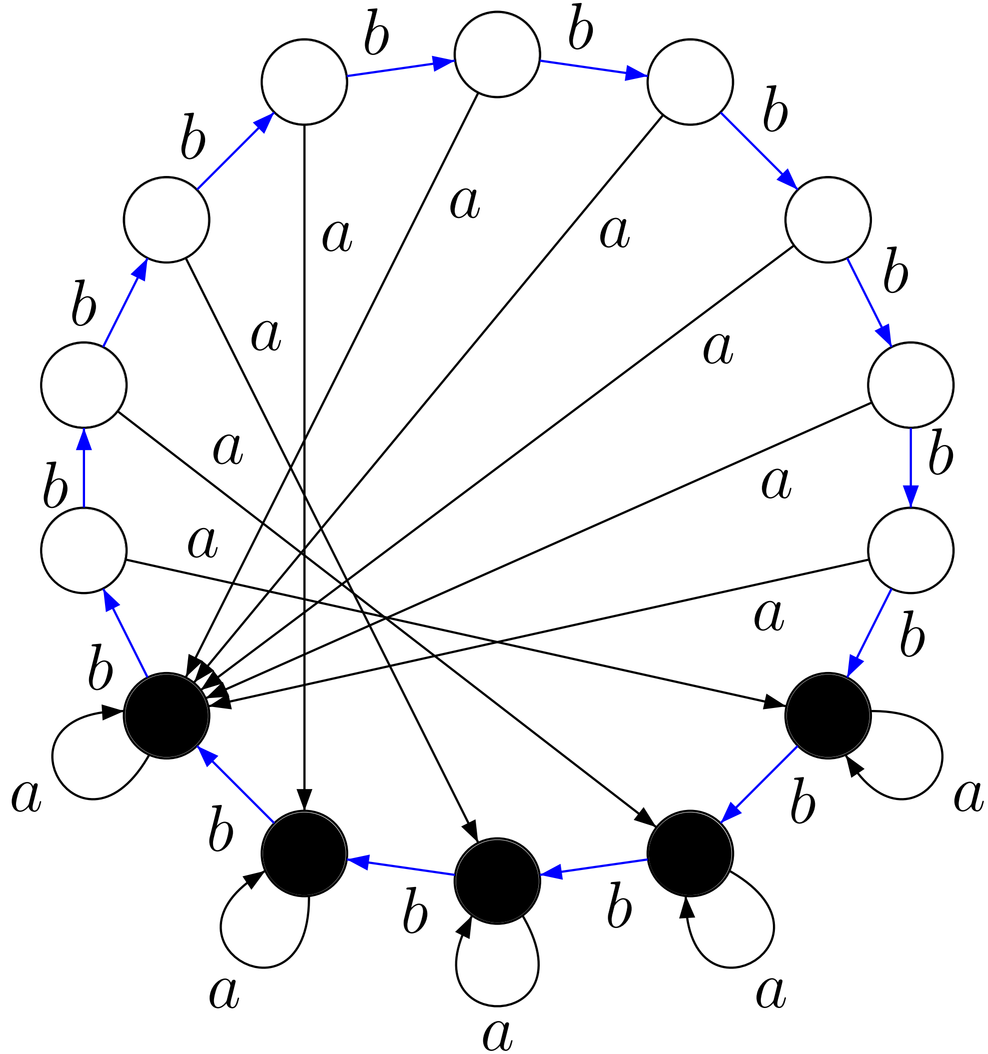

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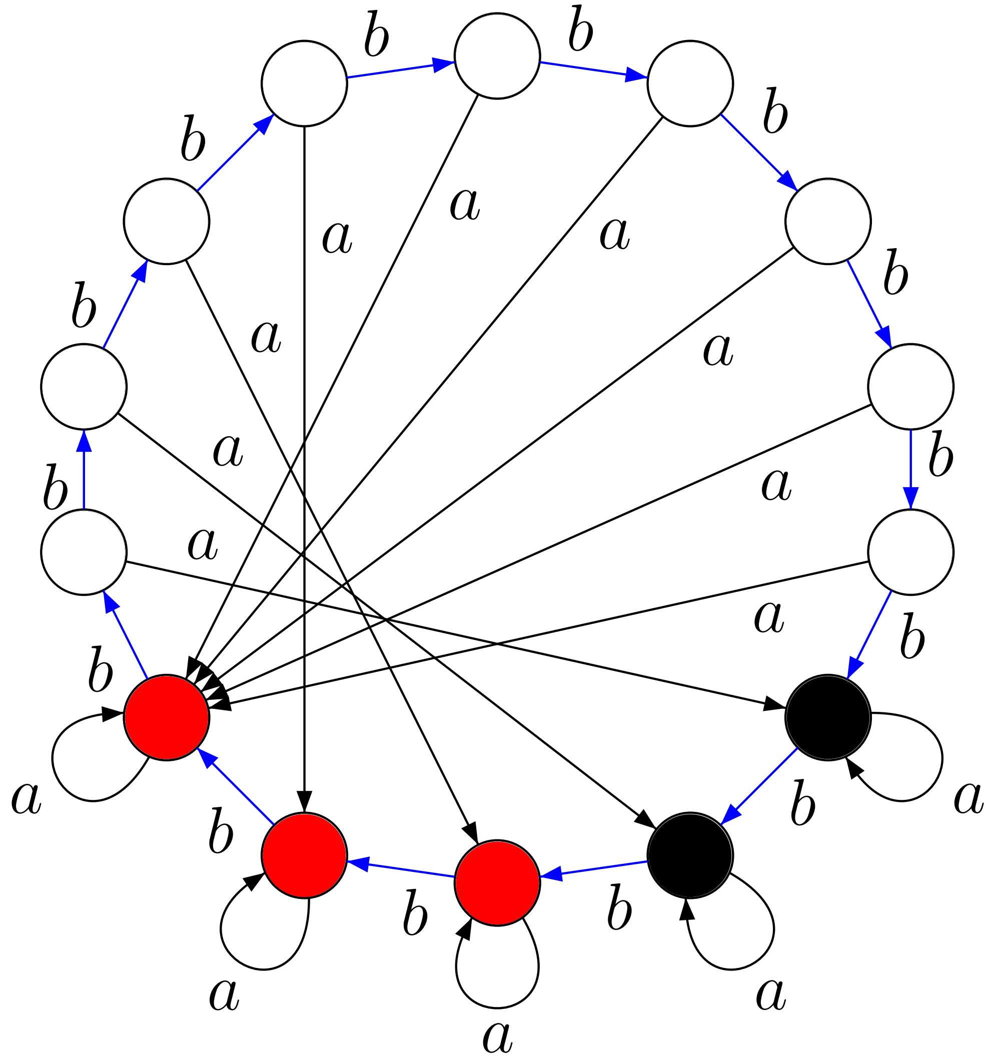

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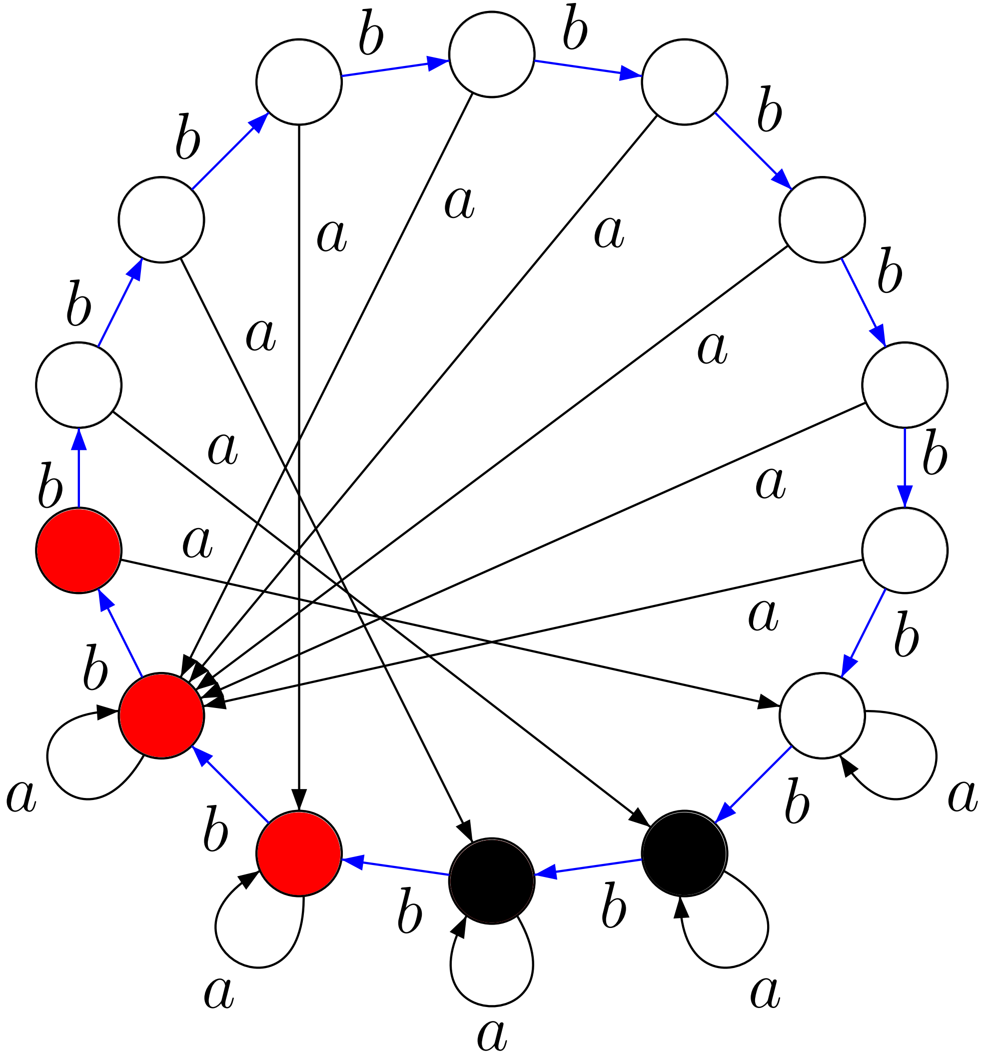

