Discrete time two-sided mate choice problem with age preferences

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Best-choice problem

matching model, job search model, mating model, buyer-seller model

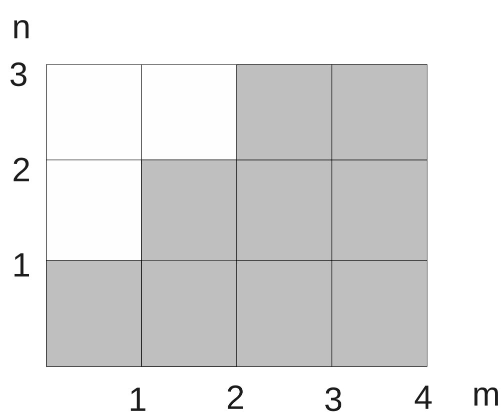
Two-sided mate choice problem

- Alpern S., Reyniers D.J. (1999, 2005) Homotypic and common preferences
- Mazalov V., Falko A. (2008) Common preferences, arriving flow
- Alpern S., Katrantzi I., Ramsey D. (2010) Age preferences: discrete time model
- Alpern S., Katrantzi I., Ramsey D. (2013) Age preferences: continuous time model

Alpern S., Katrantzi I., Ramsey D. (2010)

- Males have m periods for mating, females — n periods, $m > n$.
- It is assumed that the total number of unmated males is greater than the total number of unmated females.
- Each group has steady state distribution for the age of individuals.
- In the game unmated individuals from different groups randomly meet each other in each period. If they accept each other, they form a couple and leave the game, otherwise they go into the next period unmated and older.
- Payoff of mated player is the number of future joint periods with selected partner: payoff of male age i and female age j is equal to $\min\{m - i + 1; n - j + 1\}$
- The aim of each player is to maximize the expected payoff.

- $a = (a_1, ..., a_m),\ b = (b_1, ..., b_n)$.
- a_i — the number of unmated males of age i relative to the number of females of age 1.
- b_j — the number of unmated females of age j relative to the number of females of age 1 ($b_1 = 1$).
- R — the ratio of the rates at which males and females enter the adult population $R = \dfrac{a_1}{b_1} = a_1$.
- $A = \sum_{i=1}^{m} a_i,\ B = \sum_{i=1}^{n} b_j,\ r = \dfrac{A}{B},\ r > 1$.
- $F = [f_1, ..., f_m],\ G = [g_1, ..., g_n]$
- $f_i = k$, $k = 1, ..., n$ — to accept a female of age $1, ..., k$
- $g_j = l$, $l = 1, ..., m$ — to accept a male of age $1, ..., l$



F=[1,2,3,3],   G=[4,4,4]

The equilibrium age distributions are equal to

$$a_{i+1} = a_i \left(1 - \sum_{i \leftrightarrow j} \frac{b_j}{A} \right), \; i = 1, ..., m-1;$$

$$b_{j+1} = b_j \left(1 - \sum_{i \leftrightarrow j} \frac{a_i}{A} \right), \; j = 1, ..., n-1.$$

- U_i, $i = 1, ..., m$ — the expected payoff of male of age i.
- V_j, $j = 1, ..., n$ — the expected payoff of female of age j.
- $\frac{a_i}{A}$ — the probability a female is matched with a male of age i,
- $\frac{B}{A}$ — the probability a male is matched.
- $\frac{b_j}{A} = \frac{b_j}{B} \cdot \frac{B}{A}$ — the probability a male is matched with a female of age j.

i accepts j if $\min\{m-i+1, n-j+1\} \geq U_{i+1}$;

j accepts i if $\min\{m-i+1, n-j+1\} \geq V_{j+1}$.

Model 1: Maximum age of males $m > 2$, maximum age of females $n = 2$

Strategies: $F = [f_1, ..., f_m]$, $G = [g_1, g_2]$

The expected payoffs of females are equal to

$$\begin{cases} V_2 = \sum_{i=1}^{m-1} \frac{a_i}{A} I\{f_i = 2\} + \frac{a_m}{A} \leq 1, \\ \\ V_1 = \sum_{i=1}^{m-1} 2\frac{a_i}{A} + \frac{a_m}{A} \max\{1, V_2\} = 2 - \frac{a_m}{A}. \end{cases}$$

