preamble

St. Petersburg, SPbSU

The choice of the strategy of tax control with the use of statistical information about taxpayers

The International Conference
«Stochastic Optimal Stopping»
Petrozavodsk, Russia

12 – 16 September, 2010

A game-theoretical model

Consider a game-theoretical model of tax control:

◇ Hierarchical game;
◇ Players: the tax authority and n taxpayers;
◇ Interaction between players corresponds to the scheme "principal-to-agent";
◇ The players: risk-neutral.

The players

◇ The k-th taxpayer ($k = \overline{1, n}$):

- True income's level i_k;
- Declared income's level r_k (where $r_k \leq i_k$).

◇ The tax authority: p_k is the probability of the k-th taxpayer:

Let's suppose that

- taxpayers have some assumptions about the expected values of these probabilities;
- auditing is effective always.

Penalties

Four kinds of penalties are known from [4, 5] (A. Vasin):

1. the net penalty is proportional to evasion;
2. the penalty is proportional to difference between true and payed tax;
3. the penalty is restricted by the given level of the agent's minimal income in the case of his nonoptimal behaviour;
4. the post-audit payment is proportional to the revealed evaded income.

Case1: the net penalty is proportional to evasion

- t is the tax rate;
- π is the penalty rate;

Then the k-th taxpayer's postaudit payment is

$$(t + \pi)(i_k - r_k)$$

Taxpayers' payoffs

The k-th taxpayer's expected payoff is

$$b_k = i_k - u_k = i_k - tr_k - p_k(t + \pi)(i_k - r_k),$$

where u_k is the k-th taxpayer's expected tax payment.

The k-th taxpayer's strategy is $r_k = i_k$ or $r_k < i_k$.

Every taxpayer's aim:

$$\max_{r_k} b_k(p_k, r_k) \quad \text{or} \quad \min_{r_k} u_k(p_k, r_k).$$

The tax authority's payoff

◇ c_k is the cost of the audit of the k-th taxpayer.

The tax authority's payoff function:

$$R = \sum_{k=1}^{n} R_k = \sum_{k=1}^{n} (u_k - p_k c_k) = \sum_{k=1}^{n} (tr_k + p_k(t+\pi)(i_k - r_k) - p_k c_k).$$

The tax authority's strategy is contract (t, π, p), where t and π are the parameters of long-term tax control, and p is the vector $p = (p_1, \ldots, p_n)$ for each tax period.

The tax authority's aim:

$$\max_p R(p, r_1, r_2, ..., r_n), \text{ where } p = (p_1, p_2, ..., p_n).$$

Compare parametres t, π and c_k:

1. for each $k = \overline{1, n}$
$$(t + \pi) i_k \geq c_k; \tag{1}$$
2. for each $k = \overline{1, n}$
$$(t + \pi) i_k < c_k. \tag{2}$$
3. (1) is satisfied for the part of taxpayers and (2) is satisfied for the another part.

The first case

(1) is satisfied for each $k = \overline{1, n}$.

Theorem 1 The optimal strategy of the tax authority (due to maximize its income) is $p^* = \frac{t}{t+\pi}$, the optimal strategy of the k-th taxpayer is

$$r_k^*(p_k) = \begin{cases} 0, & \text{if } p_k < p^*, \\ i_k, & \text{if } p_k \geq p^* . \end{cases}$$

(r_k^*, p^*) is the Nash equilibrium.

The second case

(2) is satisfied for each $k = \overline{1, n}$.

Theorem 2 The optimal strategy of the tax authority (due to maximize its income) is $p^* = 0$, the optimal strategy of the k-th taxpayer is $r_k^*(p_k) = 0$. (r_k^*, p^*) is the Nash equilibrium.

The third case

(1) is satisfied for the part of taxpayers and (2) is satisfied for the another part.