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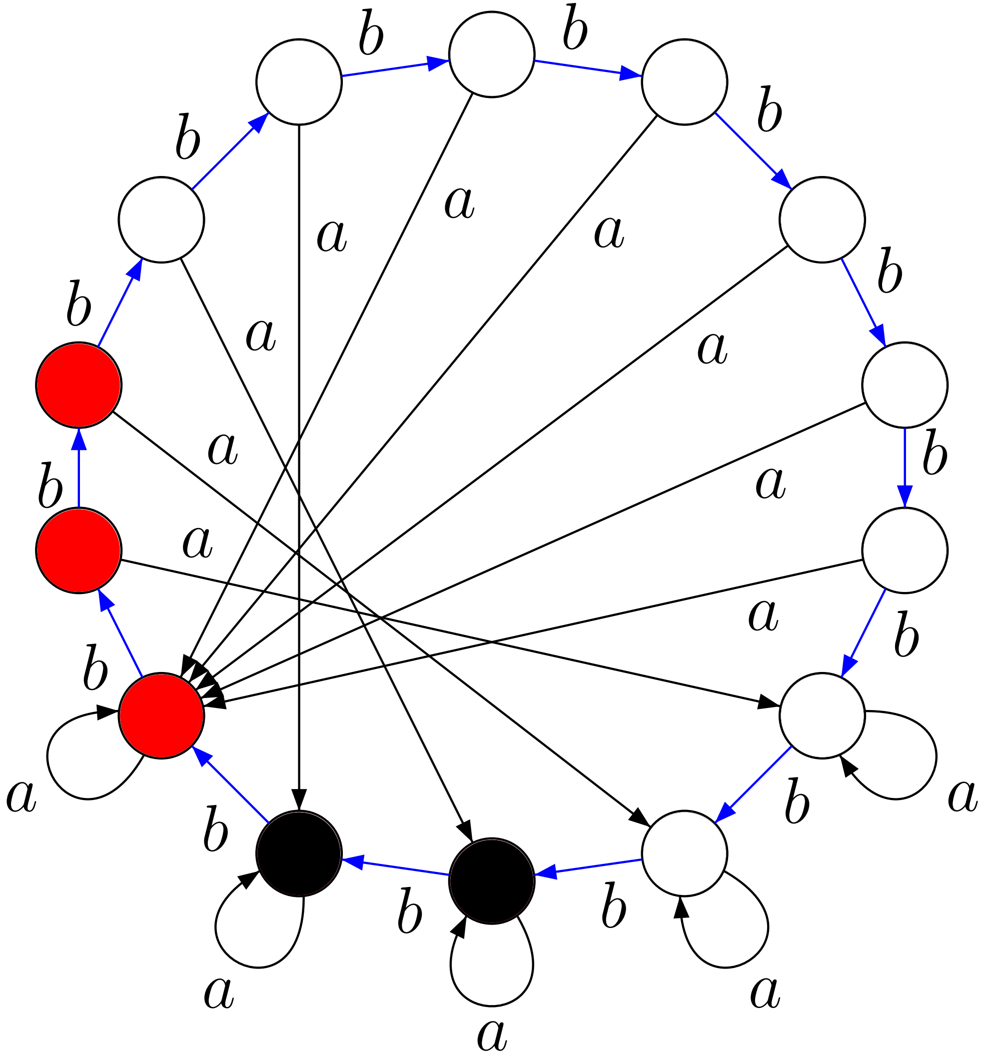

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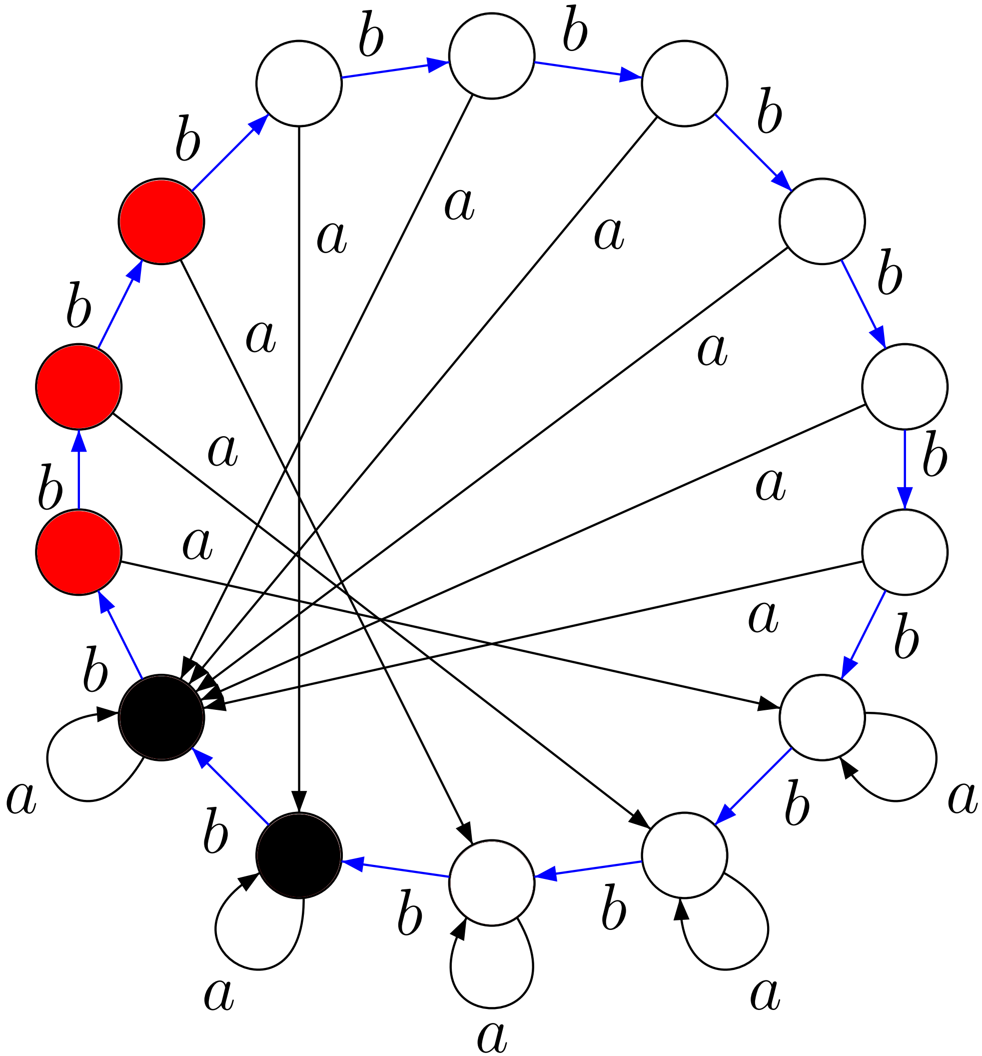

