# On property $B$ of hypergraphs

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# Introduction

In 1961 Erdős and Hajnal introduced the quantity $m(n)$ as the minimum number of edges in an $n$-uniform hypergraph with chromatic number at least 3.

The first nontrivial estimate was obtained by Erdős:

**Theorem (Erdős, 1963)**

$2^{n-1} \leqslant m(n) \leqslant e(\ln 2)n^2 2^{n-2}(1 + \bar{o}(1)).$

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# Improvements of the lower bound

**Theorem (Schmidt, 1964)**

$m(n) \geqslant \frac{n}{n+2} 2^n$ *for any* $n \geqslant 2$.

**Theorem (Beck, Spencer, 1977-1981)**

$m(n) \geqslant c \left(\frac{n}{\ln n}\right)^{1/3} 2^n$ *for any* $n \geqslant 2$.

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# Pluhár's approach

Consider a hypergraph $H = (V, E)$. Let $\sigma$ denote an ordering of the vertices $V$. A couple of edges $A_1, A_2 \in E$ is called *ordered* $2$-*chain* in $\sigma$, if the following conditions hold:

1. $|A_1 \cap A_2| = 1$;
2. for any $v \in A_1$, $u \in A_2$, we have $\sigma(v) \leqslant \sigma(u)$.

**Theorem (Pluhár, 2009)**

*The chromatic number of a hypergraph $H = (V, E)$ does not exceed $2$ if and only if there exists an ordering $\sigma$ of $V$ such that there are no ordered $2$-chains in $H$.*

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# The new algorithm

Fix some $p \in [0, 1]$.
Color each vertex independently in red with probability $(1-p)/2$ or in blue with the same probability.
Note that with probability $p$ vertex has no color.

For every edge $A_i$ consider its colorless part $B_i$.
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$Q \subset 2^W$ is the set of all subsets $B_i \subset W$ such that $B_i$ is a colorless part of an edge $A_i$, in which all colored vertices got the same color.

We use greedy coloring for the new hypergraph $(W, Q)$.
Surprisingly, substituting $p = 0.5 \ln n / n$ we get exactly the Radhakrishnan and Srinivasan's bound (with the same constant):

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# Multiple colors case

**Theorem (Shabanov, 2009)**

$m(n,r) \geqslant \frac{1}{2} n^{1/2} r^{n-1}$ *for any* $n \geqslant 3$, $r \geqslant 3$.

**Theorem (Kostochka, 2006)**

$m(n,r) \geqslant e^{-4r^2} \left(\frac{n}{\ln n}\right)^{a/(a+1)} r^n$, *where* $a = \lfloor \log_2 r \rfloor$ *and* $r < \sqrt{1/8 \ln 1/2 \ln n}$.

**Theorem (Pluhar, Shabanov, 2009)**

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# Multiple colors case. New algorithm part 2

We use modified Pluhár argument.

Consider a hypergraph $H = (V, E)$ and a map $f$ from $E$ to $\{1, \ldots, r\}$.

Ordered $r$-chain is called strong ordered if $f(A_i) = i$ for $i = 1 \ldots r$.

**Theorem**

*The following statements are equivalent:*
*(i) there is a coloring of $V$ in $r$ colors such that no edge $A$ in $E$ consists only of vertices of color $f(A)$;*
*(ii) there is an order of elements of $V$ without strong ordered $r$-chains.*

Substituting $p = \frac{r-1}{r}\frac{\ln n}{n}$ we get the theorem

**Theorem (DC, Kozik, 2013)**

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# The new algorithm. Counting.

Denote by $P_1$ the probability of the appearance of a monochromatic edge without colorless vertices.

$$P_1 \leqslant |E| \left( \frac{1-p}{2} \right)^n \leqslant |E| e^{-pn} 2^{-n} \leqslant |E| \frac{1}{\sqrt{n} 2^n}$$

Denote by $P_2$ the probability that the greedy algorithm fails.

$$\frac{P_2}{|E|^2} \leqslant p \left( \frac{1-p}{2} \right)^{2n} \sum_{k,l} \frac{(k-1)!(l-1)!}{(k+l-1)!} C_{n-1}^{k-1} \left( \frac{2p}{1-p} \right)^{k-1} C_{n-1}^{l-1} \left( \frac{2p}{1-p} \right)^{l-1}$$

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# Thank you!

Thank you for your patience!