Then renumber the set of n taxpayers so that:

- (1) is satisfied for the k-th taxpayer, where $k = \overline{1, n_0}$ (the Theorem 1 is fulfilled);
- (2) is satisfied for the k-th taxpayer, where $k = \overline{n_0 + 1, n}$ (the Theorem 2 is fulfilled).

Case2: the penalty is proportional to difference between true and payed tax

The expected tax payment of the k-th taxpayer:

$$u_k = tr_k + p_k(1+\pi)t(i_k - r_k).$$

The tax authority's payoff function:

$$R = \sum_{k=1}^{n} R_k = \sum_{k=1}^{n} (tr_k + p_k(1+\pi)t(i_k - r_k) - p_k c_k).$$

(1) becomes

$$(1+\pi)ti_k \geq c_k. \tag{3}$$

1. If (3) is satisfied, then theorem 1 is fulfilled for $p^* = \dfrac{1}{1+\pi}$;
2. If (3) is not satisfied, then theorem 2 is fulfilled.

The choice of the strategy with the use of statistical information about taxpayers

- W_k is the random quantity, which characterize the k-th taxpayer's disposition to evade.
- Suppose: it is beta-distributed.
- A tax story: a characteristic of the taxpayer's behaviour in the previous periods.

Beta distribution

The random quantity X is beta distributed with parametres α and β ($\alpha > 0,\ \beta > 0$), if X is distributed absolutely continously with the density

$$f(x|\alpha,\beta) = \begin{cases} \frac{x^{\alpha-1}(1-x)^{\beta-1}}{B(\alpha,\beta)}, & \text{when } 0 < x < 1, \\ 0, & \text{in other cases,} \end{cases}$$

where $B(\alpha,\beta) = \int\limits_0^1 x^{\alpha-1}(1-x)^{\beta-1}dx$ is the beta-function.

j is the number of current tax period. Let $j = 1$:

1. there wasn't any audits of this taxpayer in previous periods;
2. a priori information is absent;
3. consider uniform equiprobability distribution of W_k (beta distribution with $\alpha = 1$, $\beta = 1$);
4. the tax authority makes the audit with some fixed probability p_0;
5. the tax story is a result of observation (audit), presented as a Bernoulli- distributed random quantity X_1:

$$X_1 = \begin{cases} 1, & \text{if there isn't evasion} \\ 0, & \text{if there is an evasion.} \end{cases}$$

Consider the theorem about conjugate families [7].

Theorem. Let $X_1, X_2, \ldots, X_n$ be a sample from the Bernoulli distribution with unknown value of parameter W. Suppose, that a priori distribution of W is the beta distribution with parametres α and β ($\alpha > 0$, $\beta > 0$). Then a posteriori distribution W при $X_i = x_i$ $(i = \overline{1, n})$ is the beta distribution with parametres $\alpha + y$ and $\beta + n - y$, where $y = \sum\limits_{i=1}^{n} x_i$.

I.e. the family of beta distributions is conjugate to the family of Bernoulli distribution.

Using the feature of conjugate families

In the period $j = 1$: a posteriori W_k is beta-distributed with parametres

- $\alpha_{1_k} = 1$, $\beta_{1_k} = 2$, if there was an evasion;
- $\alpha_{1_k} = 2$, $\beta_{1_k} = 1$, if there was no evasion.

For the next tax periods $(j > 1)$:

- Only X_{j-1} is considered as a tax story;
- A posteriori distribution of W_k, obtained in $j-1$ period, is considered as a priori distribution for the j period.