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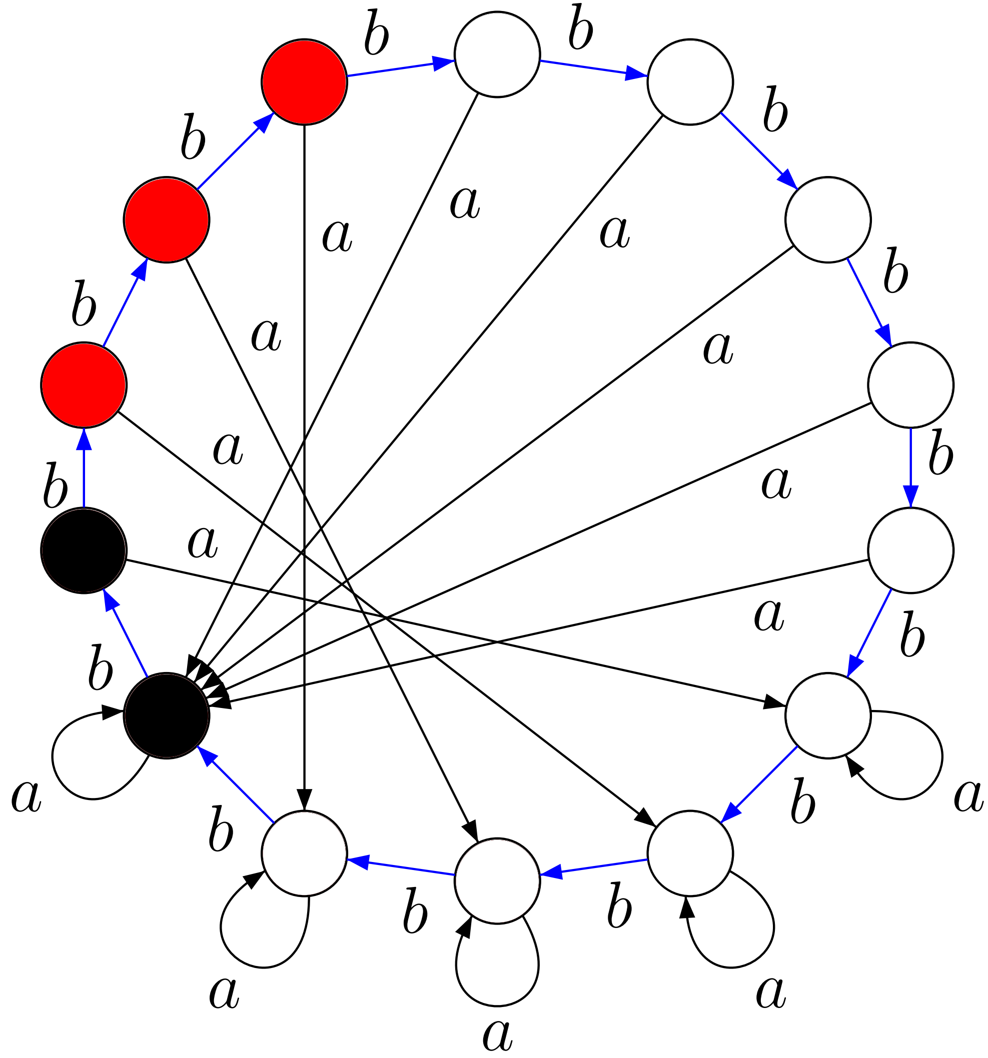

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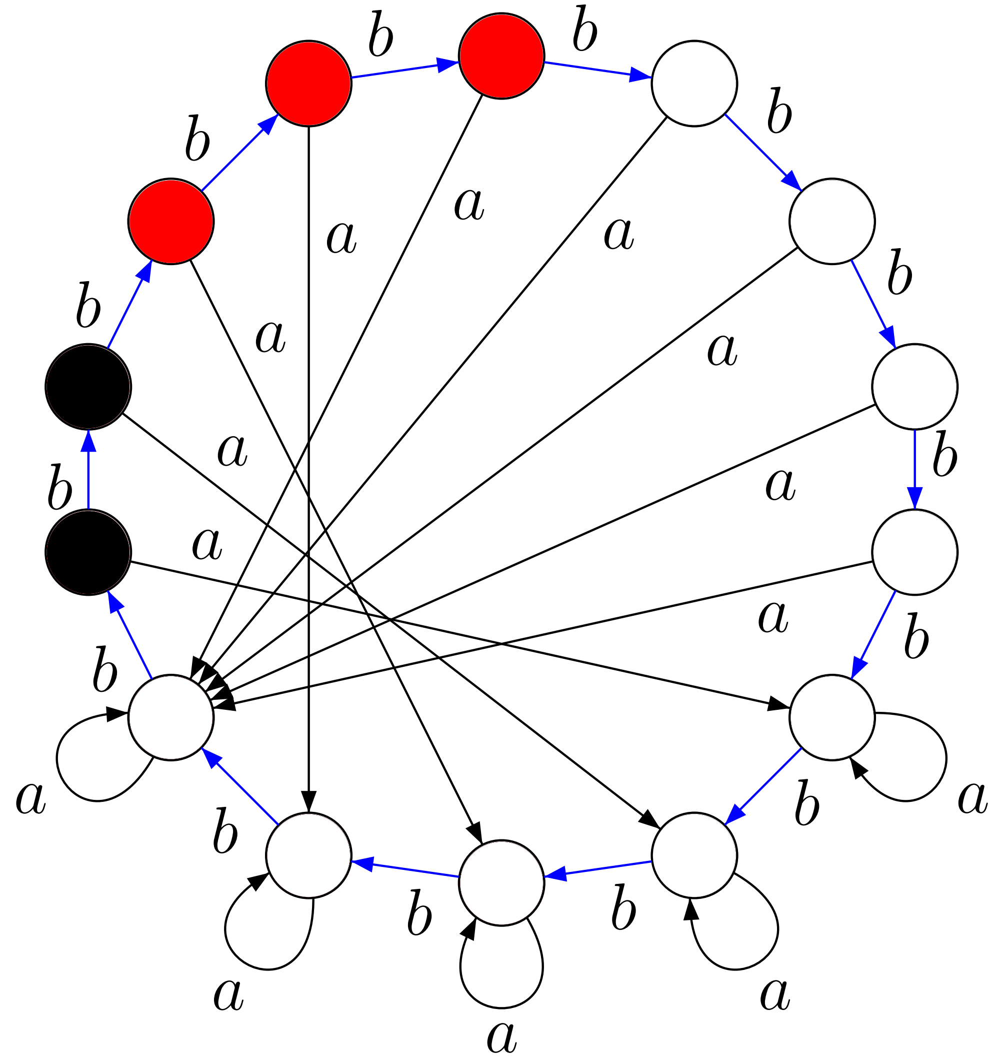

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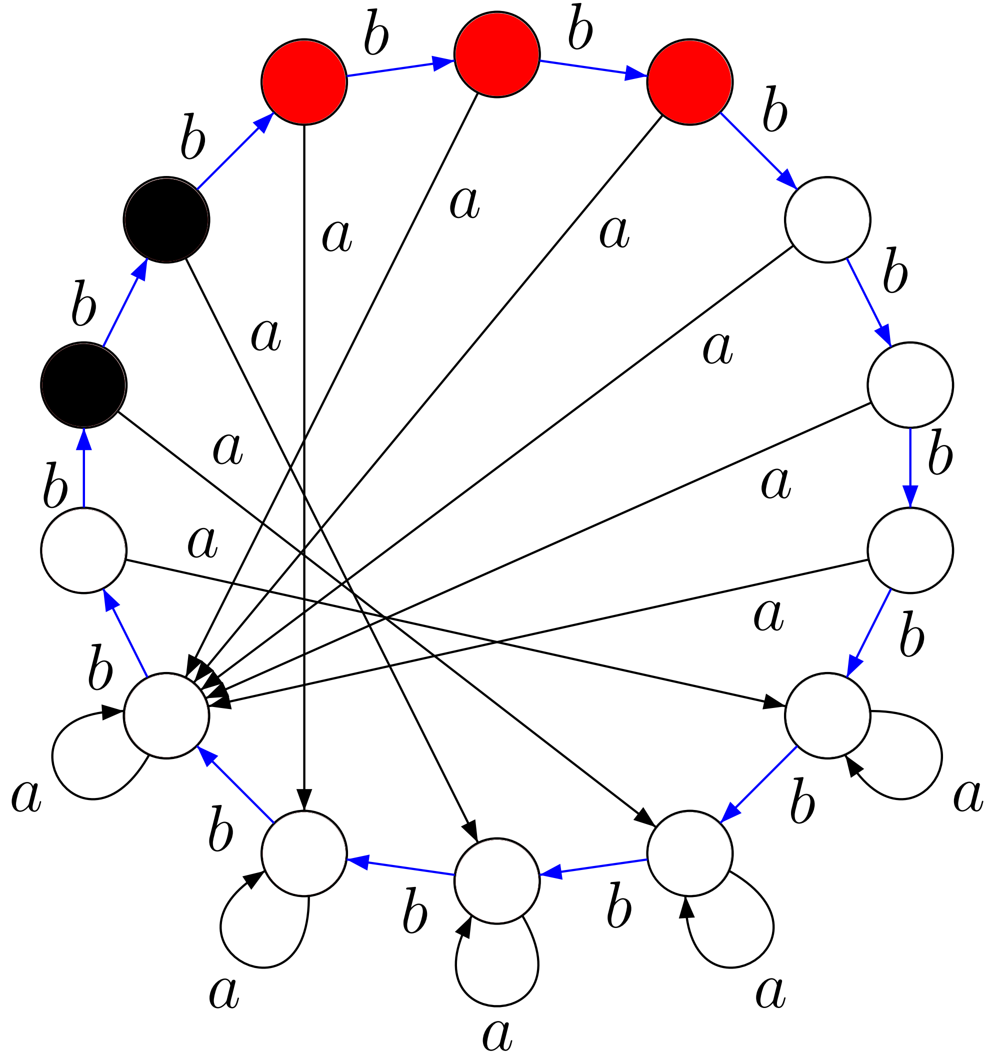

