Математическая Теория Игр и её Приложения, т.2, в.3, с. 137–149

УДК 518.9 + 517.9
ББК 65.050.2

TIME CONSISTENT SHAPLEY VALUE IMPUTATION FOR COST-SAVING JOINT VENTURES

David W.K. Yeung
SRS Consortium for Advanced Study in Cooperative Dynamic Games
Shue Yan University
Hong Kong
and
Center of Game Theory
St. Petersburg State University
198504, Saint-Petersburg, Universitetskii pr., 35
e-mail: dwkyeung@hksyu.edu

As markets become increasingly globalized and firms become more multinational, corporate joint ventures are likely to yield opportunities to quickly create economies of scale and critical mass, and facilitate rational resource sharing. A major source of gain from joint venture is from cost savings. However, it is often observed that after a certain time of cooperation, some firms may gain sufficient skills and technology that they would do better by breaking up from the joint venture. This is the well-known problem of time inconsistency. In this paper, we consider a dynamic cost saving joint venture which adopts the Shapley value as its profit allocation scheme. A compensation mechanism distributing payments to participating firms at each instant of time is devised to ensure the realization of the Shapley value imputation throughout the venture duration. Hence time-consistency will be attained, and a dynamically stable joint venture can be formed.

Keywords: corporate joint venture, the Shapley value, cost saving, dynamic stability.

1. Introduction

With joint ventures becoming a powerful force shaping global corporate strategy, partnerships between firms have significantly increased. D'Aspremont and Jacquemin [6], Kamien et al [7] and Suzumura [11] have studied cooperative R&D with spillovers in joint ventures under a static framework. Cellini and Lambertini [4], [5] considered cooperative solutions to investment in product differentiation in a dynamic approach. Moreover, as markets become increasingly globalized and firms become more multinational, corporate joint ventures are likely to yield opportunities to quickly create economies of scale and critical mass, and facilitate rational resource sharing (see [1]). A major source of gain from joint venture is from cost savings. Cost saving opportunities are created under joint venture, for instance, savings in joint R&D, administration, marketing, customer services, purchasing, financing, and economy of scales and scope. Despite their purported benefits, however, joint ventures are highly unstable and have a consistently high rate of failure ([3], [8]). After a certain time of cooperation, some firms may gain sufficient managerial and technological expertise that they would do better by breaking away from the joint venture. Thus a major source of instability is the lack of *dynamical stable* or *time consistent* cooperative solutions to the joint-venture. Time consistency is a fundamental element in dynamic cooperation, and it ensures that: (i) the extension of the solution policy to a later starting time and a state brought about by prior optimal behavior of the players would remain optimal, and (ii) all participating firms do not have incentive to deviate from the initial plan (see [12], [13]). Petrosyan and Zaccour [9] provided a time consistent solution to a class of differential games involving pollution cost reduction. Yeung and Petrosyan [14] presented a dynamically stable joint venture involving cooperative R&D with spillovers. Yeung and Petrosyan [15] developed a cooperative differential game of transboundary industrial pollution and derived a dynamically stable solution.

In this paper, we consider a joint venture which results in cost saving. The Shapley value [10] is adopted to be the profit allocation scheme to reflect the relative contributions of the firms in cost saving. Since joint venture is a continual arrangement, a dynamic specification of the Shapley value is provided. To fulfill time-consistency, the Shapley value imputation has to be throughout the venture duration. A compensation mechanism distributing payments to participating firms at each instant of time ensuring the realization of the Shapley value imputation throughout the venture duration is devised.