Choice of the audit probabilities

The tax authority can use obtained distribution of W_k to choose the value of p_k for $k = \overline{1, n}$ as:

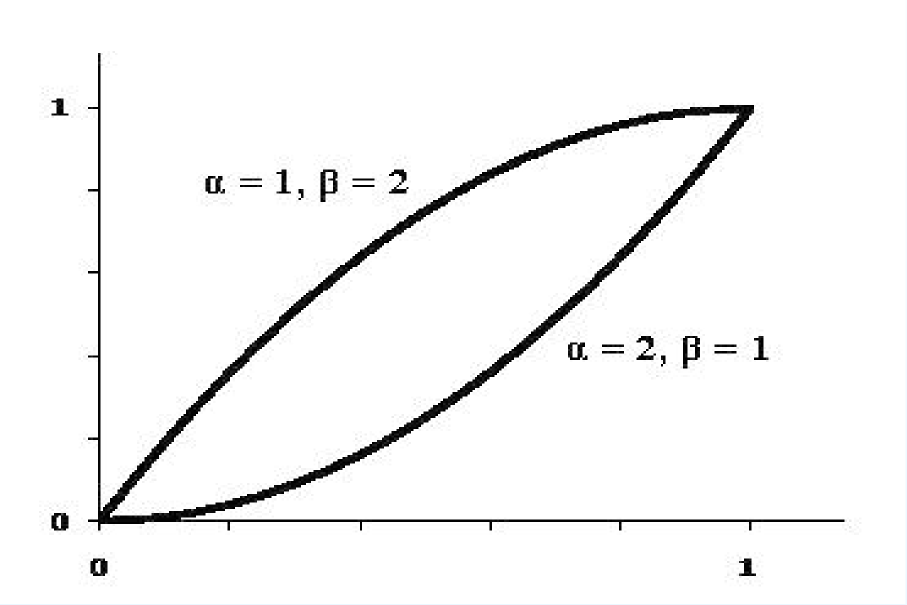
- mode;
- median;
- another quantile.

Numerical experiment

Consider $j = 1$:

- W_k is uniform-distributed: $\alpha_{0_k} = 1$, $\beta_{0_k} = 1$;
- quartile: $0, 25$;
- median: $0, 5$;
- mode: doesn't exist.

The first audit's result



Picture 1.
The curves from up to bottom: beta distribution with
$\alpha = 1,\ \beta = 2;\ \alpha = 2,\ \beta = 1.$

$j = 2$:

For those, who avoided in the first tax period:

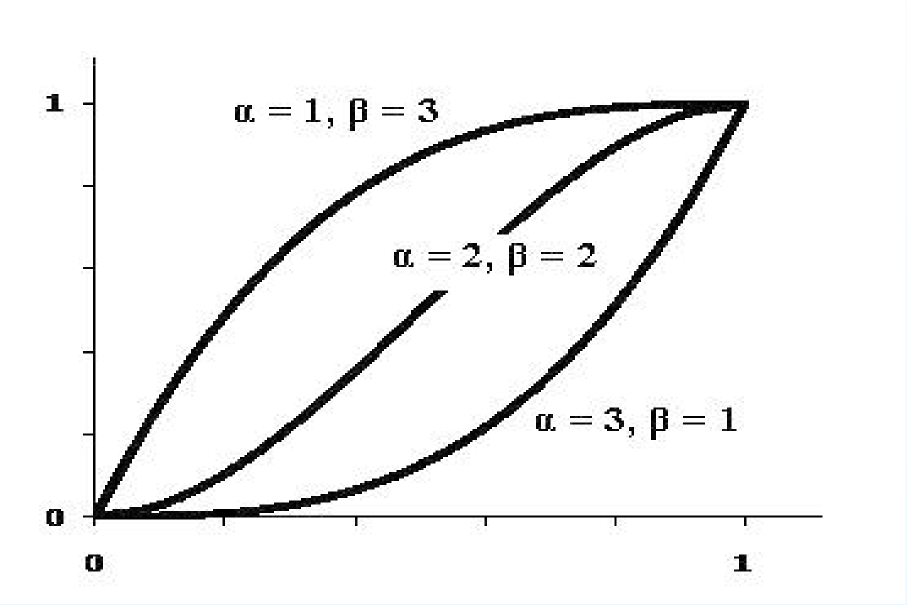
- a priori distribution: $\alpha_{1_k} = 1$, $\beta_{1_k} = 2$;
- quartile: $0,435$;
- median: $0,750$.
- a posteriori distribution:
  - for those, who avoided again: $\alpha_{2_k} = 1$, $\beta_{2_k} = 3$;
  - for those, who didn't avoid in the second period: $\alpha_{2_k} = 2$, $\beta_{2_k} = 2$.