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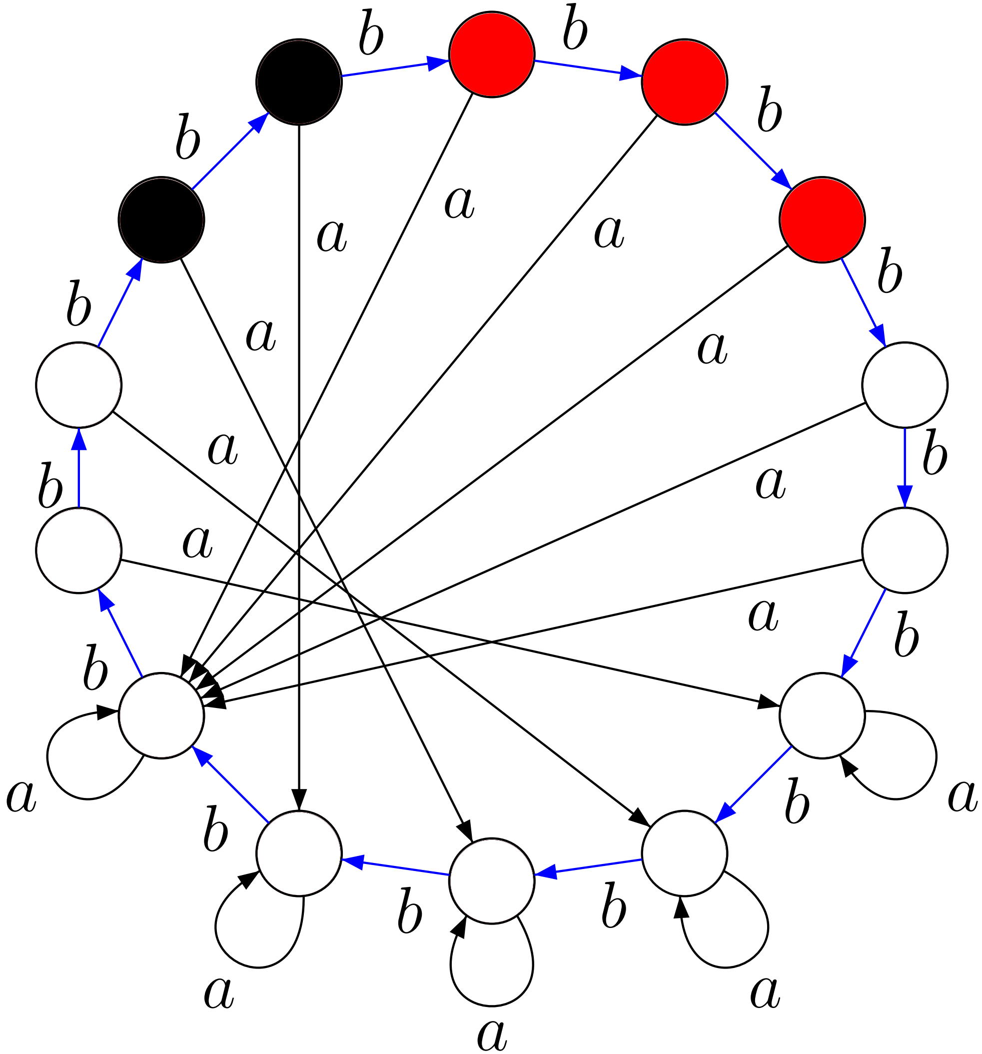

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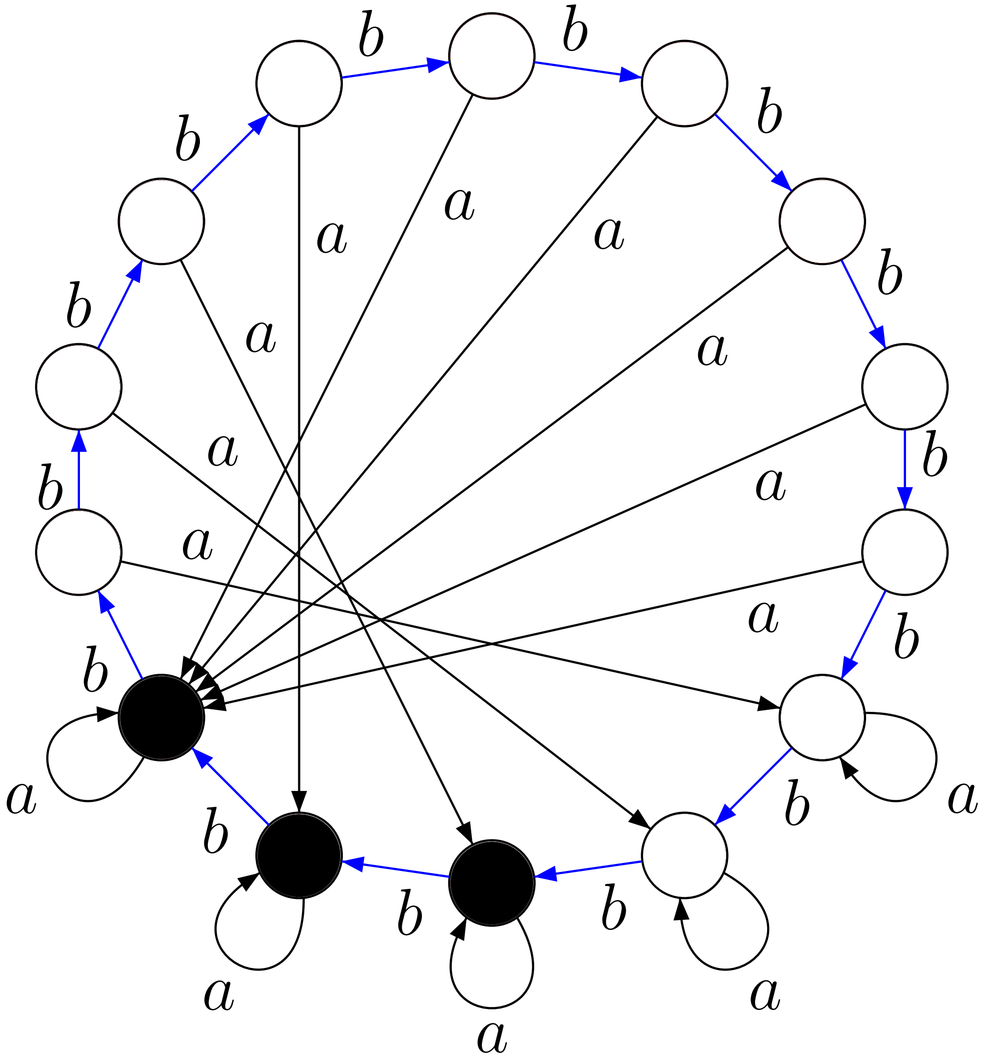