$G = [m, m]$: the expected payoffs of males are equal to

$$\begin{cases} U_m = \frac{b_1}{A}1 + \frac{b_2}{A}1 + (1 - \frac{B}{A})0 = \frac{1}{r} < 1, \Rightarrow f_m = 2,\ f_{m-1} = 2 \\ \\ U_{m-1} = \frac{b_1}{A}2 + \frac{b_2}{A}\max\{1, U_m\} + \left(1 - \frac{1}{r}\right) U_m = \frac{2}{r} - \frac{b_2}{A} + \left(1 - \frac{1}{r}\right) U_m < 2, \\ \\ U_{m-2} = \frac{b_1}{A}2 + \frac{b_2}{A}\max\{1, U_{m-1}\} + \left(1 - \frac{1}{r}\right) U_{m-1} \\ \ldots \\ U_1 = \frac{b_1}{A}2 + \frac{b_2}{A}\max\{1, U_2\} + \left(1 - \frac{1}{r}\right) U_2 \end{cases}$$

Theorem 1. *Equilibrium strategy of female is to accept any partner. Equilibrium strategy of male of age i is $f_i = 1$, if $U_{i+1} > 1$, $i = 1, ..., m-2$, and $f_i = 2$, if $U_{i+1} \leq 1$, $i = 1, ..., m-2$.*

Steady state distributions for the age of males and females:

$a = \left(R, Rz, ..., Rz^{m-1}\right)$,

$$b = (1, 0),\ R = a_1 = \frac{1}{(1-z)(1+z+z^2+...+z^{m-1})}$$

$$\begin{cases} U_m = 1 - z, \\ U_{m-i} = 2(1-z) + zU_{m-i+1},\ i = 1, ..., m-2, \end{cases}$$

where $z = 1 - 1/r$.

Equilibrium for $m = 4$, $n = 2$	$r = \frac{A}{B}$
$([1,1,2,2],[4,4])$	$(1, 2.618)$
$([1,2,2,2],[4,4])$	$[2.618, 4.079)$
$([2,2,2,2],[4,4])$	$[4.079, +\infty)$

Model 2: Maximum age of males $m \geq 3$, maximum age of females $n = 3$

The expected payoff of female has the following form

$$V_3 = \sum_{i=1}^{m-1} \frac{a_i}{A} I\{f_i = 3\} + \frac{a_m}{A} \leq 1,$$

$$V_2 = \sum_{i=1}^{m-2} \frac{a_i}{A} 2I\{f_i \geq 2\} + \frac{a_{m-1}}{A} 2 + \frac{a_m}{A} 1 \leq 2,$$

$$V_1 = \sum_{i=1}^{m-2} \frac{a_i}{A} 3 + \frac{a_{m-1}}{A} 2 + \frac{a_m}{A} \max\{1, V_2\},$$

Females have two equilibrium strategies $G_1 = [m-1, m, m]$, $G_2 = [m, m, m]$ for different values of operational sex ratio r.

Males use strategy $F = [f_1, ..., f_m]$ — to accept only those females who are not older than f_i, $i = 1, ..., m$.

There are three forms of strategies:

$G_2 = [m, m, m]$	$G_1 = [m-1, m, m]$
I. $F_1 = [\underbrace{1,...,1}_{k}, \underbrace{2,...,2}_{l}, \underbrace{3,...,3}_{m-k-l}]$ $k = 1,...,m-3,\ l = 1,...,m-3$	II. $F_3 = [\underbrace{2,...,2}_{k}, \underbrace{3,...,3}_{m-k}]$ $k = 1,...,m-2$
	III. $F_2 = [\underbrace{1,...,1}_{k}, \underbrace{2,...,2}_{l}, \underbrace{3,...,3}_{m-k-l}]$ $k = 1,...,m-3,\ l = 1,...,m-3$

I. Players use strategy profile (F_1, G_2),
where $G_2 = [m, m, m]$ (to accept any partner), $F_1 = [\underbrace{1, ..., 1}_{k}, \underbrace{2, ..., 2}_{l} \underbrace{3, ..., 3}_{m-k-l}]$

Theorem 2. *If players use strategy profile* (F_1^*, G_2^*),
where $G_2^* = [m, m, m]$, $F_1^* = [\underbrace{1, ..., 1}_{k}, \underbrace{2, ..., 2}_{l} \underbrace{3, ..., 3}_{m-k-l}]$, *then male's payoffs are equal to*

$$\begin{cases} U_m = 1 - z, \\ U_{m-1} = 2 - z^2 - z, \\ U_{m-i} = 3 - z^{i+1} - z^i - z^{i-1},\ i = 2, ..., m-2. \end{cases}$$