$j = 2$:

For those, who didn't avoid in the first tax period:

- a priori distribution: $\alpha_{1_k} = 2$, $\beta_{1_k} = 1$;
- quartile: $0,065$;
- median: $0,250$;
- a posteriori distribution:
  - for those, who didn't avoid again: $\alpha_{2_k} = 3$, $\beta_{2_k} = 1$;
  - for those, who avoided in the second period: $\alpha_{2_k} = 2$, $\beta_{2_k} = 2$;

The second audit's result:



Picture 2.
The curves from up to bottom: beta distributions with
$\alpha = 1,\ \beta = 3;\ \alpha = 2,\ \beta = 2;\ \alpha = 3,\ \beta = 1.$

$j = 3$:

For twice-avoided:

- a priori distribution: $\alpha_{2_k} = 1$, $\beta_{2_k} = 3$;
- quartile: $0,5725$;
- median: $0,8750$;
- a posteriori distribution:
  - for those, who avoided again: $\alpha_{3_k} = 1$, $\beta_{3_k} = 4$;
  - for those, who didn't avoid in the third period: $\alpha_{3_k} = 2$, $\beta_{3_k} = 3$;

$j = 3$:

For those, who changed the decision whether to evade or not:

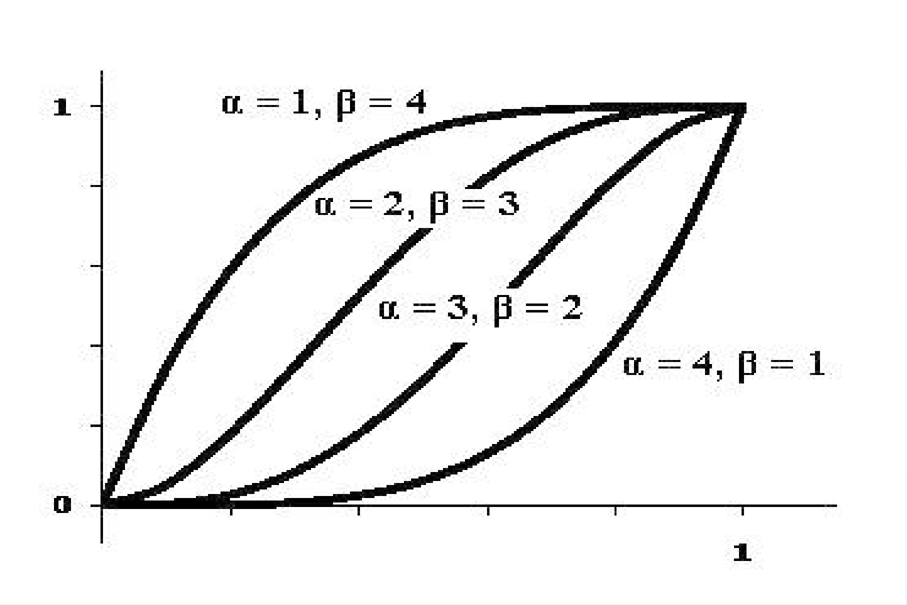
- a priori distribution: $\alpha_{2_k} = 2$, $\beta_{2_k} = 2$;
- quartile: $0,16$;
- median: $0,50$;
- a posteriori distribution:
  - for those, who avoided again: $\alpha_{3_k} = 2$, $\beta_{3_k} = 3$;
  - for those, who didn't avoid in the third period: $\alpha_{3_k} = 3$, $\beta_{3_k} = 2$;

$j = 3$:

For those, who didn't avoid during two periods:

