The paged representation os stacks in single level memory

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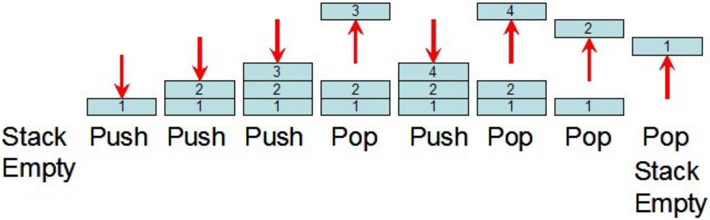
Introduction

Memory size limitations:

- The high cost of production
- Power consumption

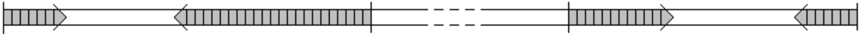
Usage of stacks:

- Calls to subroutines
- Recursive algorithms
- Problems of translation and syntax analysis

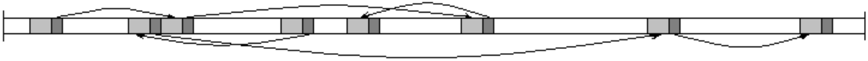


Representation of stacks

Consequtive representation



Linked representation



Paged representation

The problem

Consider n stacks in memory size m. Time is discrete and only one of operations can happen on each time step:

- p_i – probability of insertion of element into i-th stack,
- q_i – probability of deletion of element from the i-th stack,
- r – probability of read of element from any stack (without deletion).

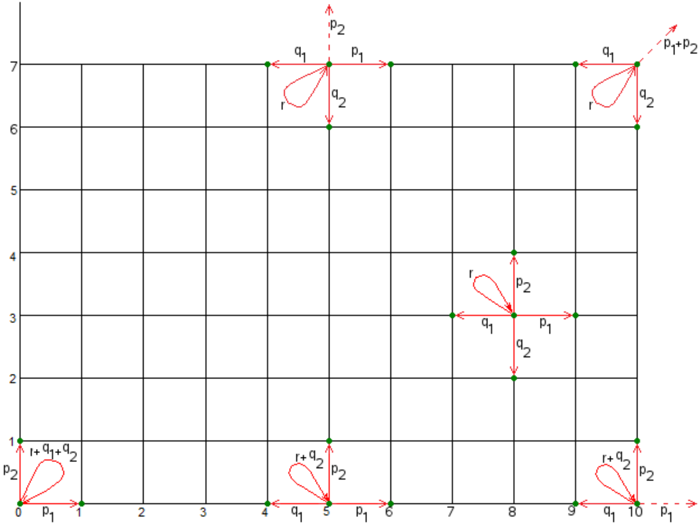
T – time until overflow (number of time steps).
There is no shutdown in the case of deletion of element from empty stack.
x_i – current length of i-th stack.
Process starts from empty stacks.

Absorbing Markov chain

$(x_1, \ldots, x_n)$ detetermines the state of Markov chain. To caclulate T we need:

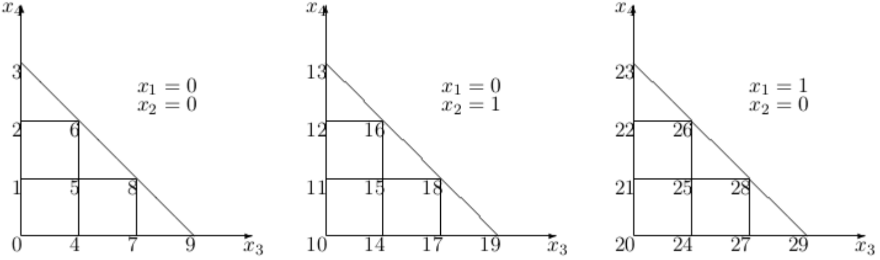
- Introduce the numbering of states.
- Construct function $F(x_1, \ldots, x_n) = I$, where I is the number of state.
- Build the transition matrix Q by iterating over states:
  $Q[F(x_1, \ldots, x_i, \ldots, x_n)][F(x_1, \ldots, x_i + 1, \ldots, x_n)] = p_i$,
  $Q[F(x_1, \ldots, x_i, \ldots, x_n)][F(x_1, \ldots, x_i - 1, \ldots, x_n)] = q_i$,
  $Q[F(x_1, \ldots, x_i, \ldots, x_n)][F(x_1, \ldots, x_i, \ldots, x_n)] = p_i$,
  Q_{ij} shows the propability of thansition in one step from state with number i to state with number j.
  Other element are equal 0.
- Calculate the fundamental matrix $N = (E - Q)^{-1}$.
- Sum the elements from row which matches the initial state.

Random walk



Consequtive representation

Consider fixed partition of memory $k_1, \dots, k_{n/2}$ $(k_1 + \dots + k_{n/2} = m)$.