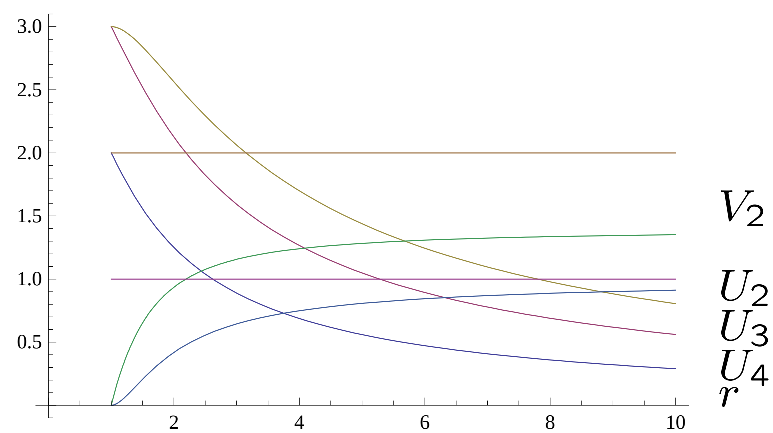
Equilibrium distributions are equal to

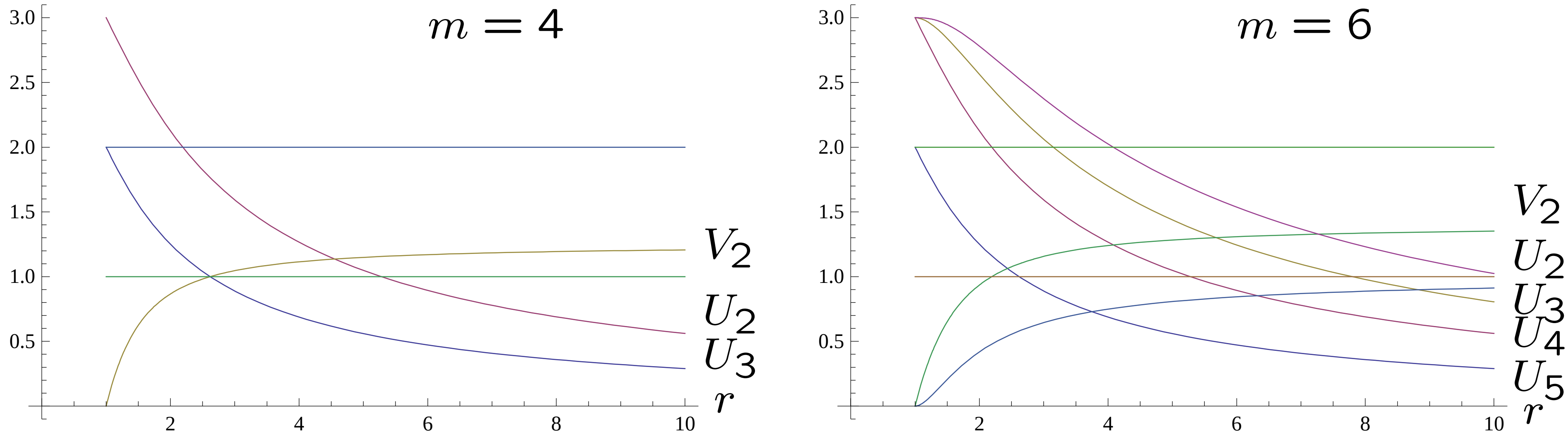
$m = 5$

$$a = (R, Rz, Rz^2, ..., Rz^{m-1});\ b = (1, 0, 0),$$

$$R = \frac{1}{(1-z)(1+z+z^2+...+z^{m-1})},$$

$A = r$,
for $z = 1 - 1/r$.





For $m = 4$ and $r = 2$, we obtain $a = \left(\frac{16}{15}, \frac{8}{15}, \frac{4}{15}, \frac{2}{15}\right)$, $b = (1, 0, 0)$, $F_1^* = [1, 2, 3, 3]$, $G_2^* = [4, 4, 4]$.

$F_1^* = [1, ..., 1, 2, 3, 3]$ for $r \in (1;\ 2.191)$ and $m \geq 4$,

$F_1^* = [1, ..., 1, 2, 2, 3, 3]$ for $r \in [2.191;\ 2.618)$ and $m \geq 6$,

$F_1^* = [1, ..., 1, 2, 3, 3, 3]$ for $r \in [2.618;\ 3.14)$ and $m \geq 6$,

$F_1^* = [1, ..., 1, 2, 2, 3, 3, 3]$ for $r \in [3.14;\ 4.079)$ and $m \geq 7$.

II. Female's strategy is $G_1 = [m-1, m, m]$ $(V_2 \geq 1)$,

male's strategy is $F_3 = [\underbrace{2, ..., 2}_{k}, \underbrace{3, ..., 3}_{m-k}]$, $k = 1, ..., m-2$.