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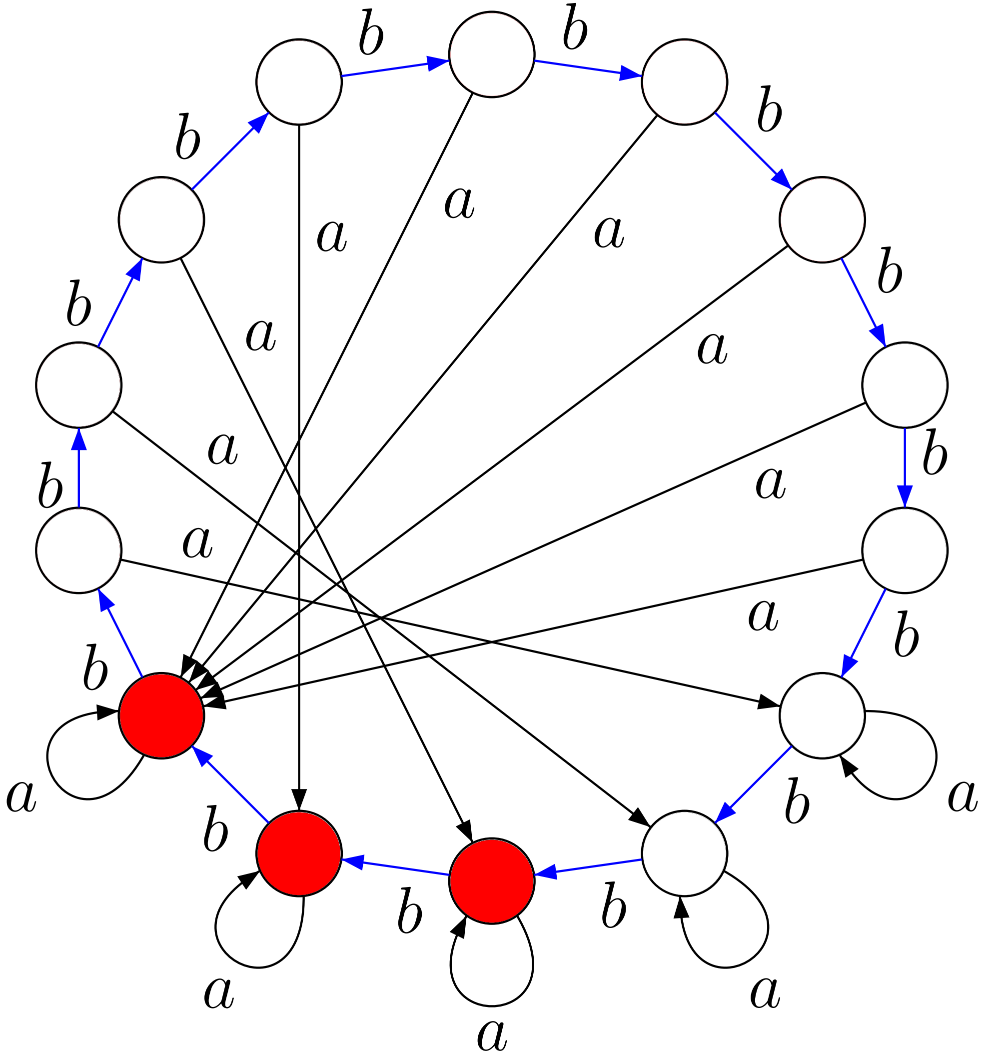

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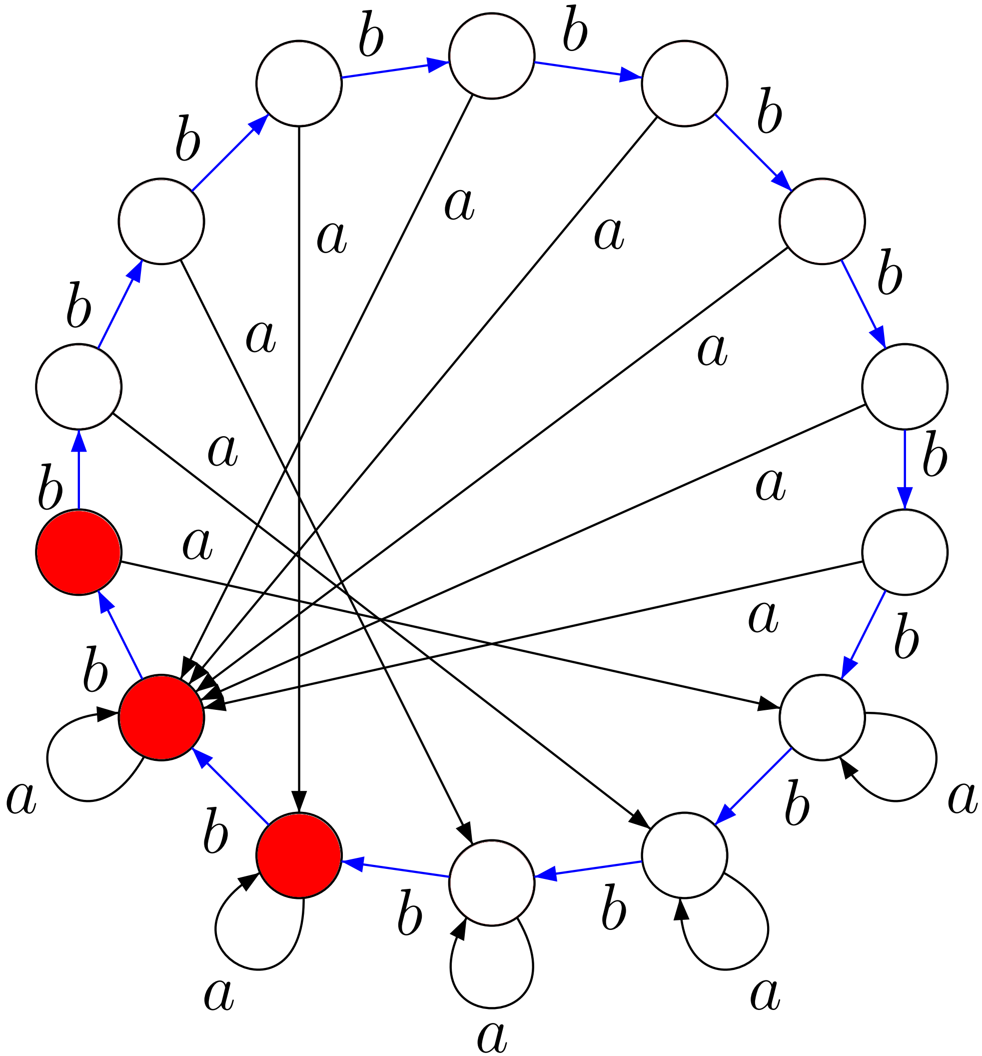

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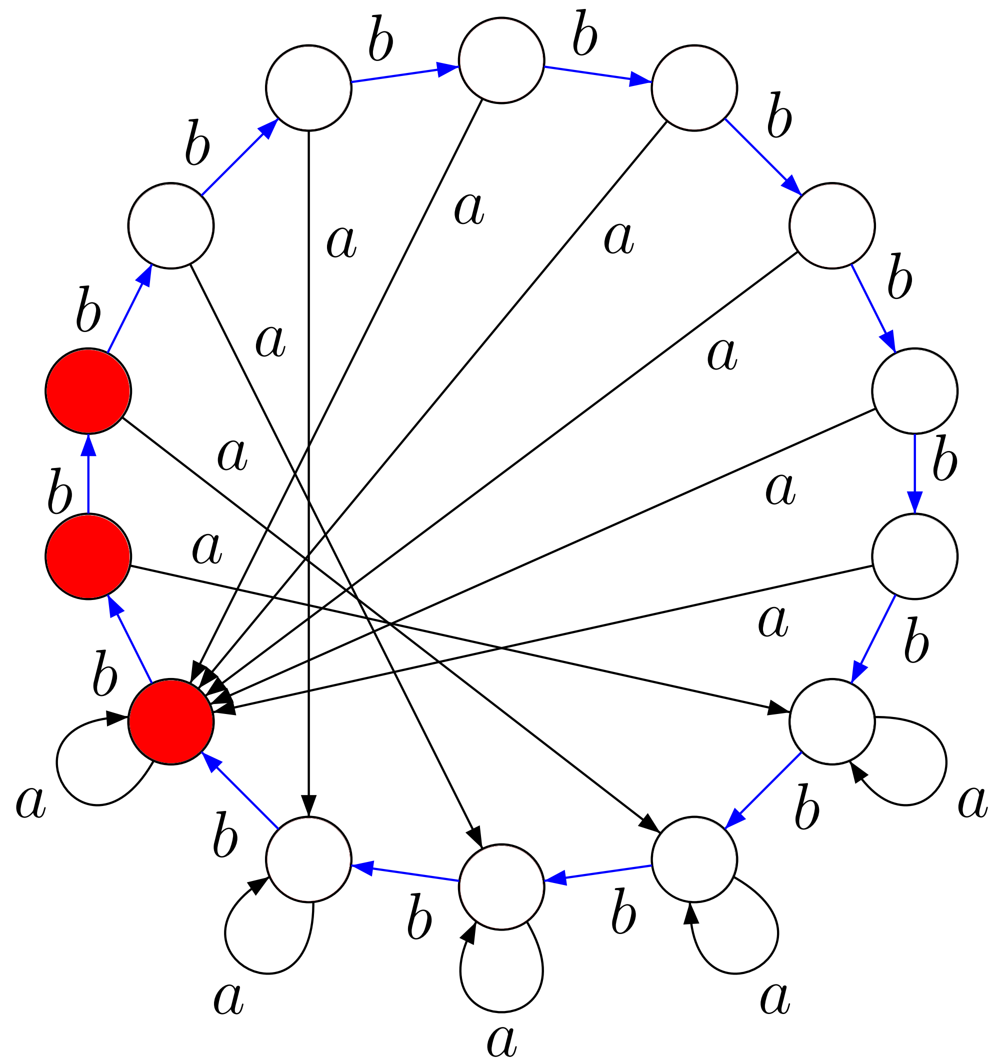