2. Dynamic cost saving joint ventures

Consider a framework of a dynamic joint venture in which there are n firms. The venture horizon is $[t_0, T]$. The objective of firm i is:

$$\int_{t_0}^{T} \left\{ g^i[s, x^i(s)] - c_i^{\{i\}}[u_i(s)] \right\} \exp\left[- \int_{t_0}^{S} r(y)dy \right] ds$$
$$+ \exp\left[- \int_{t_0}^{T} r(y)dy \right] q^i(x^i(T)), \quad \text{for } \; i \in [1, 2, \cdots, n] \equiv N, \tag{2.1}$$

where $x^i(s) \in X^i \subset R^{m_i +}$ denotes the state variables of firm i, $u_i \in U_i \subset R^{l_i +}$ is the control vector of firm i, $g^i[s, x^i(s)]$ the instantaneous revenue, $c_i^{\{i\}}[u_i(s)]$ represents the costs of the firms control $u_i(s)$ when it is operating on its own, $\exp\left[- \int_{t_0}^{t} r(y)dy \right]$ is the discount factor, and $q^i(x^i(T))$ the terminal payment. In particular, the firm's revenue $g^i[s, x^i]$ is affected by the state variables like capital stock, special skills, productive resources and technologies.

The state dynamics of the ith firm is characterized by the set of vector-valued differential equations:

$$\dot{x}^i(s) = f^i[s, x^i(s), u_i(s)], \quad x^i(t_0) = x^{i(0)}, \qquad \text{for } \; i \in N. \tag{2.2}$$

Consider a joint venture consisting of a subset of firms $K \subseteq N$. There are k firms in the subset K. The participating firms can obtain cost reduction and the profit to the joint venture K at time t_0 becomes:

$$\int_{t_0}^{T} \sum_{j \in K} \left\{ g^j[s, x^j(s)] - c_j^K[u_j(s)] \right\} \exp\left[- \int_{t_0}^{S} r(y)dy \right] ds$$
$$+ \sum_{j \in K} \exp\left[- \int_{t_0}^{T} r(y)dy \right] q^j(x^j(T)), \qquad \text{for } K \subseteq N, \tag{2.3}$$

where $c_j^K[u_j(s)]$ represents the costs of the controls of the firm j in the subset K. Cost saving opportunities are created under joint venture, for instance, savings in joint R&D, administration, marketing, customer services, purchasing, financing, and economy of scales and scope. With absolute joint venture cost advantage we have

$$c_j^K[u_j(s)] \leq c_j^L[u_j(s)], \quad \text{for} \quad j \in L \subseteq K, \tag{2.4}$$

Moreover, marginal cost advantages lead to:

$$\partial c_j^K[u_j(s)]/\partial u_j(s) \leq \partial c_j^L[u_j(s)]/\partial u_j(s), \text{ for } j \in L \subseteq K.$$

The model adopted for analysis concentrates on cost savings and the profit of an outside firm is not affected by the actions of the joint venture. Let x^K denote the concatenation of all x^j for $j \in K$. To compute the profit of the joint venture K we have to consider the optimal control problem $\varpi[K; t_0, x^{K(0)}]$ which maximizes (2.3) subject to (2.2). Using Bellman's [2] technique of dynamic programming the solution of the problem $\varpi[K; t_0, x^{K(0)}]$ can be characterized as follows.

Definition 2.1. *A set of controls* $\left\{u_j^*(t) = \psi_j^{(t_0)K*}(t, x^K),\ j \in K\right\}$ *provides an optimal solution to the problem* $\varpi[K; t_0, x^{K(0)}]$ *if there exist continuously differentiable function*

$$W^{(t_0)K}(t, x^K) : [t_0, T] \times \prod_{j \in K} R^{m_j} \to R,$$

satisfying the Bellman equation:

$$\begin{aligned}
-W_t^{(t_0)K}(t, x^K) = \max_{u_K} \Bigg\{ & \sum_{j \in K} \left\{ g^j[s, x^j(s)] - c_j^K[u_j(s)] \right\} \exp\left[-\int_{t_0}^{t} r(y)dy \right] \\
& + \sum_{j \in K} W_{x_j}^{(t_0)K}(t, x^K) f^j[t, x^j, u_j] \Bigg\}, \\
W^{(t_0)K}(T, x^K) = \exp & \left[-\int_{t_0}^{T} r(y)dy \right] \sum_{j \in K} q^j(x^j).
\end{aligned}$$