Theorem 3. *If players use the equilibrium strategy profile* (F_3^*, G_1^*)*, where* $G_1^* = [m-1, m, m]$*,* $F_3^* = [\underbrace{2, ..., 2}_{k}, \underbrace{3, ..., 3}_{m-k}]$*, for certain values of* k *(*$k = 1, ..., m-2$*) then the males' optimal payoffs are equal to*

$$U_m = 1 - z - \frac{1}{A},$$

$$U_{m-1} = 2(1-z) + zU_m,$$

$$U_{m-i} = 3 - \frac{a_m}{A^2(1-z)} - \left(1 - \frac{a_m}{A^2(1-z)}\right) z^{i-1} - \left(1 + \frac{1}{A}\right) z^i - z^{i+1},$$
$$i = 2, ..., m-2,$$

the equilibrium age distributions are equal to

$$a = \left(R, Rz, Rz^2, ..., Rz^{m-1}\right), b = \left(1, \frac{z^{m-1}}{\sum\limits_{i=0}^{m-1} z^i}, 0\right).$$

$$R = \frac{1 + z + z^2 + ... + z^{m-2} + 2z^{m-1}}{(1-z)(1 + z + z^2 + ... + z^{m-1})^2},$$

$$A = R\sum_{i=0}^{m-1} z^i,$$

where $z = 1 - 1/r$.

Equilibrium for $m = 5$	$r = \frac{A}{B}$
$([2,2,3,3,3],[4,5,5])$	$[2.85, 4.517)$
$([2,3,3,3,3],[4,5,5])$	$[4.517, 6.87)$
$([3,3,3,3,3],[4,5,5])$	$[6.87, +\infty)$

III. Female's strategy is $G_1 = [m-1, m, m]$ ($V_2 \geq 1$),

male's strategy is $F_2 = [\underbrace{1, ..., 1}_{k}, \underbrace{2, ..., 2}_{l}, \underbrace{3, ..., 3}_{m-k-l}]$, $k = 1, ..., m-3$, $l = 1, ..., m-3$.

$$V_2 = 2 - \frac{a_m}{A} - 2\sum_{i=1}^{k} \frac{a_i}{A} < 1$$

The distributions for the age of males and females have forms

$$a = (a_1, ..., a_m);\ b = \left(1,\ \frac{a_m}{A},\ \frac{a_m}{A}\sum_{i=1}^{k} \frac{a_i}{A}\right),$$

where

$$a_1 = R,\ a_i = a_{i-1}(1 - 1/A),\ i = 2, ..., k+1,$$

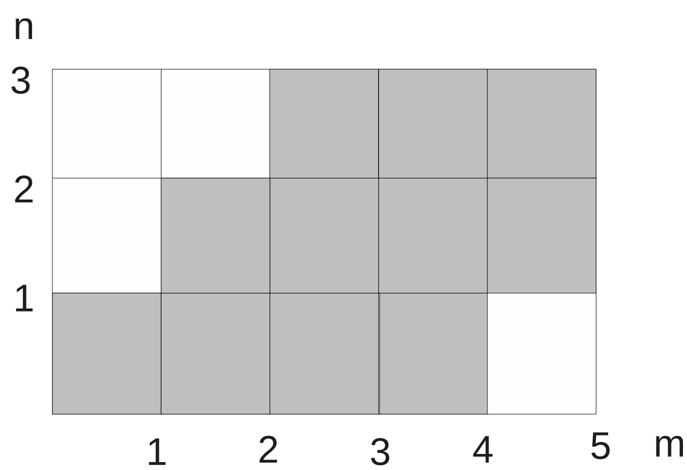
$$a_j = a_{j-1}\left(\frac{b_3}{A} + 1 - \frac{1}{r}\right),\ j = k+2, ..., k+l+1,$$

$$a_s = a_{s-1}\left(1 - \frac{1}{r}\right),\ s = k+l+2, ..., m.$$

Equilibrium for $m = 5$	$r = \frac{A}{B}$
$([1, 2, 3, 3, 3], [4, 5, 5])$	$[2.016, 2.79)$

$$m = 5$$

Equilibrium	$r = \frac{A}{B}$	R
$([1,1,2,3,3],[5,5,5])$	$(1, 2.191)$	$(1, 1.049)$
$([1,2,3,3,3],[4,5,5])$	$[2.016, 2.79)$	$[1.081, 1.191)$
$([2,2,3,3,3],[4,5,5])$	$[2.85, 4.517)$	$[1.209, 1.560)$
$([2,3,3,3,3],[4,5,5])$	$[4.517, 6.87)$	$[1.560, 2.097)$
$([3,3,3,3,3],[4,5,5])$	$[6.87, +\infty)$	$[2.097, +\infty)$



Proposition. *If* $m = n \geq 2$ *then* $U_i \leq m-(i-1)$ *and* $V_j \leq m-(j-1)$ *for* $i, j = 1, ..., m$.

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THANK YOU FOR YOUR ATTENTION