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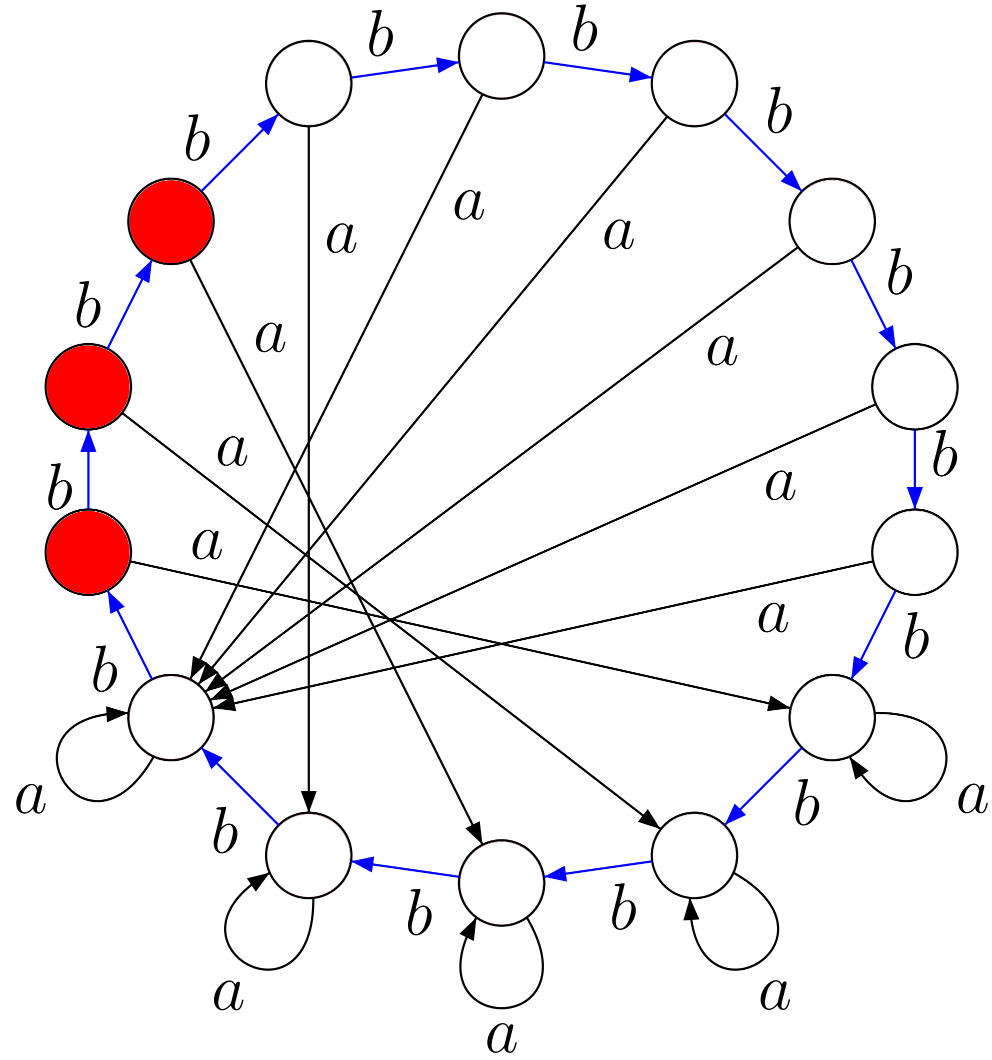

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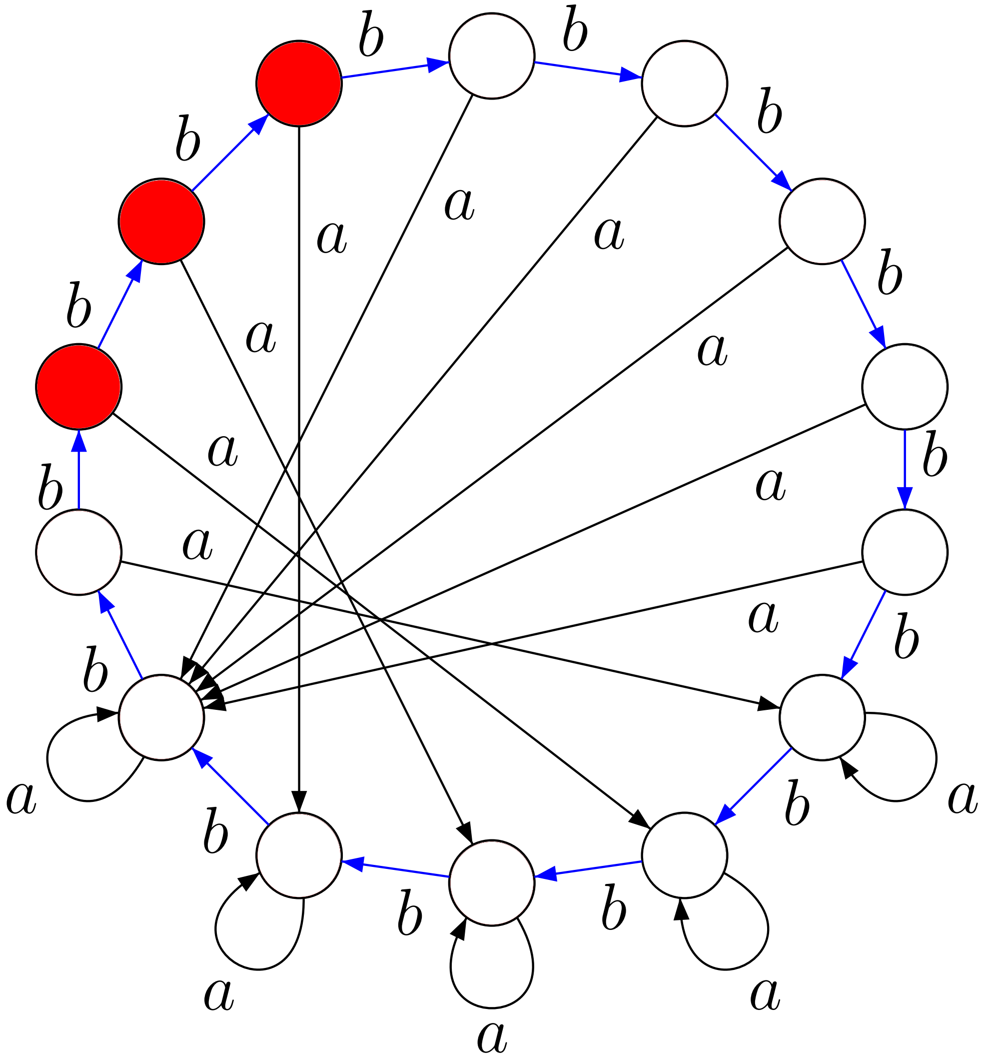