In the case when all the n firms are in the joint venture, the set of optimal controls $\left\{\psi_j^{(t_0)N^*}(s, x^N(s)), \ \ \text{for } j \in N\right\}$, will be adopted and the dynamics of the optimal state trajectory of the grand coalition can be expressed as:

$$\dot{x}^j(s) = f^j[s, x^j(s), \psi_j^{(t_0)N^*}(s, x(s))], \ x^j(t_0) = x_j^0, \qquad \text{for } j \in N. \tag{2.5}$$

Let $x^*(t) = \left\{x^{1^*}(t), x^{2^*}(t), \cdots, x^{n^*}(t)\right\}$ for $t \in [t_0, T]$ denote the solution to (2.5) which yields the optimal trajectories. In particular

$$x^{j^*}(t) = x^{j(0)} + \int_{t_0}^{t} f^j[s, x^{j^*}(s), \psi_j^{(t_0)N^*}(s, x_j^*(s))]ds, \text{ for } j = 1, 2, \ldots, n. \quad (2.6)$$

We use $x_t^{j^*}$ to denote the value of $x^{j^*}(t)$ at time $t \in [t_0, T]$, and $x_t^{L^*}$ to denote the vector containing all $x_t^{j^*}$, for $j \in L \subseteq N$. The profit of the grand coalition joint venture becomes

$$\begin{aligned} &W^{(t_0)N}(t_0, x^{N(0)}) = \\ &\int_{t_0}^{T} \sum_{j=1}^{n} \left\{g^j[s, x^{j^*}(s)] - c_j^N[\psi_j^{(t_0)N^*}(s, x_j^*(s))]\right\} \exp\left[-\int_{t_0}^{S} r(y)dy\right] ds \\ &+ \sum_{j=1}^{n} \exp\left[-\int_{t_0}^{T} r(y)dy\right] q^j(x^{j^*}(T)). \end{aligned}$$

A remark which will be used in subsequent analysis is provided below.

Remark 2.1. Consider the problem $\varpi[K; \tau, x^K]$ which starts at time $\tau \in [t_0, T]$ with initial state x_τ^K which maximizes

$$\begin{aligned} &\int_{\tau}^{T} \sum_{j \in K} \{g^j[s, x^j(s)] - c_j^K[u_j(s)]\} \exp\left[-\int_{\tau}^{S} r(y)dy\right] ds \\ &\quad + \sum_{j \in K} \exp\left[-\int_{\tau}^{T} r(y)dy\right] q^j(x^j(T)) \end{aligned}$$

subject to

$$\dot{x}^j(s) = f^j[s, x^j(s), u_j(s), \quad x^j(\tau) = x^j, \quad \text{for } j \in K.$$

One can readily show that:

$$\exp\left[\int_{\tau}^{t} r(y)dy\right] W^{(\tau)K}(t, x_t^K) = W^{(t)K}(t, x_t^K), \text{ for } t_0 \le \tau \le t \le T;$$

and

$$\Psi_j^{(\tau)K^*}(t, x_t^K) = \Psi_j^{(t)K^*}(t, x_t^K), \text{ for } t_0 \le \tau \le t \le T \text{ and } j \in K.$$

Since profit maximization by coalition K is not affected by actions of firms outside the coalition, the following superaddivity property can be obtained.

Proposition 2.1. *Coalition profits are superadditivity, that is*

$$W^{(\tau)K}(\tau, x_\tau^K) \geq W^{(\tau)L}(\tau, x_\tau^L) + W^{(\tau)K\backslash L}(\tau, x_\tau^{K\backslash L}), \quad \textit{for } L \subset K \subseteq N,$$

where $K \setminus L$ is the relative complement of L in K.

Proof. See Appendix.

3. Dynamic Shapley value imputation

The problem of sharing the cooperative gains is inescapable in virtually every joint venture. The Shapley value is one of the most commonly used sharing mechanism in static cooperation games with transferable payoffs. Besides being individually rational and group rational, the Shapley value is also unique. Specifically, the Shapley value gives an imputation rule:

$$\varphi^i(v) = \sum_{K \subseteq N} \frac{(k-1)!(n-k)!}{n!} [v(K) - v(K \setminus i)], \text{ for } i \in N, \tag{3.1}$$

where $K \setminus i$ is the relative complement of i in K, $v(K)$ is the profit of coalition K, and $[v(K) - v(K \setminus i)]$ is the marginal contribution of firm i to the coalition K.