$n = 4, m = 4, k_1 = 3, k_2 = 1.$

Consequtive representation

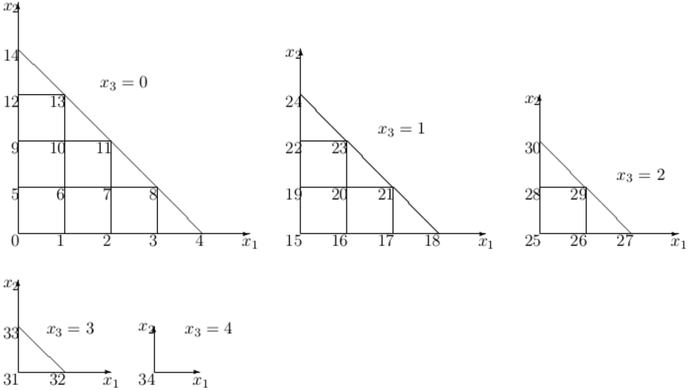
Number of states is equal to:

$$\frac{1}{2^{n/2}}\prod_{i=1}^{n/2}(k_i+1)(k_i+2)$$

$$F(x_1,\ldots,x_n) = \Big(\tfrac{(k_{n/2}+1)(k_{n/2}+2)}{2}\ldots\tfrac{(k_1+1)(k_1+2)}{2}(x_2(k_1+\tfrac{3-x_2}{2})+x_1)+ x_4(k_2+\tfrac{3-x_4}{2})+x_3)\ldots\Big)+x_n(k_{n/2}+\tfrac{3-x_n}{2})+x_{n-1}$$

Linked representation

l is the ratio of the size of pointer to the size of essential data.
$M = [\frac{m}{1+l}]$ - size of memory which is allocated to essential data.



$n = 3, M = 4$

Linked representation

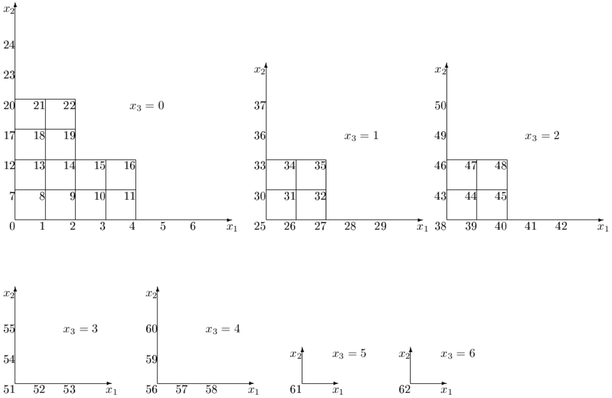
Number of states is equal to: $\dfrac{(M+n)!}{M!n!}$

$$F(x_1,\dots,x_n)=\sum_{j=1}^{n}\binom{M-w_j+j}{j}-\binom{M-w_j+j-x_j}{j}$$

where $w_j = x_{j+1}+\cdots+x_n$.

Paged representation

$M = [\frac{m}{1+l}]$ - size of memory which is allocated to essential data.
k – size of page.
$N = \left\lfloor \frac{M}{k} \right\rfloor$ – maximum number of pages.



$n = 3, m = 6, k = 2, N = 3$

Paged representation

Total number of states is equal to

$$\sum_{i=0}^{n} \binom{n}{i} \binom{N}{n-i} k^{n-i}$$

$$F(x_1, \dots, x_n) = \sum_{j=1}^{n} \binom{M - z_j + j}{j} - \binom{M - z_j + j - x_j}{j}$$

where $z_j = \left\lceil \frac{x_{j+1}}{k} \right\rceil k + \dots + \left\lceil \frac{x_n}{k} \right\rceil k$

Calculations

- $O(n)$ – calculation of index for current state.
- $\leq 2n+1$ nonzero elements in transition matrix Q.
- S – dimension of transition matrix.
- $O(S * n^2)$ – the complexity of constructing of matrix.
- We need only the first row for matrix $N = (E - Q)^{-1}$.
- Calculations were made on the cluster KarRS RAS with usage of Intel Math Kernel Library.

Table 1 : $n = 4, m = 16, l = 1/2$

p_1	p_2	p_3	p_4	q_1	q_2	q_3	q_4	T_c^1	T_c^2	T_l	T_p^2	T_p^3
0.2	0.2	0.05	0.05	0.2	0.2	0.05	0.05	84.65	69.47	60.18	62.98	45.98
0.2	0.2	0.2	0.2	0.05	0.05	0.05	0.05	21.69	21.69	16.38	17.04	14.29
0.1	0.1	0.1	0.1	0.15	0.15	0.15	0.15	225.06	225.06	153.16	158.64	90.29
0.2	0.2	0.05	0.05	0.2	0.2	0.05	0.05	50.11	36.80	37.57	39.68	32.24
0.65	0.05	0.05	0.05	0.05	0.05	0.05	0.05	21.03	13.95	15.27	16.60	14.84
0.05	0.05	0.05	0.05	0.65	0.05	0.05	0.05	270.63	251.74	204.88	211.77	146.57
0.41	0.03	0.03	0.03	0.03	0.03	0.03	0.41	84.65	23.18	26.34	28.94	26.05

T_c^1 – time for consequtive representation in the case of optimal partition of memory.

T_c^2 – time for consequtive representation when memory is divided equally between pairs of stacks.

T_l – time for linked representation.

T_p^i – time for paged representation when size of pages is equal to i.