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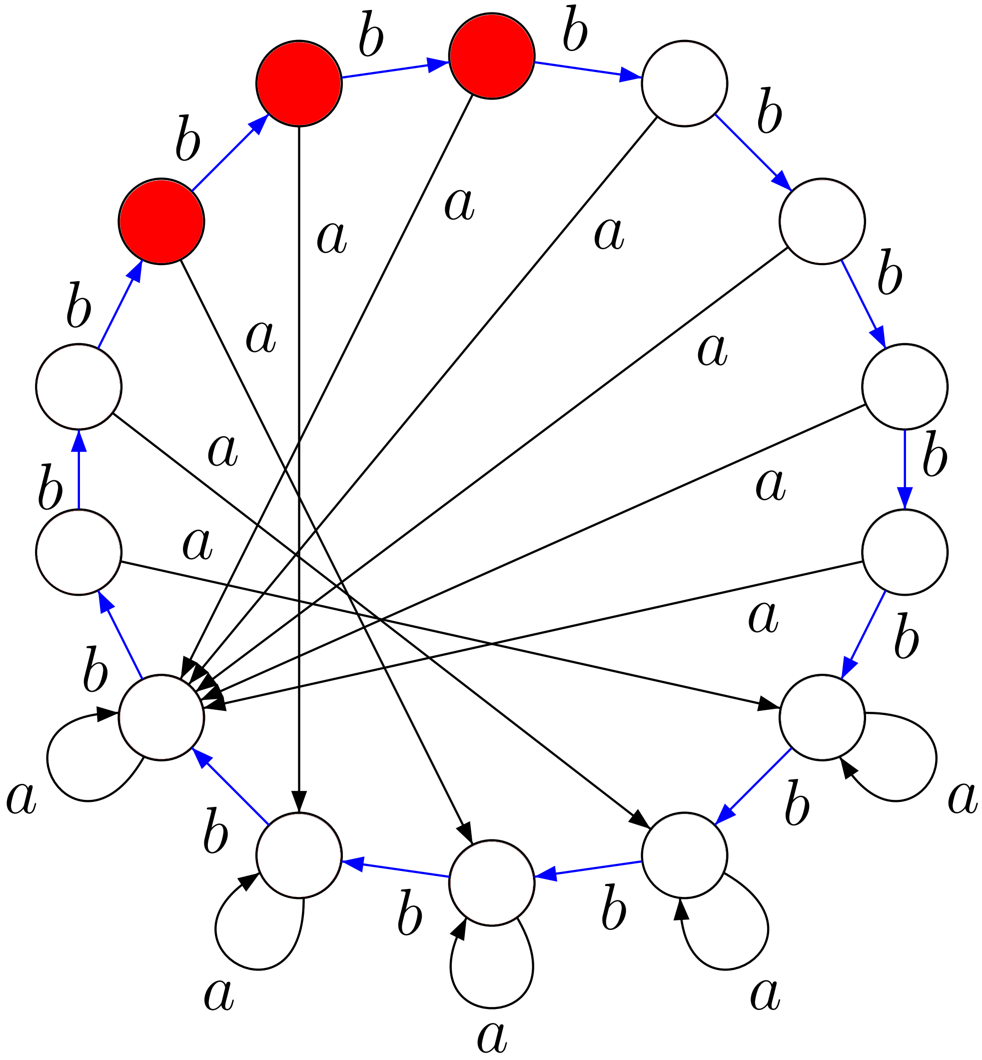

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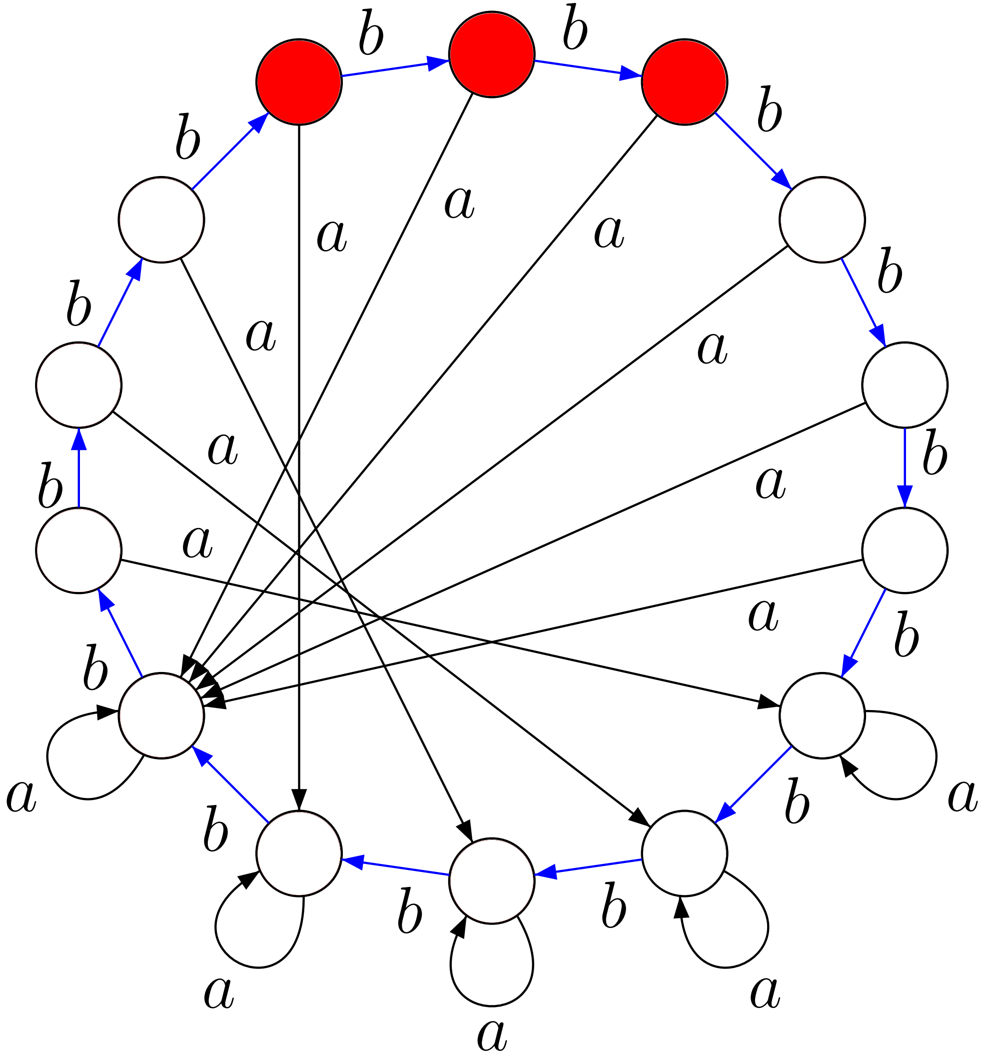

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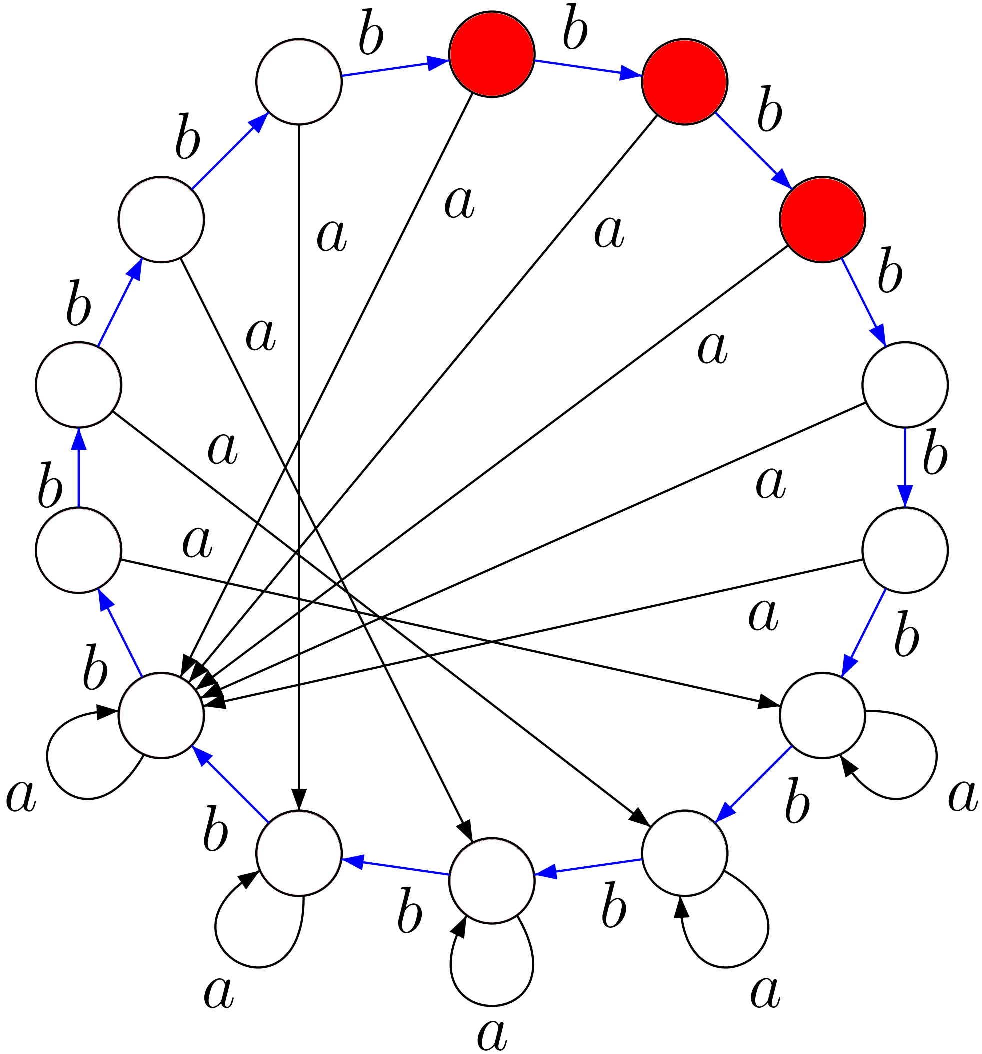

