Complexity of checking whether two automata are synchronized by the same language

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Definitions

- A *deterministic finite automaton* (DFA) is a triple $\mathscr{A} = \langle Q, \Sigma, \delta \rangle$, where
  – Q is the state set;
  – Σ is the input alphabet;
  – $\delta : Q \times \Sigma \to Q$ is totally defined transition function.
  We do not need any initial and final states.
- Σ^* stands for the set of all words over Σ including the empty word.
- The function δ uniquely extends to a function $Q \times \Sigma^* \to Q$ still denoted by δ.
- We often write $q \,.\, w$ for $\delta(q, w)$ and $P \,.\, w$ for $\delta(P, w) = \{\delta(q, w) \mid q \in P\}$.

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- A DFA $\mathscr{A} = \langle Q, \Sigma, \delta \rangle$ is called *synchronizing* if there exists a word $w \in \Sigma^*$ whose action resets $\mathscr{A}$, that is $|Q \, . \, w| = 1$.
- Any word with this property is said to be *reset* for the DFA $\mathscr{A}$.
- $\mathrm{Syn}(\mathscr{A})$ is the language of all reset words for $\mathscr{A}$.

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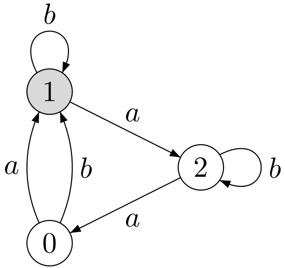
Example



A reset word is $w = baab$. Indeed, $Q\,.\,w = \{1\}$. In fact this is the shortest reset word for this automaton.

- A language $I \subseteq \Sigma^*$ is called a *two-sided ideal* (or simply an *ideal*) if $I \neq \emptyset$ and $I = \Sigma^* I \Sigma^*$.
- For every synchronizing automaton $\mathscr{A}$ it holds

$$\Sigma^* \operatorname{Syn}(\mathscr{A})\Sigma^* = \operatorname{Syn}(\mathscr{A}).$$

Ideals and Synchronizing Automata

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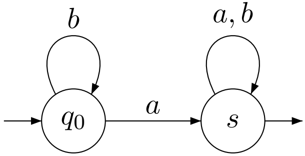
- Every regular ideal language I is a language of reset words for some synchronizing automaton $\mathscr{A}$ (e.g. for its minimal recognizing automaton)



$$L[\mathscr{A}] = I = \Sigma^* a \Sigma^* = \operatorname{Syn}(\mathscr{A})$$

Reset Complexity

Ideal I

Classical

New

$$\mathscr{A}_I = \langle Q_1, \Sigma, \delta_1, q_0, F \rangle$$

$$w \in I \Leftrightarrow q_0.w \in F$$

$$\mathscr{B} = \langle Q_2, \Sigma, \delta_2 \rangle : \mathrm{Syn}(\mathscr{B}) = I$$

$$w \in I \Leftrightarrow |Q_2 \,.\, w| = 1$$

- The *state complexity* $sc(I)$ of a regular language I is the number of sates in the minimal automaton recognizing I.
- The *reset complexity* $rc(I)$ of an ideal language I is the minimal possible number of states in a synchronizing automaton $\mathscr{B}$ such that $\mathrm{Syn}(\mathscr{B}) = I$.
  Every such automaton $\mathscr{B}$ is called *minimal synchronizing automaton* (for brevity, MSA).

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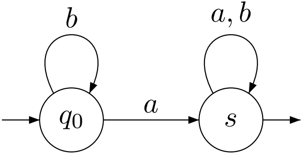
Reset Complexity VS State Complexity

- For every ideal language $rc(I) \leq sc(I)$.



- $L[\mathscr{A}] = I = \Sigma^* a \Sigma^* = \mathrm{Syn}(\mathscr{A})$
- $\mathscr{A}$ is an MSA for I
- $\mathscr{A}$ is the minimal automaton recognizing I
- $rc(I) = sc(I) = 2$

Theorem [M.,2012]

For every $n \geq 3$ there are ideals I_n s.t. $rc(I_n) = n$, and $sc(I_n) = 2^n - n$.

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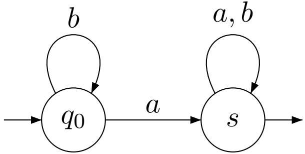

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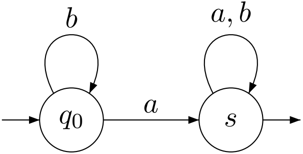

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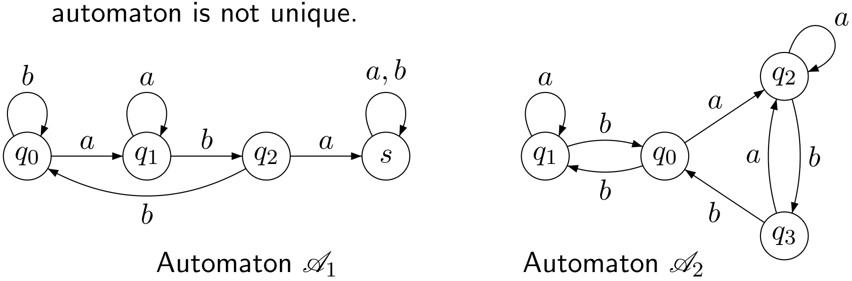
- We do not know how to compute $rc(I)$, and how to build the corresponding automaton. One difficulty is that such automaton is not unique.