In the present dynamic analysis instead of a one-time allocation of the Shapley value, we have to consider the maintenance of the Shapley value imputation over the joint venture horizon.

Again, since profit maximization by coalition K is not affected by firms outside the coalition, the function $v(K)$ can be regarded as a characteristic function.

At time t_0 with state $x^{N(0)}$, the firms agree that firm i's share of profits be:

$$\begin{aligned} &\xi^{(t_0)i}(t_0, x^{N(0)}) = \\ &\quad \sum_{K \subseteq N} \frac{(k-1)!(n-k)!}{n!} \left[W^{(t_0)K}(t_0, x^{K(0)}) - W^{(t_0)K\backslash i}(t_0, x^{K\backslash i(0)})\right], \\ &\quad \text{for } i \in N, \end{aligned} \tag{3.2}$$

However, the Shapley value has to be maintained throughout the venture horizon $[t_0, T]$ to ensure time consistency. In particular, at time τ with the state being x_τ^* the following imputation principle has to be maintained:

Condition 3.1. *At time τ, firm i's share of profits is:*

$$\xi^{(\tau)i}(\tau, x_\tau^*) = \sum_{K \subseteq N} \frac{(k-1)!(n-k)!}{n!} \left[W^{(\tau)K}(\tau, x_\tau^{K^*}) - W^{(\tau)K\backslash i}(\tau, x_\tau^{K\backslash i^*})\right], \quad \textit{for } i \in N \textit{ and } \tau \in [t_0, T]. \tag{3.3}$$

Note that $\xi^{(\tau)}(\tau, x_\tau^*) = \left[\xi^{(\tau)1}(\tau, x_\tau^*), \xi^{(\tau)2}(\tau, x_\tau^*), \cdots, \xi^{(\tau)n}(\tau, x_\tau^*)\right]$ as specified in (3.3) satisfies the basic properties of an imputation vector:

$$\text{(i)} \quad \sum_{j=1}^{n} \xi^{(\tau)j}(\tau, x_\tau^*) = W^{(\tau)N}(\tau, x_\tau^*), \quad \text{and}$$

$$\text{(ii)} \quad \xi^{(\tau)i}(\tau, x_\tau^*) \geq W^{(\tau)i}(\tau, x_\tau^*), \quad \text{for } i \in N \text{ and } \tau \in [t_0, T]. \tag{3.4}$$

Part (i) of (3.4) shows that $\xi^{(\tau)}(\tau, x_\tau^*)$ satisfies the property of Pareto optimality throughout the game interval. Part (ii) demonstrates that $\xi^{(\tau)}(\tau, x_\tau^*)$ guarantees individual rationality throughout the game interval. Crucial to the analysis is the formulation of a profit distribution mechanism that would lead to the realization of Condition 3.1. This will be done in the next section.

4. Transitory compensation to secure the Shapley value imputation

In this section, a profit distribution mechanism will be developed to compensate transitory changes so that the Shapley value principle could be maintained throughout the venture horizon. First, an imputation distribution procedure (similar to those in [9], [12], [13]) must be now formulated so that the imputation scheme in Condition 3.1 can be realized. Let the $B_i^\tau(s)$ denote the payment received by firm $i \in N$ at time $\tau \in [t_0, T]$ dictated by $\xi^{(\tau)}(\tau, x_\tau^*)$. In particular,

$$\xi^{(\tau)i}(\tau, x_\tau^*) = \sum_{K \subseteq N} \frac{(k-1)!(n-k)!}{n!} \left[W^{(\tau)K}(\tau, x_\tau^*) - W^{(\tau)K\backslash i}(\tau, x_\tau^*)\right] =$$

$$= \int_\tau^T B_i^\tau(s) \exp\left[-\int_\tau^S r(y)dy\right] ds + q^i(x^{i^*}(T)) \exp\left[-\int_\tau^T r(y)dy\right], \quad \text{for } i \in N \text{ and } \tau \in [t_0, T]. \tag{4.1}$$