- a priori distribution: $\alpha_{2_k} = 3$, $\beta_{2_k} = 1$;
- quartile: $0,0175$;
- median: $0,1250$;
- a posteriori distribution:
  - for those, who avoided: $\alpha_{3_k} = 3$, $\beta_{3_k} = 2$;
  - for those, who didn't avoid again: $\alpha_{3_k} = 4$, $\beta_{3_k} = 1$;

The third audit's result:



Picture 3.
The curves from up to bottom: beta distributions with
$\alpha = 1,\ \beta = 4;\ \alpha = 2,\ \beta = 3;\ \alpha = 3,\ \beta = 2;\ \alpha = 4,\ \beta = 1.$

For the next period ($j = 4$):

1. For those, who avoid three times during three periods:
   - a priori distribution: $\alpha_{3_k} = 1$, $\beta_{3_k} = 4$;
   - quartile: $0,67515$; median: $0,93750$;
2. For those, who avoid twice of three periods:
   - a priori distribution: $\alpha_{3_k} = 2$, $\beta_{3_k} = 3$;
   - quartile: $0,26455$; median: $0,68750$;
3. For those, who avoid once of three periods:
   - a priori distribution: $\alpha_{3_k} = 3$, $\beta_{3_k} = 2$;
   - quartile: $0,05545$; median: $0,31250$;
4. For those, who didn't avoid during three periods:
   - a priori distribution: $\alpha_{3_k} = 4$, $\beta_{3_k} = 1$;
   - quartile: $0,00485$; median: $0,06250$;

Parametres for optimality of obtained probabilities

t, π -? p_k is optimal (Theorem 1)

Consider two tax rates:

- $t = 0,2$: profit tax in Russian Federation;
- $t = 0,13$: income tax in Russian Federation.

The net penalty is proportional to evasion

experimental p_k (quantile of beta distribution)	π: p_k is optimal	
	for $t = 0,13$	for $t = 0,2$
0,01750	7,29857	11,22857
0,05545	2,21445	3,40685
0,06500	1,87000	2,87692
0,12500	0,91000	1,40000
0,16000	0,68250	1,05000
0,25000	0,39000	0,60000
0,26455	0,36140	0,55600
0,31250	0,28600	0,44000
0,43500	0,16885	0,25977
0,50000	0,13000	0,20000
0,57250	0,09707	0,14934
0,68750	0,05909	0,09091
0,75000	0,04333	0,06667
0,87500	0,01857	0,02857
0,93750	0,00867	0,01333

The penalty is proportional to difference between true and payed tax

experimental p_k (quantile of beta distribution)	π: p_k is optimal
0,01750	56,14286
0,05545	17,03427
0,06500	14,38462
0,12500	7,00000
0,16000	5,25000
0,25000	3,00000
0,26455	2,78000
0,31250	2,20000
0,43500	1,29885
0,50000	1,00000
0,57250	0,74672
0,68750	0,45455
0,75000	0,33333
0,87500	0,14286
0,93750	0,06667

References

1. Petrosyan L., Zenkevich N., Semina E. The game theory. M., 1998.
2. Kumacheva S., Petrosyan L. A game theoretical model of interaction between taxpayers and tax authority // Processes Of Conrtol And Stability. St. Petersburg, 2009.
3. Boure V., Kumacheva S. A model of audit with using of statistical information about taxpayers' income // Vestnik SPbGU. Series 10. 2005. Vol. 1-2.
4. Vasin A., Vasina, P. The optimization of tax system in condition of tax evasion: the role of penalty restriction // EERC. Series "Nauchnye doklady". 2002.
5. Vasin A., Morozov V. The Game Theory and Models of Mathematical Economics. M., 2005.
6. Chander P., Wilde L.L.. A General Characterization of Optimal Income Tax Enfocement // Rev. of Economic Studies. 1998. Vol. 65. P. 165-183.
7. De Groot M.H. Optimal Statistical Decisions. M., 1974.

THANK YOU!