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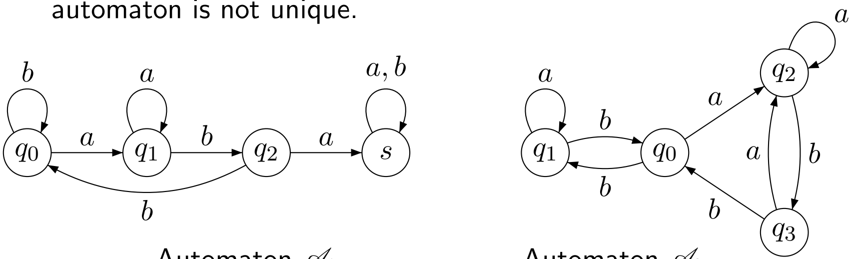

Automaton $\mathscr{A}_1$ Automaton $\mathscr{A}_2$

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Main questions

- An ideal I is presented by a DFA $\mathscr{A}$ with $\mathrm{Syn}(\mathscr{A}) = I$. Let $rc(I) \leq k$. It means that there exists some automaton $\mathscr{B}$ with at most k states s.t. $\mathrm{Syn}(\mathscr{B}) = I$.

Question 1

How hard is it to verify the equality $\mathrm{Syn}(\mathscr{A}) = \mathrm{Syn}(\mathscr{B})$?

Question 2

How hard is it to verify the inequality $rc(I) \leq k$?

- DFA's case: the equality $L[\mathscr{A}] = L[\mathscr{B}]$ can be checked easily.
- $\mathrm{Syn}(\mathscr{A})$ and $\mathrm{Syn}(\mathscr{B})$ are regular languages.
- Obstacle: the minimal automaton recognizing $\mathrm{Syn}(\mathscr{A})$ has up to $2^n - n$ states, where n is the size of $\mathscr{A}$.

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Problems

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–*Input:* synchronizing automata $\mathscr{A}$ and $\mathscr{B}$.

–*Question:* is $\mathrm{Syn}(\mathscr{A}) = \mathrm{Syn}(\mathscr{B})$?

RESET-COMPLEXITY ($\leq$)

–*Input:* a synchronizing automaton $\mathscr{A}$, $k \in \mathbb{N}$.

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FINITE AUTOMATA INTERSECTION

–*Input:* given n DFAs $M_i = \langle Q_i, \Sigma, \delta_i, q_i, F_i \rangle$, for $i = 1, \ldots, n$.

–*Question:* is $\bigcap_i L[M_i] \neq \emptyset$?

- It is assumed that $|\Sigma| = 2$.
- We construct the DFA $\mathscr{A} = \langle Q, \Delta, \varphi, \rangle$ with $Q = \bigcup_{i=1}^{n} Q_i \cup \{s, h\}$ and $\Delta = \Sigma \cup \{x, y, z\}$.
- $I = L_1 \cup L_2$; $L_1 = (\Sigma \cup \{x\})^* y \Delta^*$; $L_2 = (\Sigma \cup \{x\})^* z \Delta^+$.

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PSPACE-completeness: automaton $\mathscr{A}$

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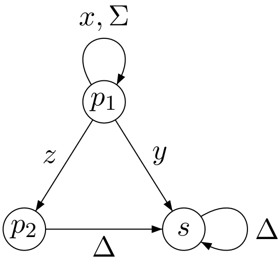

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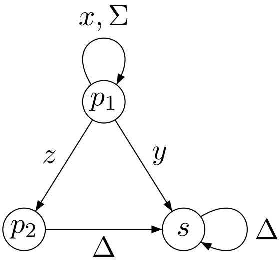

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PSPACE-completeness

Theorem 2.

SYN-EQUALITY is **PSPACE**-complete.

- Polynomial reduction from the negation of FINITE AUTOMATA INTERSECTION to SYN-EQUALITY.
- $\mathscr{B}$ is an MSA for I.
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Checking the inequality $rc(I) \leq k$

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- The inequality $rc(I) \leq 2$ can be checked in polynomial of the size of $\mathscr{A}$ time.
- RESET-COMPLEXITY $(\leq)$ is in **PSPACE**.
- We have constructed for each instance of FINITE AUTOMATA INTERSECTION the corresponding automaton $\mathscr{A}$. Let $I = \mathrm{Syn}(\mathscr{A})$.
- If $\bigcap_{i=1}^{n} L[M_i] = \emptyset$, then $I = J$, where J is the language of reset words of a 3-state automaton $\mathscr{B}$. In this case $rc(I) \leq 3$.
- If $\bigcap_{i=1}^{n} L[M_i] \neq \emptyset$, then I does not serve as the language of reset words for some automaton $\mathscr{B}$ with at most three states. In this case $rc(I) > 3$.

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RESET-COMPLEXITY $(\leq)$ is **PSPACE**-complete.

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Checking the inequality $rc(I) \leq k$

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Future work

- Computational complexity of RESET-COMPLEXITY ($\leq$) for a non-unary alphabet of size less than five.
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