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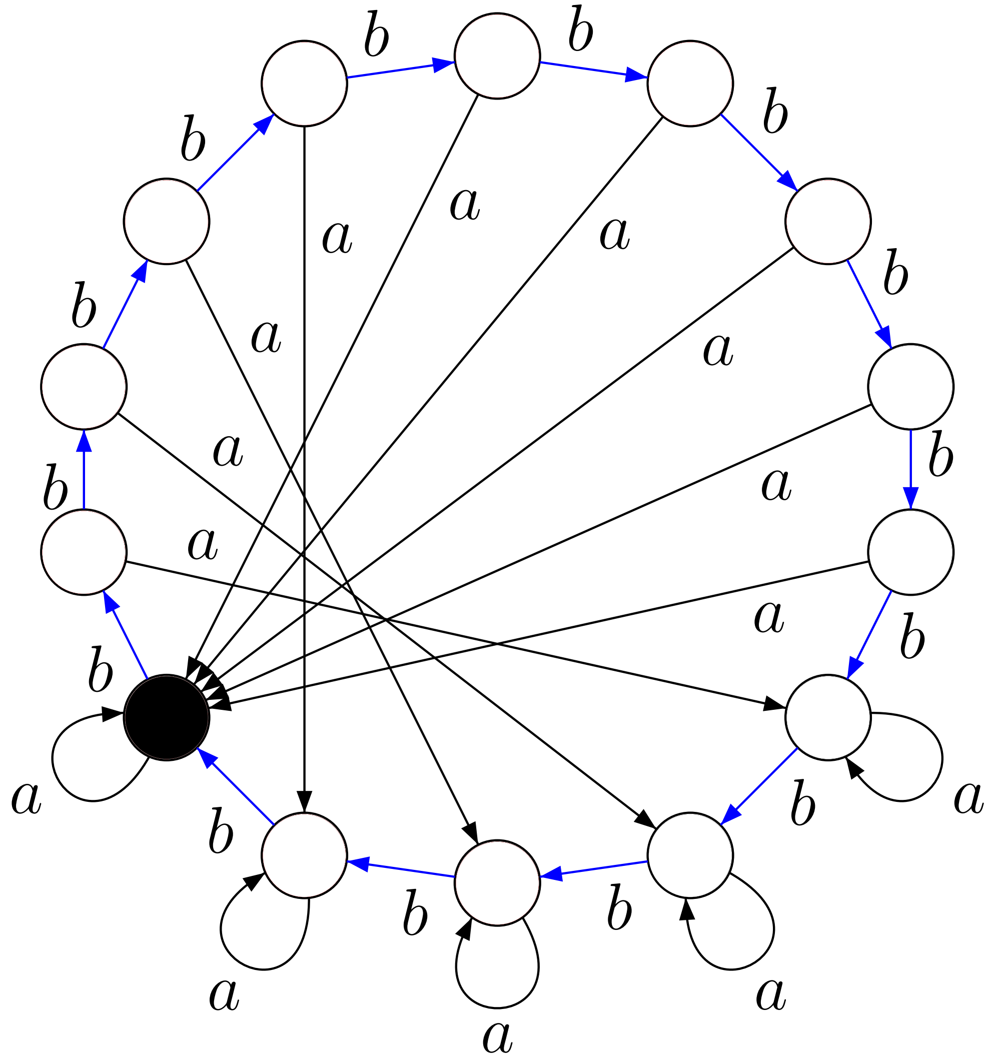

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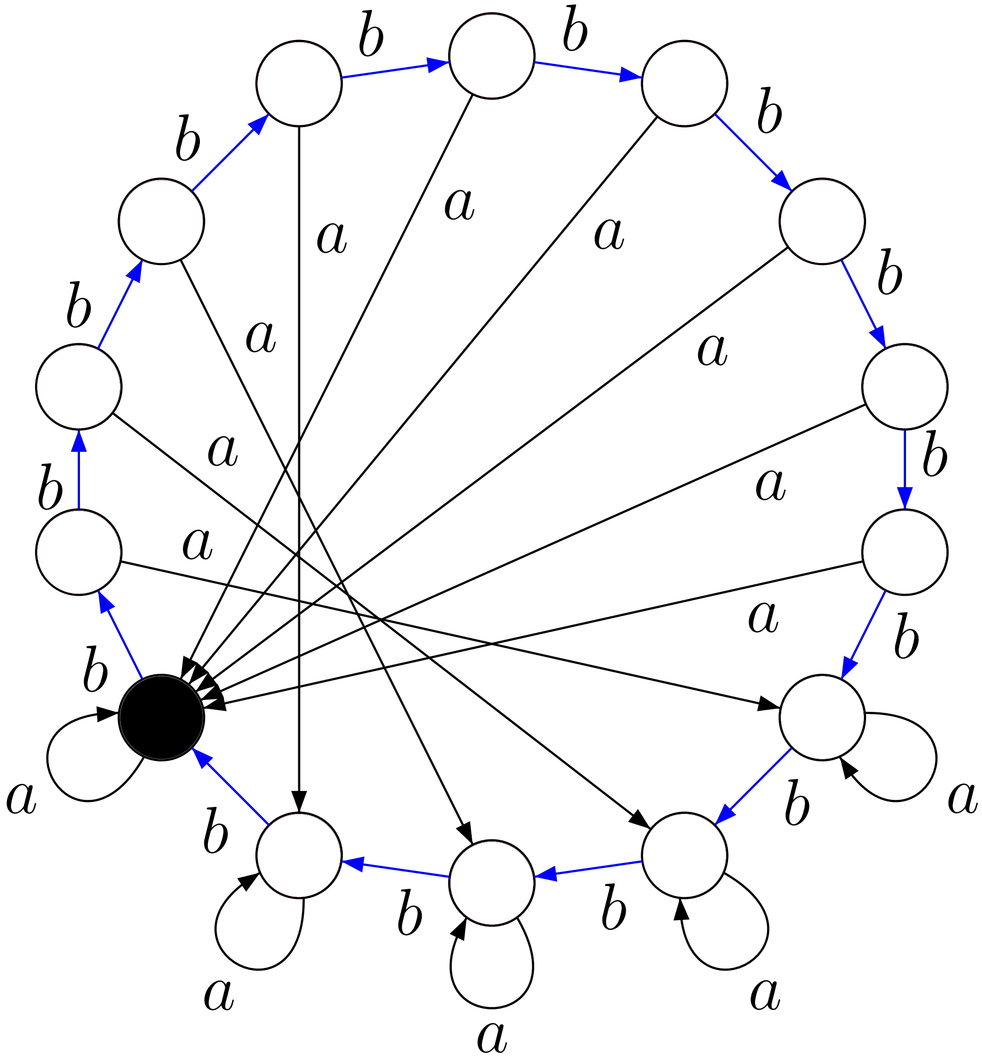

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approximation ratio $\frac{17}{11}$ or $\geq \frac{n-1}{6(k-1)}$

Theorem

Approximation ratio of $SYNCH-SUBSET(k)$ with arbitrary greedy choice of word and arbitrary greedy choice of set is at least $\frac{n}{6(k-1)}$.

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Open Problem
Is there a constant C, s.t. the approximation ratio of $SYNCH - SUBSET(k)$ with arbitrary greedy choice of word and arbitrary (not only greedy) choice of set is at least $C \cdot n$.

End

Thank you.