Moreover, for $i \in N$ and $t \in [\tau, T]$, we use

$$\xi^{(\tau)i}(t, x_t^*) = \int_t^T B_i^\tau(s) \exp\left[-\int_\tau^S r(y)dy\right] ds + q^i(x^{i^*}(T)) \exp\left[-\int_\tau^T r(y)dy\right], \quad (4.2)$$

to denote the present value of player i's cooperative profit according to $\xi^{(\tau)}(\tau, x_\tau^*)$ over the time interval $[t, T]$, given that the state is x_τ^* at time $t \in [\tau, T]$.

A necessary condition for $\xi^{(\tau)i}(t, x_t^*)$ to follow Condition 3.1 is that:

$$\xi^{(\tau)i}(t, x_t^*) = \xi^{(t)i}(t, x_t^*) \exp\left[-\int_\tau^t r(y)dy\right],$$
$$\text{for } \ i \in N, \ \ t \in [\tau, T] \ \text{ and } \ \tau \in [t_0, T]. \quad (4.3)$$

A candidate of $\xi^{(\tau)i}(t, x_t^*)$ satisfying (4.1)–(4.3) has to be found. A natural choice is

$$\xi^{(\tau)i}(t, x_t^*) = \sum_{K \subseteq N} \frac{(k-1)!(n-k)!}{n!} \left[W^{(\tau)K}(t, x_t^{K^*}) - W^{(\tau)K \backslash i}(t, x_t^{K \backslash i*})\right] \quad (4.4)$$

With Remark 2.1, one can readily that $\xi^{(\tau)i}(t, x_t^*)$ as defined in (4.4) satisfies (4.1)–(4.3).

For (4.1)–(4.4) to hold, $B_i^\tau(s)$ has to be equal to $B_i^t(s)$, for $i \in N$ and $\tau \neq t$. Therefore we adopt the notation $B_i^\tau(s) = B_i^t(s) = B_i(s)$. To fulfill the Pareto optimality property, the imputation vector $\xi^{(\tau)}(t, x_t^*)$ has to satisfy the following condition.

Condition 4.1.

$$\sum_{j=1}^n B_i(s) = \sum_{j=1}^n g^j[s, x_j^*, \psi_j^{(\tau)N^*}(s, x_s^*)], \qquad \textit{for } \ s \in [\tau, T] \ \textit{ and } \ \tau \in [t_0, T].$$

If there exist twice continuously differentiable value functions $W^{(\tau)K}(t, x_t^{K^*})$, for all $K \subseteq N$, the term $\xi^{(\tau)i}(t, x_t^*)$ is twice continuously differentiable in t and x_t^*.

Given the differentiability property of $\xi^{(\tau)i}(t, x_t^*)$, for $\Delta t \to 0$ one can use (4.3) to obtain:

$$\xi^{(\tau)i}(\tau, x_\tau^*) = \int_\tau^{\tau+\Delta t} B_i(s) \exp\left[-\int_\tau^S r(y)dy\right] ds +$$
$$\exp\left[-\int_\tau^{\tau+\Delta t} r(y)dy\right] \xi^{(\tau+\Delta t)i}(\tau+\Delta t, x_\tau^* + \Delta x_\tau^*)\Big| x(\tau) = x_\tau^*,$$
$$\text{for } i \in N, \quad t \in [\tau, T] \text{ and } \tau \in [t_0, T]. \tag{4.5}$$

where

$$\Delta x_\tau^* = \left[\Delta x_\tau^{1^*}, \Delta x_\tau^{2^*}, \cdots, \Delta x_\tau^{n^*}\right],$$
$$\Delta x_\tau^{j^*} = f^j\left[\tau, x_\tau^{j^*}, \psi_j^{(\tau)N^*}(\tau, x_\tau^*)\right]\Delta t + o(\Delta t), \qquad \text{for } j \in N,$$

and $[o(\Delta t)]/\Delta t \to 0$ as $\Delta t \to 0$.

Using (4.3), (4.4) and (4.5), one can obtain

$$B_i(\tau) = -\left[\xi_t^{(\tau)i}(t, x_t^*)\,|_{t=\tau}\right] - \sum_{j=1}^{n}\left[\xi_{x_j^{t^*}}^{(\tau)i}(t, x_t^*)\,|_{t=\tau}\right] f^j\left[\tau, x_\tau^{j^*}, \psi_j^{(\tau)N^*}(\tau, x_\tau^*)\right],$$
$$\text{for } i \in N,\ t \in [\tau, T]\ \tau \in [t_0, T]. \tag{4.6}$$

Using (4.4) and (4.6), we obtain:

Since the partial derivative of $W^{(\tau)K}(\tau, x_\tau^{K^*})$ with respect to x_j, for $j \notin K$, will vanish, a theorem characterizing the payoff distribution procedure leading to the realization of Condition 3.1 can be obtained as:

Теорема 4.1. *A payment to player $i \in N$ at time $\tau \in [t_0, T]$ leading to the realization of the Condition 3.1 can be expressed as:*

$$B_i(\tau) = -\sum_{K \subseteq N} \frac{(k-1)!(n-k)!}{n!}\left\{\left[W_t^{(\tau)K}(t, x_t^{K^*})\,|_{t=\tau}\right] - \right.$$
$$\left[W_t^{(\tau)K\backslash i}(t, x_t^{K\backslash i^*})\,|_{t=\tau}\right] + \sum_{j \in K}\left[W_{x_t^{j^*}}^{(\tau)K}(t, x_t^{K^*})\,|_{t=\tau}\right] f^j\left[\tau, x_\tau^{j^*}, \psi_j^{(\tau)N^*}(\tau, x_\tau^*)\right]$$
$$\left. -\sum_{j \in K}\left[W_{x_t^{j^*}}^{(\tau)K\backslash i}(t, x_t^{K\backslash i^*})\,|_{t=\tau}\right] f^{K\backslash i}\left[\tau, x_\tau^{K\backslash i^*}, \psi_j^{(\tau)N^*}(\tau, x_\tau^*)\right]\right\}$$

The vector $B(\tau)$ serves as a form equilibrating transitory compensation that guarantees the realization of the Shapley value imputation throughout the game horizon. Note that the instantaneous profit $B_i(\tau)$ offered to player i at time τ is conditional upon the current state x_τ^* and current time τ. One can elect to express $B_i(\tau)$ as $B_i(\tau, x_\tau^*)$. Hence an instantaneous payment $B_i(\tau, x_\tau^*)$ to player $i \in N$ yields a dynamically stable solution to the joint venture.

5. Concluding remarks

Despite all their purported benefits, joint ventures are highly unstable because of the lack of dynamical stable profit sharing schemes. In this paper, we consider a cost saving dynamic joint venture which adopts the Shapley value as its profit allocation scheme. A compensation mechanism distributing payments to participating firms at each instant of time is devised to ensure the realization of the Shapley value imputation throughout the venture duration. Hence time-consistency will be attained, and a dynamically stable joint venture can result. Finally, this paper concentrates on the establishment of dynamically stable cost saving joint ventures. Further study on joint ventures which requires particular information on the demand structures, is left to the readers.

Appendix: Proof of Proposition 2.1.

Let $\hat{x}^{j(L)}$ for $j \in L$ denote the optimal trajectory of the optimal control problem $\varpi[L; \tau, x_\tau^L]$ which maximizes

$$\int_t^T \sum_{j \in L} \left\{ g^j[s, x^j(s)] - c_j^L[u_j(s)] \right\} \exp\left[- \int_\tau^S r(y)dy \right] ds$$
$$+ \sum_{j \in L} \exp\left[- \int_{t_0}^T r(y)dy \right] q^j(x^j(T))$$

subject to $\dot{x}^j(s) = f^j[s, x^j(s), u_j(s)], \;\; x^j(\tau) = x_\tau^j, \qquad \text{for } j \in L.$

Note that

$$W^{(\tau)L}(\tau, x_\tau^L) =$$
$$\int_\tau^T \sum_{j \in L} \left\{ g^j[s, \hat{x}^{j(L)}(s)] - c_j^L[\psi_j^{(\tau)L^*}(s, \hat{x}^{L(L)}(s))] \right\} \exp\left[- \int_\tau^S r(y)dy \right] ds$$
$$+ \sum_{j \in L} \exp\left[- \int_\tau^T r(y)dy \right] q^j(\hat{x}^{j(L)}(T))$$

$$\leq \int_{\tau}^{T} \sum_{j \in L} \Big\{ g^j[s, \hat{x}^{j(L)}(s)] - c_j^K[\psi_j^{(\tau)L^*}(s, \hat{x}^{L(L)}(s))] \Big\} \exp\Big[- \int_{\tau}^{S} r(y)dy \Big] ds$$
$$+ \sum_{j \in L} \exp\left[- \int_{\tau}^{T} r(y)dy \right] q^j(\hat{x}^{j(L)}(T)),$$
because $c_j^K[u_j(s)] \leq c_j^L[u_j(s)]$, for $j \in L \subseteq K$. (A.1)

Applying the above analysis to the optimal control problem $\varpi[K \setminus L; \tau, x_\tau^{K \setminus L}]$, we have

$$W^{(\tau)K \setminus L}(\tau, x_\tau^{K \setminus L})$$
$$= \int_{\tau}^{T} \sum_{j \in K \setminus L} \left\{ g^j[s, \hat{x}^{j(K \setminus L)}(s)] - c_j^{K \setminus L}[\psi_j^{(\tau)K \setminus L^*}(s, \hat{x}^{K \setminus L(K \setminus L)}(s))] \right\}$$
$$\times \exp\left[- \int_{\tau}^{S} r(y)dy \right] ds + \sum_{j \in K \setminus L} \exp\left[- \int_{\tau}^{T} r(y)dy \right] q^j(\hat{x}^{j(K \setminus L)}(T))$$
$$\leq \int_{\tau}^{T} \sum_{j \in K \setminus L} \left\{ g^j[s, \hat{x}^{j(K \setminus L)}(s)] - c_j^{K}[\psi_j^{(\tau)K \setminus L^*}(s, \hat{x}^{K \setminus L(K \setminus L)}(s))] \right\}$$
$$\times \exp\left[- \int_{\tau}^{S} r(y)dy \right] ds + \sum_{j \in K \setminus L} \exp\left[- \int_{\tau}^{T} r(y)dy \right] q^j(\hat{x}^{j(K \setminus L)}(T)),$$
because $c_j^K[u_j(s)] \leq c_j^{K \setminus L}[u_j(s)]$, for $j \in K \setminus L \subseteq K$. (A.2)

Now consider the optimal control problem $\varpi[K; \tau, x_\tau^K]$ which maximizes

$$\int_{\tau}^{T} \sum_{j \in K} \left\{ g^j[s, x^j(s)] - c_j^K[u_j(s)] \right\} \exp\left[- \int_{\tau}^{S} r(y)dy \right] ds$$
$$+ \sum_{j \in K} \exp\left[- \int_{t_0}^{T} r(y)dy \right] q^j(x^j(T))$$

subject to $\dot{x}^j(s) = f^j[s, x^j(s), u_j(s)]$, $x^j(\tau) = x_\tau^j$, for $j \in K$.

Since $\psi_j^{(\tau)K^*}(s, \hat{x}^{K(K)}(s))$ and $\hat{x}^{K(K)}(s)$ are respectively the optimal control and optimal state trajectory of the problem $\varpi[K;\tau,x_\tau^K]$,

$$\begin{aligned}
&W^{(\tau)K}(\tau,x_\tau^K)\\
&=\int_\tau^T \sum_{j\in K}\Big\{g^j[s,\hat{x}^{j(K)}(s)]-c_j^K[\psi_j^{(\tau)K*}(s,\hat{x}^{K(K)}(s))]\Big\}\exp\Big[-\int_\tau^S r(y)dy\Big]ds\\
&\qquad+\sum_{j\in K}\exp\Big[-\int_\tau^T r(y)dy\Big]q^j(\hat{x}^{j(K)}(T))\\
&\geq\int_\tau^T \sum_{j\in L}\Big\{g^j[s,\hat{x}^{j(L)}(s)]-c_j^K[\psi_j^{(\tau)L*}(s,\hat{x}^{L(L)}(s))]\Big\}\exp\Big[-\int_\tau^S r(y)dy\Big]ds\\
&\qquad+\sum_{j\in L}\exp\Big[-\int_\tau^T r(y)dy\Big]q^j(\hat{x}^{j(L)}(T))\\
&+\int_\tau^T \sum_{j\in K\backslash L}\Big\{g^j[s,\hat{x}^{j(K\backslash L)}(s)]-c_j^K[\psi_j^{(\tau)K\backslash L*}(s,\hat{x}^{K\backslash L(K\backslash L)}(s))]\Big\}\\
&\times\exp\Big[-\int_\tau^S r(y)dy\Big]ds+\sum_{j\in K\backslash L}\exp\Big[-\int_\tau^T r(y)dy\Big]q^j(\hat{x}^{j(K\backslash L)}(T)). \qquad (A.3)
\end{aligned}$$

Invoking (A.1), (A.2) and (A.3), we can readily obtain

$$W^{(\tau)K}(\tau,x_\tau^K)\geq W^{(\tau)L}(\tau,x_\tau^L)+W^{(\tau)K\backslash L}(\tau,x_\tau^{K\backslash L}).$$

Hence Proposition 2.1 follows.

REFERENCES

1. Bleeke J., Ernst D. *Collaborating to compete.* New York: John Wiley & Sons. 1993.

2. Bellman R. *Dynamic programming.* Princeton, Princeton University Press. 1957.

3. Blodgett L.L. *Factors in the instability of international joint ventures: An event history analysis* // Strategic Management Journal. 1992. V. 13. P. 475–481.

4. Cellini R., Lambertini L. *A differential game approach to investment product differentiation* // J. of Economic Dynamics and Control. 2002. V. 27. P. 51–62.

5. Cellini R., Lambertini L. *Private and social incentives towards investment in product differentiation* // Int. Game Theory Review. 2004. V. 6. N. 4. P. 493–508.

6. D'Aspremont C., Jacquemin A. *Cooperative and noncooperative R&D in duopoly with spillovers* // The American Economic Review. 1988. V. 78. N. 5. P. 1133–1137.

7. Kamien M.I., Muller E., and Zang I. *Research joint ventures and R&D cartels* // American Economic Review. 1992. V. 82. N 5. P. 1293–1306.

8. Parkhe A. *"Messy" research, methodological predispositions and theory development in international joint ventures* // Academy of Management Review. 1993. V. 18. N. 2. P. 227–268.

9. Petrosyan L. A., Zaccour G. *Time-consistent Shapley value allocation of pollution cost reduction* // Journal of Economic Dynamics and Control. 2003. V. 27. N 3. P. 381–398.

10. Shapley L.S. *A value for n-person games*. In: Kuhn, H. W. and Tucker, A. W. (Eds.), Contributions to the theory of games. 1953. Princeton: Princeton University Press. P. 307–317.

11. Suzumura K. *Cooperative and noncooperative R&D in an oligopoly with spillovers* // The American Economic Review. 1992. V. 82. N 5. P. 1307–1320.

12. Yeung D.W.K., Petrosyan L.A. *Subgame consistent cooperative solutions in stochastic differential games* // Journal of Optimization Theory and Applications. 2004. V. 120. N 3. P. 651–666.

13. Yeung D.W.K., Petrosyan L.A. *Cooperative stochastic differential games*. New York: Springer-Verlag. 2006.

14. Yeung D.W.K., Petrosyan L.A. *Dynamically stable corporate joint ventures* // Automatica. 2006. V. 42. P. 365–370.

15. Yeung D.W.K., Petrosyan L.A. *A cooperative stochastic differential game of transboundary Industrial Pollution* // Automatica. 2008. V. 44. P. 1532–1544.